International Financial Operations
Arbitrage, Hedging, Speculation, Financing and Investment

IMAD A. MOOSA
To Nisreen and Danny
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## Currency Symbols

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<td>AUD</td>
<td>Australian dollar</td>
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<tr>
<td>CAD</td>
<td>Canadian dollar</td>
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<tr>
<td>CHF</td>
<td>Swiss franc</td>
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<tr>
<td>DEM</td>
<td>German mark (replaced by the euro)</td>
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<tr>
<td>DKK</td>
<td>Danish kroner</td>
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<tr>
<td>GBP</td>
<td>UK pound</td>
</tr>
<tr>
<td>JPY</td>
<td>Japanese yen</td>
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<tr>
<td>KWD</td>
<td>Kuwaiti dinar</td>
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<tr>
<td>NOK</td>
<td>Norwegian krone</td>
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<tr>
<td>NZD</td>
<td>New Zealand dollar</td>
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<tr>
<td>SEK</td>
<td>Swedish krona</td>
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<tr>
<td>USD</td>
<td>US dollar</td>
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List of Abbreviations

ADR  American Depository Receipt
AGS  Ashley–Granger–Schmalensee (test)
APV  Adjusted Present Value
ARCH Autoregressive Conditional Heteroscedasticity
ARIMA Autoregressive Integrated Moving Average
BFI  Baltic Freight Index
BIFFEX Baltic International Freight Futures Exchange
CAPM Capital Asset Pricing Model
CD  Certificate of Deposit
CIP  Covered Interest Parity
CP  Commercial Paper
DSV  Downside Semi-Variance
ECU  European Currency Unit
EMS  European Monetary System
ERM  Exchange Rate Mechanism
ETL  Expected Tail Loss
EU  European Union
EWMA Exponentially Weighted Moving Average
FDI  Foreign Direct Investment
FRN  Floating Rate Note
FX  Foreign Exchange
GAPM Global Asset Pricing Model
GARCH Generalised Autoregressive Conditional Heteroscedasticity
GDP  Gross Domestic Product
IMF  International Monetary Fund
IRR  Internal Rate of Return
LDC  Less Developed Country
LIBOR London Interbank Offer Rate
LOP  Law of One Price
M&As Mergers and Acquisitions
MA  Moving Average
MAD  Mean Absolute Deviation
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<tbody>
<tr>
<td>MSE</td>
<td>Mean Square Error</td>
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<tr>
<td>NAFTA</td>
<td>North American Free Trade Area</td>
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<td>NBFI</td>
<td>Non-Bank Financial Intermediary</td>
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<td>NIF</td>
<td>Note Issuance Facility</td>
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<td>NPV</td>
<td>Net Present Value</td>
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<tr>
<td>OECD</td>
<td>Organisation for Economic Co-operation and Development</td>
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<td>OLS</td>
<td>Ordinary Least Squares</td>
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<tr>
<td>OPEC</td>
<td>Organization of the Petroleum Exporting Countries</td>
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<tr>
<td>PI</td>
<td>Profitability Index</td>
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<tr>
<td>PPP</td>
<td>Purchasing Power Parity</td>
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<td>R&amp;D</td>
<td>Research and Development</td>
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<td>RIP</td>
<td>Real Interest Parity</td>
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<td>SDR</td>
<td>Special Drawing Rights</td>
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<td>SEC</td>
<td>Securities and Exchange Commission</td>
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<td>SIBOR</td>
<td>Singapore Interbank Offer Rate</td>
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<td>TRIP</td>
<td>Trade-Related Investment Performance</td>
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<td>TVP</td>
<td>Time-Varying Parameter</td>
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<td>UIP</td>
<td>Uncovered Interest Parity</td>
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<tr>
<td>UNCTAD</td>
<td>United Nations Conference on Trade and Development</td>
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<td>VAR</td>
<td>Value at Risk</td>
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Notation and Conventions

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<td>$S(x/y)$</td>
<td>The spot exchange rate between currencies $x$ and $y$ measured as the $x$ price of one unit of $y$, where $x$ is the base currency. $(x/y)$ will be dropped for convenience if it is known or specified in advance how the exchange rate is measured. If more than one exchange rate is used then it is $S(x/y)$ and $S(x/z)$.</td>
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<td>$S_t$</td>
<td>The spot exchange rate at time $t$, which is taken to be the present time (the time at which a decision is taken). Thus $S_{t+1}$ is the spot exchange rate at time $t+1$, which is the time at which the results of a decision taken at time $t$ materialise. If the underlying decision involves more than two points in time, then we have $S_t, S_{t+1}, S_{t+2}, \ldots, S_{t+n}$.</td>
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<td>$S_{t+1,i}$</td>
<td>Under various scenarios/probability distributions, the spot exchange rate at $t + 1$ may assume several values such that $i = 1, 2, \ldots, n$.</td>
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<tr>
<td>$S_b, S_a$</td>
<td>The bid and offer exchange rates respectively.</td>
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<td>$S_{a,t+1}(x/y)$</td>
<td>The offer exchange rate at time $t + 1$ measured as the price of one unit of $y$.</td>
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<td>$\dot{S}(x/y)$</td>
<td>The percentage change in the exchange rate measured as the price of one unit of $y$. $(x/y)$ may be dropped for convenience. The percentage change is measured between $t$ and $t + 1$, unless a time subscript is added to indicate otherwise. $\dot{S}_{t+1}$ means the same thing.</td>
</tr>
<tr>
<td>$E_t(S_{t+1})$</td>
<td>The expected value of the exchange rate to prevail at time $t + 1$. The expectation is made at time $t$ on the basis of the information available then. Hence, $E_{t-1}(S_t)$ is the expected value of the exchange rate to prevail at $t$, when the expectation is made at $t – 1$. The time subscript on $E$ may be deleted for</td>
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convenience, in which case it would be assumed to be \( t \). We may also have \( E[S_{t+1}(x/y)] \), and so on.

\[ F(x/y) \]

The forward exchange rate expressed as the price of one unit of \( y \). It is normally assumed that the forward contract is initiated at time \( t \) for delivery at time \( t + 1 \), in which case \( F_t \) or \( F \) are used. If forward contracts with various maturities are used, then the symbol \( F^{t+j}_{t+i} \) is used to indicate the forward rate applicable to a contract initiated at \( t + i \) for delivery at \( t + j \), such that \( j > i \).

\[ f(x/y) \]

The forward spread corresponding to \( F \). It is invariably the case that \( (x/y) \) is dropped unless there is more than one forward rate. We may also add a time subscript to get \( f_t \).

\[ i_{x,a} \]

The offer interest rate on currency \( x \). The time subscript is normally omitted, but it is invariably \( t \). When no allowance is made for the bid–offer spread, the subscript \( a \) is deleted.

\[ i_{y,b} \]

The bid interest rate on currency \( y \).
Multinational firms and (financial and non-financial) firms that indulge in cross-border transactions in general take part in a variety of international financial operations, including arbitrage, hedging, speculation, financing and investment. These operations, which involve a currency factor, are invariably interrelated in the sense that each operation has implications for some of the others. For example, the unavailability of covered arbitrage opportunities has at least two implications for other operations. First, there is no difference between money market hedging and forward hedging of transaction exposure to foreign exchange risk. Second, investing or financing in foreign currencies while simultaneously covering the exposure in the forward market produces a similar outcome to that of investing or financing in the base currency. Despite the connections, these activities are normally dealt with in a fragmented and/or superficial manner and invariably without taking into account the complexities of the real world (bid–offer spreads, transaction costs, capital rationing, market imperfections etc). The objective of this book is to present a comprehensive, concise and integrated treatment of these operations while taking into account practical realities. Hence the book provides some practical extensions to the conventional textbook operations.

This book is written for Palgrave Macmillan’s Finance and Capital Markets series, so the target readership is mainly professionals with formal training in international finance. However, it can also be useful for university libraries as a research reference for those working on topics related to International Business and Finance. This does not preclude the possibility of the book being used for teaching. It should be useful for teaching postgraduate and MBA courses as well as professional courses.

This book has come to the realm of existence as the product of the experience I gained by working as a professional economist and an investment banker for over ten years, coupled with teaching and research in international finance for another ten years. This is why this book is a blend of theoretical principles and practical applications. Some versions of the operations described in this book, which I practised during my time as an investment banker,
banker, cannot be found in any existing book (for example, arbitrage and speculation when the base currency is pegged to a basket).

The book falls into 12 chapters. The first three chapters examine arbitrage, starting with two-currency, three-currency and multi-currency arbitrage. In Chapter 2 we deal with covered and uncovered interest arbitrage by allowing for some real-life complications. In Chapter 3 we deal with other kinds of arbitrage, showing how the risk associated with uncovered arbitrage can be eliminated or minimised in special situations. We also put forward the unconventional concept of real interest arbitrage. In Chapters 4–6 we deal with the management of exposure to foreign exchange risk, explaining why and how firms hedge this exposure and demonstrating how to calculate the optimal hedge ratio. This is followed by the study of speculation in Chapters 7 and 8, where we show how speculators speculate and put forward the proposition that exchange rate volatility results from the heterogeneity of traders with respect to their trading strategies. The remaining four chapters deal with short-term and long-term financing and investment operations, placing some emphasis on the choice of currency denomination for assets and liabilities, and relating operating exposure to equity exposure. The last chapter deals with foreign direct investment and international capital budgeting.

Writing this book would not have been possible if it was not for the help and encouragement I received from family, friends and colleagues. My utmost gratitude must go to my wife and children who had to bear the opportunity cost of writing this book. My wife Afaf not only bore most of the opportunity cost of writing the book, but proved once again to be my best research assistant by producing the elegant diagrams shown in the book. My colleagues at the Department of Economics and Finance, La Trobe University, have been supportive, directly or indirectly, by providing the intellectual and social environment that is conducive, among other things, to writing a book. Out of my colleagues I must particularly thank Xiangkang Yin, who diligently checked my mathematical derivations, and Buly Cardak, whom I always resort to when I hit a wall. Buly’s strong intuition and logical thinking always come to my rescue. Lee Smith deserves a special mention here because, as always, she was so helpful with the bibliography. Samantha Booth provided much needed secretarial assistance and efficiently read the whole manuscript, coming up with suggestions for stylistic alterations. I wish Sam the best of luck as she embarks on her journey back to the UK at the conclusion of this project. Liam Lenten provided his expertise in extracting data and offered some helpful advice whenever I got stuck trying to do something on Excel (which happens quite often). My students who served as guinea pigs for this project must be mentioned here, including Sean Patterson, Leigh Cassidy and Kelly Burns. A former student of mine, Hasan Tevfik, was helpful in providing some of the data used in this study.

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Naturally, I am the only one responsible for any errors and omissions in this book. It is dedicated to my beloved children, Nisreen and Danny, who are more interested in eating Big Mac meals and watching The Simpsons than worrying about the outcome of international financial operations.

Imad A. Moosa
Melbourne, October 2002
1.1 DEFINITION OF ARBITRAGE

Arbitrage is generally defined as capitalising on a discrepancy in quoted prices, triggered by the violation of an equilibrium (pricing) condition. It is often the case that arbitrage is portrayed to be a riskless operation, in the sense that all of the decision variables are known when the decision is made, but the process invariably involves risk, such as the risk of non-delivery (Herstatt risk). The arbitrage process restores equilibrium via changes in the supply of and demand for the underlying commodity, asset or currency. These changes in supply and demand cause price changes in such a way as to restore the equilibrium no-arbitrage condition. At this point the arbitrage process comes to an end, as the operation becomes unprofitable.

In the special case of the foreign exchange market, arbitrage is defined as the simultaneous purchase and sale of currencies for the sake of making profit. Profitable arbitrage opportunities arise in the spot or forward foreign exchange market either because exchange rates differ from one financial centre to another or because they are inconsistent, violating an equilibrium pricing condition in both cases. It must be mentioned at the outset that in today’s integrated financial markets, arbitrage opportunities of this kind rarely, if at all, arise. And even if they arose, they would be quickly exploited by arbitragers to the point of “extinction”. It is, however, still important to study these operations because they provide the mechanisms whereby the equilibrium conditions are maintained. In fact, the no-arbitrage condition is typically taken to define the equilibrium price of the underlying asset(s), and hence the study of arbitrage boils down to the study of price determination in financial markets, which is a crucial element of financial economics. At a later stage, we shall challenge some of the misconceptions about arbitrage that arise from its conventional definition as stated earlier.
1.2 TWO-CURRENCY ARBITRAGE

Also known as spatial, locational or two-point arbitrage, two-currency arbitrage opportunities arise when the exchange rate between two currencies is not the same in two financial centres. Although two-currency arbitrage can be conducted in both the spot and forward markets, the discussion will be limited to the spot market.

Let us assume that there are two financial centres, A and B, and two currencies, \( x \) and \( y \), and that (for the time being) there are no transaction costs (for example, brokerage fees), no taxes and a zero bid–offer spread. If \( S(x/y) \) is the spot exchange rate between \( x \) and \( y \) measured as the price (in terms of \( x \)) of one unit of \( y \), then two-currency arbitrage will be triggered if

\[
S_A(x/y) \neq S_B(x/y)
\]  

which means that the exchange rate between the two currencies has two different values in two financial centres at the same point in time. To simplify the notation, we will for the rest of this section drop the units of measurement, \((x/y)\), from the exchange rate symbol. It is essential, however, to bear in mind that the exchange rate is measured as the price of one unit of \( y \), \( S(x/y) \), and not the other way round.

If the arbitrage condition represented by (1.1) is violated, then one possibility is that

\[
S_A > S_B
\]  

which means that currency \( y \) is cheaper in B than in A (or that currency \( x \) is cheaper in A). Two-currency arbitrage, in this case, would take the form of buying \( y \) where it is cheap (in B) and selling it where it is expensive (in A). The profit realised from this operation, \( \pi \), is the difference between the selling and buying rates, or

\[
\pi = S_A - S_B
\]  

Figure 1.1 shows how arbitrage affects the forces of supply and demand, and hence the exchange rates in both centres. As the demand for \( y \) rises in B, \( S_B \) rises, and as the supply of \( y \) rises in A, \( S_A \) falls, reducing arbitrage profit. This operation continues until profit declines to zero (\( \pi = 0 \)). Thus the no-arbitrage condition, which is obtained when arbitrage profit falls to zero, is given by

\[
S_A = S_B
\]  

Hence, any violation of the condition represented by (1.4) triggers (profitable) arbitrage. The no-arbitrage condition is represented by the points falling on the no-arbitrage line, which is a 45° line passing through the origin (Figure 1.2). Points falling off the line represent violation of the no-arbitrage condition. Those falling above the line indicate a violation of the no-arbitrage condition as in (1.2). Arbitrage causes a movement towards the line either by an
increase in $S_B$, a decrease in $S_A$, or (more likely) both. Points below the line indicate a violation of the form $S_A < S_B$. In the following subsections some real-life complications are introduced.
Two-currency arbitrage with brokerage fees

Let us now assume that buyers and sellers have to pay brokerage fees on the transactions involving the buying and selling of currencies. Assume initially that brokerage fees are fixed and independent of the size of the transactions. Suppose now that the arbitrager wants to make profit by buying \( y \) in B and selling it in A. In the presence of fixed brokerage fees, the profit realised from this operation is

\[
\pi = S_A - S_B - (\beta_A + \beta_B) \tag{1.5}
\]

where \( \beta_A \) and \( \beta_B \) are the brokerage fees in financial centres A and B respectively. For the arbitrage operation to be profitable in this case the following condition must be satisfied

\[
S_A - S_B > (\beta_A + \beta_B) \tag{1.6}
\]

which means that the difference between the selling and buying rates must be greater than the sum of the brokerage fees incurred in the buying and selling transactions. Hence the no-arbitrage condition in the presence of fixed brokerage fees is given by

\[
S_A = S_B + (\beta_A + \beta_B) \tag{1.7}
\]

Likewise, if the arbitrager is to make profit by buying \( y \) in A and selling it in B then the following condition must be satisfied

\[
S_B - S_A > (\beta_A + \beta_B) \tag{1.8}
\]

and the no-arbitrage condition becomes

\[
S_A = S_B - (\beta_A + \beta_B) \tag{1.9}
\]

Figure 1.3 shows the effect of two-currency arbitrage in the presence of fixed brokerage fees when the arbitrager buys \( y \) in B and sells it in A. Demand increases in B and supply increases in A, leading to a rise in the exchange rate in B and a fall in A. In this case, however, arbitrage does not come to an end when the exchange rates are equal in the two financial centres, but when the difference between them is equal to the sum of brokerage fees incurred in both financial centres, \( (\beta_A + \beta_B) \). Figure 1.4 shows what happens to the no-arbitrage line when there are fixed brokerage fees. A band, \( 2(\beta_A + \beta_B) \) wide, will be created around the original no-arbitrage line. The upper limit of the band is defined by equation (1.7), whereas the lower band is defined by equation (1.9). Points within the band but off the original no-arbitrage line indicate that while the exchange rates are not equal across financial centres, arbitrage is not profitable because arbitrage profit will be consumed by brokerage fees. Points falling outside the band define profitable arbitrage operations. Above the upper limit, arbitrage is profitable by buying \( y \) in B and selling it in A. Below the lower limit, arbitrage is profitable by buying \( y \) in A and selling it in B.
Assume now that brokerage fees depend on the size of the transactions, such that they are charged at the rates of $\beta_A$ and $\beta_B$ in financial centres A and B respectively. If $S_A > S_B$, then arbitragers will buy $y$ in B and sell it in A. In this case the profit realised from arbitrage is
\[ \pi = S_A (1 - \beta_A) - S_B (1 + \beta_B) \]  
(1.10)

which means that arbitrage will be profitable \((\pi > 0)\) if

\[ S_A > S_B \left[ \frac{1 + \beta_B}{1 - \beta_A} \right] \]  
(1.11)

Alternatively, if \(S_A < S_B\), then arbitragers will buy \(y\) in \(A\) and sell it in \(B\). In this case the profit realised from arbitrage is

\[ \pi = S_B (1 - \beta_B) - S_A (1 + \beta_A) \]  
(1.12)

which means that arbitrage will be profitable if

\[ S_A < S_B \left[ \frac{1 - \beta_B}{1 + \beta_A} \right] \]  
(1.13)

Hence the no-arbitrage lines associated with (1.11) and (1.13) respectively are

\[ S_A = S_B \left[ \frac{1 + \beta_B}{1 - \beta_A} \right] \]  
(1.14)

and

\[ S_A = S_B \left[ \frac{1 - \beta_B}{1 + \beta_A} \right] \]  
(1.15)

Figure 1.5 shows what happens to the no-arbitrage line in this case. Notice that since \(0 < \beta_A < 1\) and \(0 < \beta_B < 1\), it follows that \((1 + \beta_B)/(1 - \beta_A) > 1\), while \((1 - \beta_B)/(1 + \beta_A) < 1\). Diagrammatically, equations (1.14) and (1.15) are represented in Figure 1.5 by two lines intersecting with the original no-arbitrage line at the origin, with one being steeper (1.14) and the other flatter (1.15). Points within the triangular area define unprofitable arbitrage, as any profit realised from the difference in the exchange rates across the financial centres will be consumed by brokerage fees. Any point above or below the triangular area indicates a profitable arbitrage opportunity.

**Two-currency arbitrage in the presence of taxes**

Here we consider two kinds of tax: capital gains tax and Tobin tax. We start with the former. Suppose that capital gains tax is imposed on the profits realised from two-currency arbitrage in the financial centre where the profit is realised. It is easy to show that the presence of capital gains tax has no effect on the no-arbitrage line, because the only effect of the tax is to reduce the profit received by the arbitrager. As long as there is a discrepancy between the exchange rates, arbitrage will be profitable, though less so than in the absence of the tax. Arbitrage will not come to an end unless the discrepancy between the exchange rates disappears.
1.2 Two-currency Arbitrage

If the rate of capital gains tax is \( \tau \), then the after-tax arbitrage profit obtained by buying \( y \) in B and selling it in A is

\[
\pi = (1-\tau)[S_A - S_B]
\]

in which case the no-arbitrage line is given by

\[
(1-\tau)[S_A - S_B] = 0
\]

which is equivalent to (1.4). Notice that \( \frac{\partial \pi}{\partial \tau} < 0 \).

Tobin tax was suggested by a Noble laureate, James Tobin, as a measure that would reduce the volatility in the foreign exchange market. It is imposed as a percentage of the value of the transaction, and hence it has the same effect as imposing brokerage fees on the buying and selling operations.

**Two-currency arbitrage with capital controls**

What happens if capital controls are imposed in one financial centre. If the transfer of capital is not allowed for financial transactions then arbitrage is not possible, and the divergence between the exchange rates in the two financial centres will persist. Nothing will happen to shift the supply and demand curves as in Figure 1.1. However, if capital controls are partial, the amount of capital allowed to be transferred from one financial centre to another may be inadequate to shift the supply and demand curves to the extent necessary to eliminate the discrepancy between the exchange rates. In this case, the
exchange rates in the two financial centres will approach each other, but they will not be equal. This situation is explained in Figure 1.6.

**Two-currency arbitrage in the presence of the bid–offer spread**

Let $S_{b,A}$ and $S_{b,B}$ be the bid rates, and $S_{a,A}$ and $S_{a,B}$ the offer rates in financial centres A and B respectively (still measured as $S(x/y)$). In the presence of the bid–offer spread, arbitragers buy (from market makers) at the higher offer rate and sell (to market makers) at the lower bid rate (note that $S_b < S_a$). Thus, the bid rate is determined by the demand of market makers and the supply of arbitragers. Conversely, the offer rate is determined by the demand of arbitragers and the supply of market makers. The bid and offer rates are related by the equation

$$S_a = S_b(1 + m) \tag{1.18}$$

where $m$ is the bid–offer spread expressed as a percentage of the bid rate. For simplicity, we will assume that the bid–offer spread in financial centre A is equal to that prevailing in financial centre B.

Suppose now that the arbitrager wants to buy $y$ in B and sell it in A. In this case arbitrage will be profitable if

$$\pi = S_{b,A} - S_{a,B} > 0 \tag{1.19}$$

which means that the no-arbitrage condition is given by

\[ S(x/y) \]

**Figure 1.6** The effect of two-currency arbitrage in the presence of partial capital controls.
\[ S_{b,A} = S_{a,B} \quad (1.20) \]

This situation is illustrated in Figure 1.7. Arbitragers buy \( y \) in B at \( S_{a,B} \) and sell it in A at \( S_{b,A} \). The process leads to a shift in the arbitragers’ demand curve in B, causing a rise in \( S_{a,B} \) and to a shift in the arbitragers’ supply curve in A, causing a fall in \( S_{b,A} \). The process continues until the two rates are equal. If only these changes take place, the bid–offer spread must rise in both A and B, and there is no reason why this should happen. In order that the spread stays at the same level, \( S_{b,B} \) must rise and \( S_{a,A} \) must fall. The following line of reasoning explains why this could take place. As \( S_{a,B} \) rises, market makers find it profitable to increase the supply of \( y \). To do this, they must obtain larger quantities of \( y \) by

**FIGURE 1.7** The effect of two-currency arbitrage in the presence of bid–offer spread.
buying it from customers. Thus the market makers’ demand curve shifts, leading to an increase in $S_{b,B}$. Similarly, as $S_{b,A}$ declines, the market maker finds it cheaper to buy $y$, and the supply curve will shift to the left, leading to a fall in $S_{a,A}$.

Let us now consider the no-arbitrage condition in the presence of the bid–offer spread. Figure 1.8 shows a four-quadrant diagram, in which quadrants 1 and 3 show the no-arbitrage condition, whereas quadrants 2 and 4 show the relationship between the bid and offer rates. Notice that the line representing the relationship between the bid and offer rates (passing through the second and fourth quadrants) is less steep than the $45^\circ$ line because the bid rate is always lower than the offer rate. The first quadrant shows the no-arbitrage line represented by equation (1.20). Any point above

**FIGURE 1.8** The no-arbitrage condition in the presence of bid–offer spread.
the line implies profitable arbitrage, with profit given by equation (1.19). In
the third quadrant, points below the no-arbitrage line represent profitable
arbitrage opportunities taking the form of buying \( y \) in A and selling it in B. In
this case the profit is

\[ \pi = S_{b,B} - S_{a,A} \]  

(1.21)

The effect of the bid–offer spread is to reduce the profitability of arbitrage,
since the spread is a transaction cost. Recall that equation (1.3) defines the arbi-
trage profit as the difference between the exchange rate in A (the sell rate) and
the exchange rate in B (the buy rate). These rates were not defined as bid or
offer rates, so let us assume that they are the mid-rates, which means

\[ S_A = \frac{1}{2} [S_{b,A} + S_{a,A}] \]  

(1.22)

\[ S_B = \frac{1}{2} [S_{b,B} + S_{a,B}] \]  

(1.23)

Arbitrage profit in the absence and presence of the bid–offer spread is given
by equations (1.3) and (1.19) respectively. Since by definition

\[ S_{b,A} < S_A \]  

(1.24)

and

\[ S_{a,B} > S_B \]  

(1.25)

it follows that

\[ S_{b,A} - S_{a,B} < S_A - S_B \]  

(1.26)

which means that the presence of the bid–offer spread reduces the profit-
ability of arbitrage, because the arbitrager has to buy at a higher rate and sell at
a lower rate than otherwise.

**Putting things together**

Let us now consider the profitability of two-currency arbitrage in the presence
of (i) bid–offer spread, (ii) fixed brokerage fees and (iii) Tobin tax. Consider the
situation when the arbitrager buys \( y \) in B and sells it in A (equations 1.19 and
1.20). In the presence of fixed brokerage fees, \( \beta_A \) and \( \beta_B \), and a Tobin tax, \( \tau \),
which is assumed to be equal in both financial centres, arbitrage profit is
reduced to

\[ \pi = S_{b,A} (1-\tau) - \beta_A - S_{a,B}(1+\tau) - \beta_B \]

\[ = [S_{b,A} - S_{a,B}](1-\tau) - (\beta_A + \beta_B) \]  

(1.27)

which means that profit is reduced further (that is, on top of the reduction
resulting from the bid–offer spread). Equation (1.27) implies a no-arbitrage
condition that is expressed as
\[ S_{b,A} - S_{a,B} = \frac{\beta_A + \beta_B}{1-\tau} \] 

(1.28)

which means that, for profitable two-currency arbitrage, the gap between the bid rate in A and the offer rate in B must be greater than \((\beta_A + \beta_B) / (1-\tau)\).

### 1.3 THREE-CURRENCY ARBITRAGE

Three-currency arbitrage, also known as triangular arbitrage and three-point arbitrage, works as follows. Given three currencies \(x, y\) and \(z\), three possible exchange rates exist: \(S(x/y)\), \(S(x/z)\) and \(S(y/z)\). Since we are in this case dealing with three exchange rates, we will resort to the original exchange rate notation, which shows the units of measurement, as above. We say that the three exchange rates are consistent if

\[ \left( \frac{1}{S(x/y)} \right) \left( \frac{1}{S(x/z)} \right) = \frac{1}{S(y/z)} \] 

(1.29)

Now, let us see what happens if an arbitrager tries to make profit by moving from one currency to another, ending up with the first currency. If the arbitrager ends up with one unit of the currency he or she started with, then arbitrage profit will be made. In general, if the condition (1.29) is violated then arbitrage profit can be made by moving in a particular direction and a loss will be made by moving in the opposite direction.

So, let us start with one unit of currency \(x\), conducting arbitrage in the following manner:

1. Selling \(x\) and buying \(y\) to obtain \(1/[S(x/y)]\) units of \(y\).
2. Selling \(y\) and buying \(z\) to obtain \(1/[S(x/y)S(y/z)]\) units of \(z\).
3. Selling \(z\) and buying \(x\) to obtain \(S(x/z)/[S(x/y)S(y/z)]\) units of \(x\).

The profit realised from this operation (measured in units of \(x\)) is given by

\[ \pi = \frac{S(x/z)}{S(x/y)S(y/z)} - 1 \] 

(1.30)

If the condition represented by (1.29) is valid, it follows that \(\pi = 0\), which means that (1.29) is the no-arbitrage condition. However, if

\[ S(x/y) < \frac{S(x/z)}{S(y/z)} \] 

(1.31)

it follows that

\[ S(x/z) > S(x/y)S(y/z) \] 

(1.32)
which means that \( \pi > 0 \). Hence, if the no-arbitrage condition (1.29) is violated, such that (1.31) is valid, then three-currency arbitrage will be profitable by the following sequence: \( x \to y \to z \to x \).

Now, let us see what happens if the arbitrager follows the sequence \( x \to y \to z \to x \), starting with one unit of \( x \). This operation consists of the following steps:

1. Selling \( x \) and buying \( z \) to obtain \( 1/\left[S(x/z)\right] \) units of \( z \).
2. Selling \( z \) and buying \( y \) to obtain \( S(y/z)/\left[S(x/z)\right] \) units of \( y \).
3. Selling \( y \) and buying \( x \) to obtain \( S(y/z)S(x/y)/\left[S(x/z)\right] \) units of \( x \).

The profit realised from this operation is given by

\[
\pi = \frac{S(y/z)S(x/y)}{S(x/z)} - 1 \quad \text{(1.33)}
\]

Again, it is obvious that if (1.29) is valid then \( \pi = 0 \). In this case, profitable arbitrage is indicated by the violation of (1.29) such that

\[
S(x/y) > \frac{S(x/z)}{S(y/z)} \quad \text{(1.34)}
\]

because (1.34) implies that

\[
S(y/z)S(x/y) > S(x/z) \quad \text{(1.35)}
\]

which means that \( \pi > 0 \).

Just like two-currency arbitrage, three-currency arbitrage leads to a restoration of the no-arbitrage condition via changes in the supply of and demand for the three currencies. Let us trace what happens in the first case, as represented by (1.31). With the aid of Figure 1.9, we can see that each of the three steps results in changes in the forces of supply and demand as follows:

1. An increase in the demand for \( y \) (the supply of \( x \)), so \( S(x/y) \) rises.
2. An increase in the demand for \( z \) (the supply of \( y \)), so \( S(y/z) \) rises.
3. An increase in the demand for \( x \) (the supply of \( z \)), so \( S(x/z) \) falls.

These changes in supply and demand will restore the equilibrium condition.

**Three-currency arbitrage in the presence of bid–offer spreads**

Let us see what happens if an arbitrager wants to follow the sequence \( x \to z \to y \to x \) in the presence of bid–offer spreads. The operation consists of the following steps:

1. Buying \( z \) against \( x \) at \( S_a(x/z) \) to obtain \( 1/[S_a(x/z)] \) units of \( z \).
2. Buying \( y \) against \( z \) to obtain \( S_b(y/z)/[S_a(x/z)] \) units of \( y \).
3. Buying \( x \) against \( y \) at \( S_a(x/y) \) to obtain \( S_b(y/z)S_a(x/y)/[S_a(x/z)] \) units of \( x \).

For this operation to be profitable, the following condition must be satisfied:
FIGURE 1.9 The effect of three-currency arbitrage.

\[ \frac{S_b(y/z)S_a(x/y)}{S_a(x/z)} > 1 \]  \hspace{1cm} (1.36)

which gives
\[
\pi = \frac{S_b(y/z)S_a(x/y)}{S_a(x/z)} - 1
\]

(1.37)

in which case, the no-arbitrage condition is

\[
S_a(x/y) = \frac{S_a(x/z)}{S_b(y/z)}
\]

(1.38)

Likewise, it can be shown that the sequence \( x \rightarrow y \rightarrow z \rightarrow x \) can be profitable if

\[
\frac{S_b(x/z)}{S_b(x/y)S_a(y/z)} > 1
\]

(1.39)

which gives

\[
\pi = \frac{S_b(x/z)}{S_b(x/y)S_a(y/z)} - 1
\]

(1.40)

in which case, the no-arbitrage condition is

\[
S_b(x/y) = \frac{S_b(x/z)}{S_a(y/z)}
\]

(1.41)

Equations (1.38) and (1.41) are used to calculate the bid and offer cross exchange rates when currency \( z \) is the numeraire.

1.4 MULTI-CURRENCY ARBITRAGE

Consider arbitrage involving four currencies: \( x_1, x_2, x_3 \) and \( x_4 \) by following the sequence \( x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow x_4 \rightarrow x_1 \). Arbitrage consists of the following steps:

1. Buying \( x_2 \) and selling \( x_1 \) at \( S(x_1/x_2) \) to obtain \( 1/[S(x_1/x_2)] \) units of \( x_2 \).
2. Buying \( x_3 \) and selling \( x_2 \) at \( S(x_2/x_3) \) to obtain \( 1/[S(x_1/x_2)S(x_2/x_3)] \) units of \( x_3 \).
3. Buying \( x_4 \) and selling \( x_3 \) at \( S(x_3/x_4) \) to obtain \( 1/[S(x_1/x_2)S(x_2/x_3)S(x_3/x_4)] \) units of \( x_4 \).
4. Buying \( x_1 \) at \( S(x_1/x_4) \) to obtain \( S(x_1/x_4)/[S(x_1/x_2)S(x_2/x_3)S(x_3/x_4)] \) units of \( x_1 \).

This operation will be profitable if

\[
\pi = \frac{S(x_1/x_4)}{S(x_1/x_2)S(x_2/x_3)S(x_3/x_4)} - 1 > 0
\]

(1.42)

in which case the no-arbitrage condition is

\[
S(x_1/x_4) = S(x_1/x_2)S(x_2/x_3)S(x_3/x_4)
\]

(1.43)

or
In general, an \( n \)-currency arbitrage is profitable if the following no-arbitrage condition is violated:

\[
S(x_1/x_2)S(x_2/x_3)S(x_3/x_4)\ldots S(x_{n-1}/x_n)S(x_n/x_1) = 1 \tag{1.45}
\]

which means that even two-currency arbitrage can be represented as a special case of (1.45). If \( n = 2 \), the no-arbitrage condition reduces to

\[
S(x_1/x_2)S(x_2/x_1) = 1 \tag{1.46}
\]

Chacholiades (1971) has shown that if three-currency arbitrage is not profitable, then \( n \)-currency arbitrage is not profitable either. This means that for equation (1.45) to be satisfied, a necessary and sufficient condition is

\[
S(x_1/x_2)S(x_2/x_3)S(x_3/x_1) = 1 \tag{1.47}
\]

The proof of this proposition is based on mathematical induction. If \((n-1)\)-currency arbitrage is not profitable, then \( n \)-currency arbitrage is not profitable either. For unprofitable \( n \)-currency arbitrage, equation (1.45) must hold. Since \((n-1)\)-currency arbitrage is not profitable by assumption, the following equation must be satisfied

\[
S(x_1/x_2)S(x_2/x_3)S(x_3/x_4)\ldots S(x_{n-2}/x_{n-1})S(x_{n-1}/x_1) = 1 \tag{1.48}
\]

Dividing (1.44) by (1.48) we obtain

\[
\frac{S(x_{n-1}/x_n)S(x_n/x_1)}{S(x_{n-1}/x_1)} = 1 \tag{1.49}
\]

which, for \( n = 3 \), is equivalent to (1.47) because \( S(x_1/x_2) = 1/[S(x_2/x_1)] \). Hence, if (1.47) and (1.48) are satisfied, (1.45) must also be satisfied, which proves the proposition.

**Multi-currency arbitrage with bid–offer spreads**

In the presence of bid–offer spreads, the operation takes the following form:

1. Buying \( x_2 \) and selling \( x_1 \) at \( S_a(x_1/x_2) \) to obtain \( 1/[S_a(x_1/x_2)] \) units of \( x_2 \).
2. Buying \( x_3 \) and selling \( x_2 \) at \( S_a(x_2/x_3) \) to obtain \( 1/[S_a(x_1/x_2)S_a(x_2/x_3)] \) units of \( x_3 \).
3. Buying \( x_4 \) at \( S_a(x_3/x_4) \) to obtain \( 1/[S_a(x_1/x_2)S_a(x_2/x_3)S_a(x_3/x_4)] \) units of \( x_4 \).
4. Buying \( x_1 \) at \( S_b(x_1/x_4) \) to obtain \( S_b(x_1/x_4)/[S_a(x_1/x_2)S_a(x_2/x_3)S_a(x_3/x_4)] \) units of \( x_1 \).

This operation will be profitable if

\[
\pi = \frac{S_b(x_1/x_4)}{S_a(x_1/x_2)S_a(x_2/x_3)S_a(x_3/x_4)} - 1 > 0 \tag{1.50}
\]
in which case the no-arbitrage condition is

\[ S_b(x_1/x_4) = S_a(x_1/x_2)S_a(x_2/x_3)S_a(x_3/x_4) \]  

(1.51)

or

\[ S_a(x_1/x_2)S_a(x_2/x_3)S_a(x_3/x_4)S_a(x_4/x_1) = 1 \]  

(1.52)

because \( S_b(x_1/x_4) = 1/[S_a(x_4/x_1)] \). If the bid–offer spread is the same for all exchange rates, the condition becomes

\[ S_b(x_1/x_2)S_b(x_2/x_3)S_b(x_3/x_4)S_b(x_1/x_4)(1 + m)^4 = 1 \]  

(1.53)

Hence the \( n \)-currency no-arbitrage condition in the presence of bid–offer spread is given by

\[ S_a(x_1/x_2)S_a(x_2/x_3)S_a(x_3/x_4)...S_a(x_n/x_1) = 1 \]  

(1.54)

or

\[ S_b(x_1/x_2)S_b(x_2/x_3)S_b(x_3/x_4)...S_b(x_n/x_1)(1 + m)^n = 1 \]  

(1.55)

### 1.5 EXAMPLES

Table 1.1 reports some bilateral exchange rates as on 16 December 2001. We can use these figures to check whether or not the no-arbitrage conditions associated with three-currency, four-currency and five-currency arbitrage are valid.

Table 1.2 lists possible sequences for three-currency, four-currency and five-currency arbitrage, the associated conditions and whether or not the conditions are satisfied. The numbers appearing in the third column are the products of the exchange rates as implied by the general no-arbitrage condition (1.45). The no-arbitrage condition will be satisfied if the product is 1, indicating zero profit. This is because unity signifies that the no-arbitrage condition implies that when the arbitrager starts with one unit of a particular

**Table 1.1** Exchange rates on 16 December 2001.

<table>
<thead>
<tr>
<th>( x/y )</th>
<th>USD</th>
<th>SEK</th>
<th>DKK</th>
<th>NZD</th>
<th>EUR</th>
<th>AUD</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SEK</td>
<td>10.54</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DKK</td>
<td>8.2449</td>
<td>0.7819</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NZD</td>
<td>2.3941</td>
<td>0.2271</td>
<td>0.2904</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EUR</td>
<td>1.1073</td>
<td>0.1050</td>
<td>0.1343</td>
<td>0.4625</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>AUD</td>
<td>1.9296</td>
<td>0.1830</td>
<td>0.2340</td>
<td>0.8060</td>
<td>1.7246</td>
<td>1</td>
</tr>
</tbody>
</table>

Source: Bloomberg.
TABLE 1.2 Examples of $n$-currency arbitrage.

<table>
<thead>
<tr>
<th>Arbitrage</th>
<th>Sequence</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three-currency</td>
<td>SEK → USD → NZD → SEK</td>
<td>0.9998</td>
</tr>
<tr>
<td>Three-currency</td>
<td>AUD → EUR → DKK → AUD</td>
<td>0.9898</td>
</tr>
<tr>
<td>Four-currency</td>
<td>USD → NZD → DKK → SEK → USD</td>
<td>0.9997</td>
</tr>
<tr>
<td>Four-currency</td>
<td>EUR → NZD → SEK → AUD → EUR</td>
<td>0.9898</td>
</tr>
<tr>
<td>Five-currency</td>
<td>SEK → USD → DKK → NZD → EUR → SEK</td>
<td>0.9994</td>
</tr>
<tr>
<td>Five-currency</td>
<td>AUD → EUR → NZD → SKK → DKK → AUD</td>
<td>0.9900</td>
</tr>
</tbody>
</table>

currency, she ends up with one unit of the same currency. The calculation of the figures in the third column can be illustrated by reference to the first arbitrage operation. In this case we have

\[
S(\text{SEK/USD}) \times S(\text{USD/NZD}) \times S(\text{NZD/SEK}) = 10.5400 \times \frac{1}{2.3941} \times 0.2271 \\
= 0.9998
\]

We can see that all of the numbers are close to one, implying the absence of profitable arbitrage operations if we assume that the slight difference between unity and the figures shown in the table is due to rounding. If it is not due to rounding, then there is still no possibility for profitable arbitrage because the difference is so small that it is bound to be consumed by transaction costs.

Let us now assume that there is a 0.1% bid–offer spread in all exchange rates, such that we have the following information:

\[
S(\text{SEK/USD}) = 10.5295 - 10.5505 \\
S(\text{NZD/USD}) = 2.3917 - 2.3965 \\
S(\text{NZD/SEK}) = 0.2269 - 0.2273
\]

In this case, the no-arbitrage condition is checked as follows

\[
S_a(\text{SEK/USD}) \times S_a(\text{USD/NZD}) \times S_a(\text{NZD/SEK}) = 10.5505 \times \frac{1}{2.3965} \times 0.2273 \\
= 1.0008
\]

which is again close to unity, implying the absence of profitable arbitrage.
Covered and Uncovered Interest Arbitrage

2.1 COVERED INTEREST ARBITRAGE WITHOUT DISTORTIONS

Covered interest arbitrage is an operation that is conducted in four markets involving two currencies: (i) the spot foreign exchange market, (ii) the forward foreign exchange market, (iii) the money market in currency \( x \), and (iv) the money market in currency \( y \). The objective is to make profit by going short on one currency and long on the other, while covering the long position in the forward market. When it matures, the long position is unwound and the proceeds are converted into the other currency at the forward rate agreed upon in advance. The proceeds are then used to meet the obligations arising from the short position, and any left over would then represent net arbitrage profit. Notice that the operation is risk-free in the sense that the decision variables are known at the time when the transaction is initiated.

Let \( S \) and \( F \) be the spot and forward exchange rates between currencies \( x \) and \( y \) measured as \( S(x/y) \) and \( F(x/y) \). Also let \( i_x \) and \( i_y \) be the interest rates on \( x \) and \( y \) respectively, such that the maturities of the assets and liabilities underlying \( i_x \) and \( i_y \) (for example, deposits and loans) are identical to the maturity of the forward contract. We will assume a two-period model where \( t \) is the present time at which the operation is initiated and \( t + 1 \) is the future when the long position, short position and the forward contract mature. Whether the arbitrager goes short on \( x \) and long on \( y \) or the other way round depends on the configuration of interest and exchange rates. A covered arbitrage operation by going short on \( x \) and long on \( y \) \((x \rightarrow y)\) consists of the following steps:

1. At time \( t \), the arbitrager borrows one unit of \( x \) at \( i_x \) for a period extending between \( t \) and \( t + 1 \), when the forward contract matures.
2. The amount borrowed is converted at \( S \), obtaining \( 1/S \) units of \( y \). This amount is then invested at \( i_y \).
3. At \( t + 1 \), the value of the investment is \( (1/S)(1 + i_y) \) units of \( y \).
4. The \( x \) currency value of the investment converted at the forward rate is \( (F/S)(1 + i_y) \).

5. At \( t + 1 \) the loan matures, and the amount \( (1 + i_x) \) has to be repaid.

The net profit arising from this operation, which is also called the covered margin, is given by

\[
\pi = \frac{F}{S} (1 + i_y) - (1 + i_x)
\]

Hence the no-arbitrage condition is

\[
\frac{F}{S} (1 + i_y) = (1 + i_x)
\]

The equality of the gross return on \( y \) and the cost of borrowing \( x \) (principal plus interest) after covering the foreign exchange risk by selling \( y \) forward, as represented by (2.2), is called covered interest parity (CIP). This relationship is an application of the law of one price to financial markets (identical financial assets should produce identical returns after covering the foreign exchange risk).

Figure 2.1 shows the no-arbitrage condition represented by equation (2.2). The no-arbitrage line is represented by a 45° line passing through the origin. Any point above the line represents profitable arbitrage by going short on \( x \) and long on \( y \) (\( x \to y \)), whereas points below the line represent profitable arbitrage by going short on \( y \) and long on \( x \) (\( y \to x \)).

![FIGURE 2.1 The no-arbitrage condition implied by CIP.](image-url)
Suppose now that the interest rate and exchange rate configuration is such that the no-arbitrage condition is violated, as represented by a point above the no-arbitrage line. In this case, arbitrage will lead to changes in the forces of supply and demand as illustrated in Figure 2.2. The following will happen:

1. Demand declines in the money market for $x$-denominated assets, leading to a rise in $i_x$.
2. Demand rises in the money market for $y$-denominated assets, leading to a decline in $i_y$.
3. The demand for currency $y$ increases in the spot market, leading to a rise in the spot exchange rate, $S$.
4. The supply of currency $y$ rises in the forward market, leading to a decline in the forward exchange rate, $F$.

These changes combined lead to a decline in the covered return on $y$ and an increase in the cost of borrowing $x$. When they are equal, the covered margin is

---

**FIGURE 2.2** The effect of covered arbitrage.
equal to zero, and the no-arbitrage condition is re-established. Arbitrage comes to an end, as the new configuration of interest and exchange rates is represented by a point falling on the no-arbitrage line.

**Other forms of the no-arbitrage condition**

The no-arbitrage condition can be expressed differently by manipulating equation (2.2). First of all we could rewrite this equation in terms of the net amounts, by subtracting 1 from both sides of the equation, to obtain

$$\frac{F}{S}(1+i_y)-1=i_x$$

(2.3)

Another specification of the no-arbitrage condition can be obtained by deriving the value of the forward rate consistent with CIP, the so-called equilibrium or interest parity forward rate, from equation (2.2). In order to distinguish between the actual forward rate (which prevails whether or not CIP holds) and the equilibrium rate, the latter is denoted $\bar{F}$. Thus the CIP no-arbitrage condition may be written as

$$F = \bar{F}$$

(2.4)

where

$$\bar{F} = S \left[ \frac{1+i_x}{1+i_y} \right]$$

(2.5)

which means that the interest parity forward rate, as represented by equation (2.5), is calculated by adjusting the spot rate for a factor reflecting the interest rate differential. Since

$$\frac{F}{S} = 1 + f$$

(2.6)

where $f$ is the forward spread, it follows that

$$(1 + f)(1 + i_y) = 1 + i_x$$

(2.7)

which can be manipulated to obtain an approximate but useful expression for the no-arbitrage condition by ignoring the (small) term $i_yf$. The approximate expression is

$$i_x - i_y = f$$

(2.8)

which tells us that if the interest differential is equal to the forward spread, then there is no possibility for profitable covered arbitrage. Equation (2.8) implies that the currency offering the higher interest rate must sell at a forward discount and vice versa. This is because if $i_x > i_y$, then $f > 0$, which means that currency $y$ (offering a lower interest rate) sells at a forward premium, whereas currency $x$ (offering a higher interest rate) sells at a
forward discount. If, on the other hand, $i_x < i_y$, then $f < 0$, implying that currency $y$ sells at a discount whereas currency $x$ sells at a premium. That the interest rate differential and the forward spread have similar signs must be a necessary condition for (no-arbitrage) equilibrium, because no investor would want to hold a currency that offers a low interest rate and sells at a discount, whereas everyone would want to hold a currency that offers a high interest rate and sells at a premium. The sufficient condition is that the interest differential and forward spread are equal.

The no-arbitrage condition, as represented by (2.8) can be represented diagrammatically by a 45° line passing through the origin, as shown in Figure 2.3. Any point off the no-arbitrage line represents a profitable arbitrage operation by going short on $x$ and long on $y$ or vice versa, depending on whether the point is above or below the line. Notice that points falling in the second and fourth quadrants represent a more serious violation of the no-arbitrage condition because they imply that the currency with the higher interest rate sells at a premium or vice versa.

**A corollary**

It can be shown that, in the absence of bid–offer spreads, if there is no covered arbitrage opportunity in one direction, then there is no arbitrage opportunity
in the opposite direction. Consider the no-arbitrage condition from $x$ to $y$ (equation 2.2), which may now be written as

$$\frac{F(x/y)}{S(x/y)}(1+i_y) = (1+i_x)$$

(2.9)

Since $F(x/y) = 1/[F(y/x)]$ and $S(x/y) = 1/[S(y/x)]$, it follows that

$$\frac{S(y/x)}{F(y/x)}(1+i_y) = (1+i_x)$$

(2.10)

which can be manipulated to produce

$$\frac{F(y/x)}{S(y/x)}(1+i_x) = (1+i_y)$$

(2.11)

which is the no-arbitrage condition for going from $y$ to $x$.

### 2.2 THE NO-ARBITRAGE CONDITION WITH BID–OFFER SPREADS

Let us now reconsider the no-arbitrage condition when there are bid–offer spreads in exchange and interest rates. Remember that an arbitrager in the foreign exchange market buys at the (higher) offer exchange rate and sells at the (lower) bid exchange rate. In the money market, the arbitrager borrows at the (higher) offer interest rate and lends at the (lower) bid interest rate.

Let us first consider arbitrage from $x$ to $y$ in the presence of bid–offer spreads. The operation consists of the following steps:

1. Borrowing one unit of $x$ at the offer interest rate, $i_{x,a}$.
2. Converting the borrowed funds, buying $y$ at the spot offer rate, $S_a$, obtaining $1/S_a$ units of $y$. This amount is invested at the bid interest rate, $i_{y,b}$.
3. The $y$ value of the invested amount at the end of the investment period is $(1/S_a)(1 + i_{y,b})$.
4. This amount is reconverted into $x$ at the bid forward rate, $F_b$, to obtain $(F_b/S_a)(1 + i_{y,b})$.
5. The value of the loan plus interest is $(1 + i_{x,a})$.

The covered margin is given by

$$\pi = \frac{F_b}{S_a} (1 + i_{y,b}) - (1 + i_{x,a})$$

(2.12)

Since $S_a = S_b(1 + m)$, where $m$ is the bid–offer spread measured as a percentage of the bid rate, it follows that
2.2 The No-Arbitrage Condition with Bid–Offer Spreads

\[ \pi = \frac{F_b}{S_b (1 + m)} (1 + i_{y,b}) - (1 + i_{x,a}) \]  

(2.13)

Assuming that the forward spread is equal on both the bid and offer sides, we have

\[ \frac{F_b}{S_b} = 1 + f \]  

(2.14)

By substituting equation (2.14) into equation (2.13) we obtain

\[ \pi = \frac{(1 + f) (1 + i_{y,b})}{(1 + m)} - (1 + i_{x,a}) \]  

(2.15)

which gives

\[ \pi = \frac{(1 + f) (1 + i_{y,b}) - (1 + m)(1 + i_{x,a})}{(1 + m)} \]  

(2.16)

Since \( 1 + m \approx 1 \), and by ignoring the small cross products, an approximate expression for the covered margin can be written as

\[ \pi = i_{y,b} - i_{x,a} + f - m \]  

(2.17)

It can be demonstrated that the profitability of arbitrage from \( x \) to \( y \) is smaller in the presence of the bid–offer spreads. Assuming that in the absence of bid–offer spreads, arbitragers act on the basis of the mid-rates, \( i_x \) and \( i_y \), in which case the covered margin is

\[ \pi = i_y - i_x + f \]  

(2.18)

Since \( i_{x,b} < i_x < i_{x,a} \) and \( i_{y,b} < i_y < i_{y,a} \), it follows that the covered margin as represented by (2.17) is smaller than the covered margin as represented by (2.18).

Let us now consider arbitrage from \( y \) to \( x \). The operation in this case consists of the following steps:

1. Borrowing one unit of \( y \) at the offer interest rate, \( i_{y,a} \).
2. Converting the borrowed funds, selling \( y \) at the spot offer rate, \( S_b \), obtaining \( S_b \) units of \( x \). This amount is invested at the bid interest rate, \( i_{x,b} \).
3. The \( x \) value of the invested amount at the end of the investment period is \( S_b (1 + i_{x,b}) \).
4. This amount is reconverted at the offer forward rate, \( F_a \), to obtain \( (S_b/F_a) (1 + i_{x,b}) \) units of \( y \).
5. The value of the loan plus interest is \( (1 + i_{y,a}) \).

The covered margin is

\[ \pi = \frac{S_b}{F_a} (1 + i_{x,b}) - (1 + i_{y,a}) \]  

(2.19)
Since $S_b = S_a/(1 + m)$ and $S_a/F_a = 1/(1 + f)$, it follows that

$$\pi = \frac{(1 + i_{x,b})}{(1 + f)(1 + m)} - (1 + i_{y,a})$$

(2.20)

Since $(1 + f)(1 + m) \approx 1$, an approximate expression for arbitrage profit is

$$\pi = i_{x,b} - i_{y,a} - f - m$$

(2.21)

It can be shown that net profit from arbitrage from $y$ to $x$ is smaller than that resulting from the use of mid-rates.

### 2.3 DEVIATIONS FROM THE CIP NO-ARBITRAGE CONDITION

Figure 2.4 shows the behaviour of the covered margin (in percentage points) resulting from covered arbitrage involving the US, British and Canadian currencies. It is obvious that the covered margin can deviate from zero, particularly when arbitrage involves the pound against either the Canadian dollar or the US dollar. These deviations imply the availability of profitable arbitrage operations, which begs the question as to why they are not exploited to the extent that reduces the margins to zero. One answer to this question is that these observed deviations may be due to measurement errors. The data used to calculate the covered margins shown in Figure 2.4 are published data, not the data on which transactions could have been conducted. Moreover, they are taken from various sources, which means that they are probably measured or observed at different points in time (for example, mid-day rather than closing).

Economists have repeatedly tested CIP to find out if there are profitable arbitrage opportunities. Two important studies of the empirical validity of CIP were conducted in the 1980s by Taylor (1987b, 1989). What was different about these studies was that they were not based on published data, which have some measurement errors. There are certain requirements for proper testing of CIP, including the following: (i) observations on exchange and interest rates must be recorded at the same point in time; (ii) they must include the bid–offer spreads and transaction costs; and (iii) they must represent the data on which arbitragers take decisions. Obviously, these three requirements are not satisfied by published data, which had been used to test CIP. Taylor (1987b) overcame all of these problems by collecting the data himself from dealers operating in the London foreign exchange market. He found no deviations from CIP (that is, the absence of profitable covered arbitrage opportunities) or zero covered margins. In his subsequent paper, however, he concluded that small but potentially exploitable opportunities of profitable arbitrage emerged occasionally during periods of turbulence in the foreign exchange market.
2.3 Deviations from the CIP No-Arbitrage Condition

![Graphs of USD/GBP, USD/CAD, and GBP/CAD deviations from the CIP no-arbitrage condition from March 1978 to March 2002.](image)

**Figure 2.4** The covered margin (percentage points).
market, but not during periods of tranquillity. He also found that the degree of reduction in the size and persistence of arbitrage increased with the passage of time. Furthermore, he established the notion of the term structure of arbitrage opportunities, indicating that profitable arbitrage opportunities are positively related to the length of the forward contract (the shorter the maturity, the smaller the profitable arbitrage opportunity). One explanation that Taylor presented for observed deviations from covered interest parity is the size and extent of credit limits. If banks impose restrictions on the amounts, maturities and the counterparties they deal with, this will operate as a liquidity constraint on covered arbitrage, in which case profitable opportunities would arise. In another study, Committeri et al. (1993) cast doubt on Taylor’s results on the following grounds. First, his analysis was based on data collected in a specific segment of the Eurocurrency market. Second, the data used in his first study did not cover a long period of time. Third, the data did not represent those that the dealers were actually prepared to deal on. By doing their own analysis, they found no profitable arbitrage opportunities.

Apart from measurement errors, some other factors lead to the distortion of the simple no-arbitrage condition given by (2.8), leading to the belief that there may be a profitable arbitrage operation. The fact of the matter is that there are certain factors that affect the simple no-arbitrage condition just like the case with two-currency arbitrage. These factors will be considered in turn.

**Transaction costs**

Deviations from CIP have been attributed to transaction costs, which are represented by the bid–offer spread of exchange rates (the cost of transacting in the foreign exchange market), the bid–offer spread of interest rates (the cost of transacting in the money market) and brokerage fees. Keynes (1923) asserted that the covered margin must exceed some minimum amount before arbitrage becomes profitable, estimating it to be half a percentage point. Branson (1969) put forward two reasons why such a minimum should exist: (i) each transaction in financial markets requires a payment of brokerage fees, and (ii) banks may require their foreign exchange departments to earn a higher yield than that of their domestic departments.

The effect of transaction costs is to create a band around the CIP no-arbitrage line within which arbitrage is not profitable, as shown in Figure 2.5. Points falling off the CIP line but within the band indicate that although arbitrage is possible it is not profitable, since the covered margin is not sufficiently large to offset transaction costs. Points falling outside the band indicate profitable arbitrage opportunities, because the covered margin is large enough to cover transaction costs and leave out some profit.

To be more precise, recall that the equation of the no-arbitrage line in Figure 2.5 is $i_x - i_y = f$. Without transaction costs, arbitrage from $x$ to $y$ is profitable if $i_x - i_y < f$, whereas arbitrage from $y$ to $x$ is profitable if $i_x - i_y > f$. Let transaction costs (measured in percentage points) be $t$. The equations of the lines
representing the lower and upper limits of the band are \( i_x - i_y = f + t \) and \( i_x - i_y = f - t \) respectively. Arbitrage from \( x \) to \( y \) is not profitable within the lower part of the band because \( i_x - i_y \geq f - t \). Similarly, arbitrage from \( y \) to \( x \) is not profitable within the upper part of the band if \( i_x - i_y \leq f + t \). Arbitrage from \( x \) to \( y \) is profitable if \( i_x - i_y < f - t \), in which case the covered margin is \( i_x - i_y + f - t \). Arbitrage from \( y \) to \( x \) is profitable if \( i_x - i_y > f + t \), in which case the covered margin is \( i_x - i_y - f - t \). Table 2.1 summarises these possibilities.

**Political risk**

Another explanation for deviations from CIP is political risk, which involves the uncertainty that while the funds are invested abroad they may be frozen, become inconvertible or be confiscated. In a less extreme case they may face new or higher taxes. Aliber (1973) argues that the comparability criterion, which is critical for choosing the money market assets used to test CIP, requires assets to be identical in terms of political risk. Accordingly, he argues that while Eurocurrency assets satisfy the comparability criterion, domestic assets do not because they are issued under different political jurisdictions.

In diagrammatic terms, political risk creates a band, as described by Figure 2.6, because arbitragers require a risk premium and hence some minimum
TABLE 2.1 The effect of transaction costs.

<table>
<thead>
<tr>
<th></th>
<th>$x \rightarrow y$</th>
<th>$y \rightarrow x$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Without transaction costs</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No arbitrage</td>
<td>$i_x - i_y = f$</td>
<td>$i_x - i_y = f$</td>
</tr>
<tr>
<td>Profitable arbitrage</td>
<td>$i_x - i_y &lt; f$</td>
<td>$i_x - i_y &gt; f$</td>
</tr>
<tr>
<td>Covered margin</td>
<td>$i_y - i_x + f$</td>
<td>$i_x - i_y - f$</td>
</tr>
<tr>
<td><strong>With transaction costs</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No arbitrage</td>
<td>$i_x - i_y \geq f - t$</td>
<td>$i_x - i_y \leq f + t$</td>
</tr>
<tr>
<td>Profitable arbitrage</td>
<td>$i_x - i_y &lt; f - t$</td>
<td>$i_x - i_y &gt; f + t$</td>
</tr>
<tr>
<td>Covered margin</td>
<td>$i_y - i_x + f - t$</td>
<td>$i_x - i_y - f - t$</td>
</tr>
</tbody>
</table>

covered margin. This band, however, does not have to be of equal width on either side of the CIP line. For example, if investors from country $y$ view country $x$ as being politically more risky than investors from country $x$ view country $y$, then there will be a larger political risk premium on $x$-denominated securities, and the band will be wider to the left of the no-arbitrage line. Moosa (1996a) attributes deviations from CIP between Australia and New Zealand in the period immediately following the abolition of capital controls in the mid-1980s to political risk, in the sense that investors on both sides of the Tasman Sea could have been worried about the possibility of either government reimposing capital controls.

Let us examine Figure 2.6 with some scrutiny. Suppose that $y$-based investors require a risk premium of $\rho_x$ to invest in $x$-denominated assets, whereas $x$-based investors require a risk premium of $\rho_y$ to invest in $y$-denominated assets such that $\rho_x > \rho_y$. The equations of the lines defining the lower and upper limits of the band created by political risk are $i_x - i_y = f - \rho_y$ and $i_x - i_y = f + \rho_x$. Arbitrage from $x$ to $y$ is not initiated within the lower part of the band because $i_x - i_y \geq f - \rho_y$. Similarly, arbitrage from $y$ to $x$ is not initiated within the upper part of the band because $i_x - i_y \leq f + \rho_x$. Arbitrage from $x$ to $y$ is initiated when $i_x - i_y < f - \rho_y$. Notice that the covered margins are equal in both cases because the risk premium is not an actual cost or a source of revenue. Therefore, it only determines the no-arbitrage zone. Table 2.2 summarises the situation.

**Tax differentials**

Levi (1977) explains deviations from the no-arbitrage condition in terms of differences in tax rates. If tax rates on interest income ($\tau_n$) and foreign exchange or capital gains ($\tau_g$) are different ($\tau_n \neq \tau_g$), the no-arbitrage line will no longer be a 45° line. Rather, it will have the equation $i_x - i_y = \theta f$, where

$$\theta = \frac{1 - \tau_g}{1 - \tau_n}$$

(2.22)
If the capital gains tax rate is higher than the income tax rate, the line will be flatter than the 45° line, as shown in Figure 2.7(a). Otherwise, it will be steeper as shown in Figure 2.7(b). Hence, points falling on the line $i_x - i_y = \theta f$ (and hence off the line $i_x - i_y = f$) represent deviations from the no-arbitrage condition in the absence, but not in the presence, of taxes. Table 2.3 shows all of the possibilities.

**FIGURE 2.6** Covered arbitrage in the presence of political risk.

**TABLE 2.2** The effect of political risk.

<table>
<thead>
<tr>
<th></th>
<th>$x \rightarrow y$</th>
<th>$y \rightarrow x$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Without political risk</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No arbitrage</td>
<td>$i_x - i_y = f$</td>
<td>$i_x - i_y = f$</td>
</tr>
<tr>
<td>Arbitrage initiated</td>
<td>$i_x - i_y &lt; f$</td>
<td>$i_x - i_y &gt; f$</td>
</tr>
<tr>
<td>Covered margin</td>
<td>$i_y - i_x + f$</td>
<td>$i_x - i_y - f$</td>
</tr>
<tr>
<td><strong>With political risk</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No arbitrage</td>
<td>$i_x - i_y \geq f - \rho_y$</td>
<td>$i_x - i_y \leq f + \rho_x$</td>
</tr>
<tr>
<td>Arbitrage initiated</td>
<td>$i_x - i_y &lt; f - \rho_y$</td>
<td>$i_x - i_y &gt; f + \rho_x$</td>
</tr>
<tr>
<td>Covered margin</td>
<td>$i_y - i_x + f$</td>
<td>$i_x - i_y - f$</td>
</tr>
</tbody>
</table>
TABLE 2.3 The effect of tax differentials.

<table>
<thead>
<tr>
<th></th>
<th>( x \to y )</th>
<th>( y \to x )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Without tax differentials (( \theta = 1 ))</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No arbitrage</td>
<td>( i_x - i_y = f )</td>
<td>( i_x - i_y = f )</td>
</tr>
<tr>
<td>Profitable arbitrage</td>
<td>( i_x - i_y &lt; f )</td>
<td>( i_x - i_y &gt; f )</td>
</tr>
<tr>
<td>Covered margin</td>
<td>( i_y - i_x + f )</td>
<td>( i_x - i_y - f )</td>
</tr>
<tr>
<td><strong>With tax differentials (( \theta \neq 1 ))</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No arbitrage</td>
<td>( i_x - i_y = \theta f )</td>
<td>( i_x - i_y = \theta f )</td>
</tr>
<tr>
<td>Profitable arbitrage</td>
<td>( i_x - i_y &lt; \theta f )</td>
<td>( i_x - i_y &gt; \theta f )</td>
</tr>
<tr>
<td>Covered margin</td>
<td>( i_y - i_x + \theta f )</td>
<td>( i_x - i_y - \theta f )</td>
</tr>
</tbody>
</table>

TABLE 2.4 The combined effect of transaction costs, political risk and tax differentials (\( \theta > 1 \)).

<table>
<thead>
<tr>
<th></th>
<th>( x \to y )</th>
<th>( y \to x )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Without the effect</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No arbitrage</td>
<td>( i_x - i_y = f )</td>
<td>( i_x - i_y = f )</td>
</tr>
<tr>
<td>Profitable arbitrage</td>
<td>( i_x - i_y &lt; f )</td>
<td>( i_x - i_y &gt; f )</td>
</tr>
<tr>
<td>Covered margin</td>
<td>( i_y - i_x + f )</td>
<td>( i_x - i_y - f )</td>
</tr>
<tr>
<td><strong>With the effect</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No arbitrage</td>
<td>( i_x - i_y \geq \theta f - t - \rho_y )</td>
<td>( i_x - i_y \leq \theta f + t + \rho_x )</td>
</tr>
<tr>
<td>Profitable arbitrage</td>
<td>( i_x - i_y &lt; \theta f - t - \rho_y )</td>
<td>( i_x - i_y &gt; \theta f + t + \rho_x )</td>
</tr>
<tr>
<td>Covered margin</td>
<td>( i_y - i_x + \theta f - t )</td>
<td>( i_x - i_y - \theta f - t )</td>
</tr>
</tbody>
</table>

**Combining the three factors**

It is possible to combine the three factors mentioned so far: transaction costs, political risk and tax differentials. The effect of this combination of factors is represented by Figure 2.8 when \( \theta > 1 \). In this case, the no-arbitrage line is steeper than the 45° line with an equation \( i_x - i_y = \theta f \). The effect of transaction costs and political risk is to create a band around this line. The unequal band width is due to differences in the risk premia, as we are assuming that investment in country \( x \) is more risky as viewed by investors from country \( y \). The no-arbitrage line in the absence of these factors is the 45° line with the equation \( i_x - i_y = f \). Table 2.4 illustrates all of the possibilities.

**Liquidity differences**

Liquidity differences may also cause deviations from the no-arbitrage condition. The liquidity of an asset can be judged by how quickly and cheaply it can be converted into cash. The more uncertainty there is concerning future needs
and alternative sources of short-term financing, the higher will be the premium that should be received before choosing to invest in the non-base currency, \( y \). This again implies the existence of a band around the no-arbitrage line, and this band may have different widths on the two sides of the line.

**Other factors**
Some other factors have been suggested to explain deviations from the CIP no-arbitrage condition because they hinder the movement of arbitrage
funds when the possibility for arbitrage arises. One of these factors is the existence of inelastic (or less than perfectly elastic) supply and demand for arbitrage funds (Pippenger, 1978), which amounts to a violation of one of the basic assumptions of CIP. A related factor is capital market imperfections (Prachowny, 1970), which is not important at the present time given the increasing degree of market perfection. Another factor is capital controls, a factor that has lost importance because of the worldwide tendency to abolish these controls and implement financial deregulation.

These factors prevent the forces of supply and demand from moving to the extent required to restore the equilibrium condition. We have seen from Figure 2.2 that equilibrium is restored when arbitrage causes some changes that take interest and exchange rates to the levels required to achieve the no-arbitrage condition. Let us denote the initial levels of the variables $i_{x,0}$, $i_{y,0}$, $S_0$ and $F_0$. Consider now the situation illustrated by Figure 2.9 assuming that the levels of the variables required to restore equilibrium are $i_{x,2}$ ($>i_{x,1}$), $i_{y,2}$ ($<i_{y,1}$), $S_2$ ($>S_1$) and $F_2$ ($<F_1$). If these factors prevent the forces of supply and demand

**FIGURE 2.8** Covered arbitrage in the presence of transaction costs, political risk and tax differentials.
from achieving the levels of the variables consistent with the no-arbitrage condition, then deviations from this condition will persist. Figure 2.9 shows that changes in supply and demand can only achieve the levels $i_{x,1}$, $i_{y,1}$, $S_1$ and $F_1$. In terms of the CIP diagram represented by Figure 2.10, changes in the forces of supply and demand lead to a fall in the forward spread from $f_0$ to $f_1$ and to a rise in the interest differential from $(i_x - i_y)_0$ to $(i_x - i_y)_1$, which is inadequate to restore the no-arbitrage condition. In other words, arbitrage causes a move towards the no-arbitrage line from A to B. To achieve equilibrium, however, a further move from B to C is required, as indicated by the dotted arrow. Because of these factors, changes in supply and demand reduce, but do not eliminate, deviations from the no-arbitrage condition.
2.4 COMBINING COVERED ARBITRAGE WITH THREE-CURRENCY ARBITRAGE

Three-currency arbitrage is normally viewed as the activity that maintains the consistency of the cross spot exchange rates (the no-arbitrage condition). There is no reason why the same is not valid for the forward market. It is important, however, to bear in mind that consistency of the cross spot rates does not necessarily imply consistency of the cross forward rates. It will be shown later that consistency of the cross spot rates is a necessary but not sufficient condition for the consistency of the cross forward rates. This means that the absence of profitable three-currency arbitrage opportunities from the spot market does not necessarily imply the absence of such opportunities from the forward market.

In what follows, it is shown that covered interest arbitrage and three-currency arbitrage in the spot market can maintain the consistency of the cross forward rates, eliminating three-currency arbitrage opportunities from the forward market. Put differently, the objective is to show that the effect of three-currency arbitrage in the forward market is equivalent to the combined effect of covered interest arbitrage (which involves four markets) and three-currency arbitrage in the spot market.
To simplify the exposition we assume that there are no bid–offer spreads and that transactions are conducted on the basis of the mid rates. Assume that there are three currencies \((x, y\) and \(z)\) and three exchange rates. Let currency \(z\) be the numeraire. Consistency of the cross forward rates (the no-arbitrage condition in the forward market) requires the following condition to be satisfied:

\[
F(x/y) = \frac{F(x/z)}{F(y/z)}
\]  

(2.23)

where \(F(x/y)\) is the forward exchange rate between \(x\) and \(y\), and so on. This no-arbitrage condition is maintained by three-currency arbitrage in the forward market. It can be shown that the condition (2.23) can be established by a combination of covered interest arbitrage and three-currency arbitrage in the spot market.

From the previous discussion, we have

\[
F(x/z) = S(x/z) \left[ \frac{1 + i_x}{1 + i_z} \right]
\]  

(2.24)

Likewise, the condition can be written for \(y\) and \(z\) and for \(x\) and \(y\) as

\[
F(y/z) = S(y/z) \left[ \frac{1 + i_y}{1 + i_z} \right]
\]  

(2.25)

\[
F(x/y) = S(x/y) \left[ \frac{1 + i_x}{1 + i_y} \right]
\]  

(2.26)

Since three-currency arbitrage in the spot market implies that \(S(x/y) = S(x/z)/S(y/z)\), it follows that

\[
F(x/y) = \frac{S(x/z)}{S(y/z)} \left[ \frac{1 + i_x}{1 + i_y} \right]
\]  

(2.27)

By multiplying and dividing the right hand side of equation (2.27) by \(1 + i_z\), we obtain

\[
F(x/y) = \frac{S(x/z)}{S(y/z)} \left[ \frac{1 + i_x}{1 + i_y} \right] \left[ \frac{1 + i_z}{1 + i_z} \right]
\]  

(2.28)

By rearranging we obtain

\[
F(x/y) = \frac{S(x/z)}{S(y/z)} \left[ \frac{(1 + i_x)/(1 + i_z)}{(1 + i_y)/(1 + i_z)} \right] = \frac{F(x/z)}{F(y/z)}
\]  

(2.29)
which is the no-arbitrage condition in the forward market. Thus, profitable three-currency arbitrage opportunities will not be present in the spot market if there are no profitable opportunities for three-currency arbitrage in the spot market and no profitable covered arbitrage.

The violation of equation (2.23) may be due to one or all of the following: (i) inconsistency of the spot exchange rates; (ii) violation of CIP between $x$ and $z$ (2.24); (iii) violation of CIP between $y$ and $z$ (2.25); and (iv) violation of CIP between $x$ and $y$ (2.26). Let us assume that equation (2.23) is violated such that

$$F(x/y) > \frac{F(x/z)}{F(y/z)}$$

(2.30)

This violation is due to one, some or all of the following violations of the no-arbitrage conditions:

$$S(x/y) > \frac{S(x/z)}{S(y/z)}$$

(2.31)

$$S(x/y) \left[ \frac{1+i_x}{1+i_y} \right] > F(x/y)$$

(2.32)

$$S(x/z) \left[ \frac{1+i_x}{1+i_z} \right] > F(x/z)$$

(2.33)

$$S(y/z) \left[ \frac{1+i_y}{1+i_z} \right] < F(y/z)$$

(2.34)

It can be demonstrated that if the forward rates are inconsistent only because the spot rates are inconsistent, then the same profit can be obtained from three-currency arbitrage in the spot market or in the forward market. Consider the violation represented by equation (2.31). If this is the case, then profit can be obtained by following the sequence $z \rightarrow y \rightarrow x \rightarrow z$. For each unit of $z$, profit realised from three-currency arbitrage in the spot market, $\pi_s$, is

$$\pi_s = \frac{S(y/z)S(x/y)}{S(x/z)} - 1$$

(2.35)

It is easy to show that the same profit can be realised by indulging in three-currency arbitrage in the forward market. In this case, profit, $\pi_f$, is

$$\pi_f = \frac{F(y/z)F(x/y)}{F(x/z)} - 1$$

(2.36)

Since there are no covered arbitrage opportunities in any of the currency pairs, it follows that
\[ \pi_f = \frac{S(y/z)((1+i_y)/(1+i_z))S(x/y)((1+i_x)/(1+i_y))}{S(x/z)((1+i_x)/(1+i_z))} - 1 \]

(2.37)

Likewise, it can be shown that profit realised from covered interest arbitrage is identical to the profit realised from three-currency arbitrage for any violation of CIP.

### 2.5 UNCOVERED INTEREST ARBITRAGE

Uncovered interest arbitrage is similar to covered arbitrage, except that it does not involve the forward market. The long position is not covered by converting the underlying currency proceeds at the forward rate that is known in advance, but rather at the spot rate prevailing on the maturity of the long position, which is not known in advance. If the currency underlying the long position does not depreciate against the currency underlying the short position by more than the interest rate differential, then profit will be made.

Obviously, this operation involves foreign exchange risk, because it is based on a decision variable that is not known in advance: the expected spot rate on the maturity of the long position. So, this operation does not satisfy the definition of arbitrage that it is a riskless operation, and hence it should be classified as speculation. In general, there is nothing wrong with classifying this operation as speculation, but there are some reasons why it may also be classified as arbitrage, including the following:

1. As we are going to see, it is a misconception that arbitrage is a riskless operation. Even in the simple case of two-currency arbitrage, some risk may be involved.
2. We will also find out that foreign exchange risk can be minimised or eliminated under certain conditions (for example, by choosing a pair of currencies with a relatively stable exchange rate). In this case, the operation boils down to borrowing a low-interest currency and investing in a high-interest currency, making profit out of the interest rate differential. The forward cover would be replaced by a natural hedge from the choice of a currency pair with a stable exchange rate.
3. If, and this is a big if, the arbitrager believes in the accuracy of her forecasts with a high confidence level, then she will be in a position to calculate the arbitrage profit in advance with a high degree of confidence.
4. As we have seen, covered arbitrage is the mechanism whereby the no-arbitrage condition, which is known as covered interest parity (CIP), is maintained. If the counterpart to covered interest parity in this case is uncovered
interest parity (UIP), then it is convenient to call the mechanism that maintains this condition uncovered arbitrage.

5. We will also find out that foreign exchange risk can be eliminated in some special situations. One such situation is when a short (long) position is taken on a currency that is pegged to a basket, while a corresponding long (short) position is taken on the components of the basket in such a way that it reflects the weights of the basket components. In this case, foreign exchange risk would disappear, and the arbitrager would know in advance how much profit will be made on the interest rate differentials.

The mechanism of uncovered arbitrage
Let us now see what happens when the arbitrager indulges in uncovered arbitrage from \( x \) to \( y \). The operation consists of the following steps:

1. At time \( t \), the arbitrager borrows one unit of \( x \) at \( i_x \) for a period extending between \( t \) and \( t + 1 \).
2. The amount borrowed is converted at \( S_t \), obtaining \( 1/S_t \) units of \( y \). This amount is then invested at \( i_y \).
3. At \( t + 1 \), the \( y \) value of the investment is \( (1/S_t)(1 + i_y) \).
4. The \( x \) currency value of the investment, converted at the spot rate prevailing at \( t + 1 \) is \( (S_{t+1}/S_t)(1 + i_y) \).
5. At \( t + 1 \) the loan matures, and the amount \( (1 + i_x) \) has to be repaid.

Net profit, or the uncovered margin, is given by

\[
\pi = \frac{S_{t+1}}{S_t}(1+i_y) - (1+i_x)
\]  

Hence the no-arbitrage condition is

\[
\frac{S_{t+1}}{S_t}(1+i_y) = (1+i_x)
\]  

which is one version of the uncovered interest parity condition. The UIP no-arbitrage condition can be expressed differently by manipulating equation (2.39). First of all we could rewrite equation (2.39) in terms of net returns and costs, by subtracting 1 from both sides of the equation, as

\[
\frac{S_{t+1}}{S_t}(1+i_y) - 1 = i_x
\]  

Another specification of the no-arbitrage condition can be obtained by manipulating equation (2.39) to obtain

\[
S_{t+1} = F_t
\]  

where \( F_t \) is the interest parity forward rate defined in equation (2.5). Likewise, we can derive the following expression for the UIP condition:
\[ i_x - i_y = \hat{S} \]  

(2.42)

where \( \hat{S} \) is the percentage change in the exchange rate between \( t \) and \( t + 1 \). Equation (2.42) tells us that if the (expected) percentage change in the exchange rate is equal to the interest rate differential, then there is no possibility of indulging in profitable arbitrage. This is because if the arbitrager wants to go long on the high-interest currency, any gains from the interest rate differential will be offset by the depreciation of the currency. Otherwise, if the arbitrager wants to go long on the currency that is expected to appreciate to make profit out of foreign exchange gains, then this gain will be offset by the interest rate differential in favour of the other currency. No profit will be made in either case.

Deriving an expression for the uncovered margin in the presence of bid–offer spreads is similar to that under covered arbitrage. In the case of arbitrage from \( x \) to \( y \), the uncovered margin is given by

\[ \pi = \frac{S_{b,t+1}}{S_{a,t}} (1+i_{y,b}) - (1+i_{x,a}) \]  

(2.43)

or

\[ \pi = i_{x,a} - i_{y,b} + \hat{S} - m \]  

(2.44)

If, on the other hand, arbitrage runs from \( y \) to \( x \), then

\[ \pi = \frac{S_{b,t}}{S_{a,t+1}} (1+i_{x,b}) - (1+i_{y,a}) \]  

(2.45)

or

\[ \pi = i_{x,b} - i_{y,a} - \hat{S} - m \]  

(2.46)

As we can see, the UIP condition is similar to the CIP condition with \( S_{t+1}(\hat{S}) \) replacing \( F_{t}(f) \).

The empirical validity of UIP

For UIP to be valid, indicating the absence of uncovered arbitrage opportunities, the exchange rate between \( t \) and \( t + 1 \) must change by a percentage that is equal (or at least related) to the interest rate differential at time \( t \). Formal tests of UIP have produced mixed results that are highly sensitive to the model specification. If we observe historical data, however, we can see that uncovered arbitrage would have produced some significantly positive or negative uncovered margins, as shown in Figure 2.11. It is obvious that the uncovered margin is greater in currency pairs involving the pound because of the relative stability of the exchange rate between the US and Canadian currencies and the narrower interest rate differential.
FIGURE 2.11 The uncovered margin (percentage points).
Notice that the uncovered margins move within wider ranges than the corresponding covered margins shown in Figure 2.4. This is a manifestation of the motto “there is no such thing as a free lunch”, in the sense that if you want to enjoy the possibility of the high return offered by uncovered arbitrage you must be prepared to endure the high risk associated with this operation.
3.1 COMMODITY ARBITRAGE

In the case of commodity arbitrage, arbitragers buy a commodity in the market where it is cheap and sell it where it is more expensive, making profit as the difference between the selling price and the buying price. This activity leads to a rise in the price of the commodity in the market where it is cheap and a decline in price in the market where it is expensive until profit is eliminated and the equilibrium condition is restored.

Let $P_x^j$ be the price of a commodity, $j$, in terms of $x$, $P_y^j$ the price of the same commodity in terms of $y$ and $S$ is the exchange rate measured as $S(x/y)$. The no-arbitrage condition is

$$P_x^j = S P_y^j$$

which says that the price of the commodity measured in the same currency must be equal. Notice that $P_x^j$ is the price of the commodity in terms of currency $x$, whereas $S P_y^j$ is the price in terms of $y$ converted into $x$ at the current spot rate. Equation (3.1) is also know as the law of one price (LOP), which generally says that the price of a commodity should be equal across countries when measured in the same currency.

Figure 3.1 shows how the process works, starting from a disequilibrium position described by the inequality $P_x^j > S P_y^j$. Given this situation, arbitragers buy the commodity where it is cheaper (in the country whose currency is $y$), leading to an increase in demand and a shift in the demand curve. They will also sell the commodity where it is more expensive (in the country whose currency is $x$), leading to an increase in supply. Thus, $S P_y^j$ rises, while $P_x^j$ falls, until they are equal. At this point, arbitrage profit disappears and the no-arbitrage condition is restored.
The LOP can be generalised by assuming that the latter holds for each and every commodity. Assuming that there are \( n \) commodities, the LOP can be generalised to obtain

\[
\sum_{j=1}^{n} w_x^j P_x^j = S \sum_{j=1}^{n} w_y^j P_y^j
\]  

(3.2)

where \( w_x^j \) and \( w_y^j \) are the weights of commodity \( j \) in the general price level of the two countries, with the weights presumably reflecting consumption patterns. This relationship is valid if \( w_x^j = w_y^j \) for all \( j \). Thus

\[ P_x = SP_y \]  

(3.3)

which is the same as equation (3.1) except that it is written in terms of the general price levels, \( P_x \) and \( P_y \), and not the prices of individual commodities. This relationship is purchasing power parity (PPP), which can be represented diagrammatically by a straight line passing through the origin in the \( S-P_x \) space, as shown in Figure 3.2. Changes in \( P_y \) are represented by the movement of the PPP line downwards (when \( P_y \) falls) and upwards (when \( P_y \) rises).

Commodity arbitrage may be intertemporal in the sense that a time factor is involved. In this case the arbitrager does not buy and sell at the same time, but rather after some time. Suppose that the arbitrager buys a bundle of commodities in country \( x \) at time \( t \) and sells it in the same country at time \( t+1 \). The rate of return on this transaction will be the inflation rate between \( t \) and \( t+1 \), which we shall denote \( \tilde{P}_x \). If, on the other hand, the arbitrager chooses to sell the bundle in country \( y \), then the rate of return (in terms of currency \( x \)) will consist of two components: the inflation rate in country \( y \), \( \tilde{P}_y \) and the percentage change in the spot exchange rate, \( \tilde{S} \), which is the rate of appreciation or depreciation of \( y \) against \( x \). Hence the no-arbitrage condition is
\[ P_x = P_y + \hat{S} \] (3.4)

If, for example, \( \hat{P}_x < \hat{P}_y + \hat{S} \), it will be profitable to buy commodities in country \( x \) and sell them in country \( y \) after some time. The no-arbitrage condition (3.4) can be written as

\[ \hat{S} = \hat{P}_x - \hat{P}_y \] (3.5)

which is PPP written in first differences (invariably, but perhaps inappropriately, known as relative PPP). If the expected values of the variables are used then equation (3.5) would represent the so-called \textit{ex ante} PPP.

It is possible to show that (3.5) can be derived by assuming that (3.3) holds at the points in time \( t \) and \( t + 1 \). If this is so, then

\[ P_{x,t} = S_t P_{y,t} \] (3.6)

\[ P_{x,t+1} = S_{t+1} P_{y,t+1} \] (3.7)

By dividing equation (3.7) by equation (3.6), we obtain

\[ \frac{P_{x,t+1}}{P_{x,t}} = \frac{S_{t+1} P_{y,t+1}}{S_t P_{y,t}} \] (3.8)

which can be rewritten as

\[ (1 + \hat{P}_x) = (1 + \hat{S})(1 + \hat{P}_y) \] (3.9)
which, after simplification, reduces to (3.5). Moreover, equation (3.8) can be written in the following form that makes it something like an exchange rate determination model

\[ S_{t+1} = S_t \left( \frac{P_{x,t+1}/P_{x,t}}{P_{y,t+1}/P_{y,t}} \right) \] (3.10)

If we allow for a base period, 0, to which prices are measured then this means that the exchange rate at any point in time \( t \) can be obtained by adjusting the exchange rate at 0 for differences in inflation rates. Hence we could define the exchange rate consistent with PPP, \( \bar{S} \), as

\[ \bar{S}_t = S_0 \left( \frac{P_{x,t}/P_{x,0}}{P_{y,t}/P_{y,0}} \right) = S_0 \left( \frac{1 + \dot{P}_{x,t}}{1 + \dot{P}_{y,t}} \right) \] (3.11)

Equation (3.11) is used to measure the extent of misalignment (overvaluation or undervaluation) of currencies, such that currency \( y \) would be deemed overvalued if \( S_t > \bar{S}_t \), and vice versa. Figure 3.3 shows the actual and PPP rates (\( S_t \) and \( \bar{S}_t \) respectively) of the pound and the Canadian dollar against the US dollar. It indicates that the pound has been predominantly overvalued, whereas the Canadian dollar has been predominantly undervalued against the US dollar. Exchange rate misalignment would normally indicate the possibility of commodity arbitrage.

### 3.2 Arbitrage Under the Gold Standard

The classical gold standard, which was in operation worldwide from around 1870 until 1914, when it collapsed with the outbreak of the First World War, is basically a system of fixed exchange rates. In this section we examine the process whereby currency arbitrage is conducted under a system of fixed exchange rates, where the rates are determined by the gold values of individual currencies.

Under the gold standard the fixed exchange rates are determined as follows. The monetary authority in each country fixes the price of gold in terms of the domestic currency and stands ready to buy or sell any amount of gold at that price. This would in turn establish a fixed exchange rate between any two currencies called the “mint parity”. The actual exchange rate can only vary above and below the mint parity between certain limits called the “gold points”, which are determined by the cost of shipping gold between the two countries in question.

Let us assume that the country whose currency is \( x \) is the home country, and the country whose currency is \( y \) is the foreign country. Suppose that the price of an ounce of gold is \( a \) units of \( x \) and \( b \) units of \( y \). The mint parity is then given by
\[ S^M = \frac{a}{b} \] (3.12)

Suppose now that the cost of shipping gold between the two countries is a fraction, \( e \), of the value of the gold shipped. From the perspective of the home country, the gold export point, which is the upper limit on the exchange rate, is given by

\[ S^U = \frac{a}{b} + e \left( \frac{a}{b} \right) = (1 + e) \left( \frac{a}{b} \right) \] (3.13)

The gold export point implies that no resident in the home country would pay more than \((1 + e)(a/b)\) units of \( x \) for one unit of \( y \). This is because it is possible to buy \((a/b)\) units of \( x \) worth of gold, ship it to the foreign country and
sell it for one unit of $y$. Similarly, the gold import point, which is the lower limit on the exchange rate, is given by

$$S^L = \frac{a}{b} - e\left(\frac{a}{b}\right) = (1 - e)\left(\frac{a}{b}\right)$$

The gold import point implies that no resident of the home country is willing to accept less than $(1 - e)(a/b)$ units of $x$ for one unit of $y$.

The gold export and import points have some implications for the shapes of the supply and demand for currency $y$ curves. The supply curve becomes infinitely elastic or horizontal at the gold export point, whereas the demand curve becomes horizontal at the gold import point. In between they have the normal upward and downward sloping shapes, as illustrated in Figure 3.4, which shows that the equilibrium exchange rate $S^E$ is above the mint parity rate, $S^M$.

The exchange rate can move between the gold import and export points, but not outside this range. Thus, the no-arbitrage zone is represented by a band around the mint parity rate.

### 3.3 Arbitrage Between Eurocurrency and Domestic Interest Rates

Arbitrage between Eurocurrency and domestic money markets ensures that there is a close relationship (a no-arbitrage condition) between domestic and
Eurocurrency interest rates. In the absence of the bid–offer spread, the no-arbitrage condition would take the form

\[ i = i^* \]  \hspace{1cm} (3.15)

where \( i \) is the domestic interest rate and \( i^* \) is the Eurocurrency interest rate. If the condition is violated, arbitrage would take the form of borrowing funds in the market where funds are cheap and lending in the market where they are expensive. In the presence of the bid–offer spread, the no-arbitrage condition would take the form

\[ (i_b, i_a) = (i_a^*, i_b^*) \]  \hspace{1cm} (3.16)

In reality, however, there are factors that prevent the equality of domestic and Eurocurrency rates as indicated by equations (3.15) and (3.16). The conditions become

\[ i = i^* + \alpha \]  \hspace{1cm} (3.17)

and

\[ (i_b, i_a) = (i_a^* + \beta, i_b^* + \gamma) \]  \hspace{1cm} (3.18)

where \( \alpha > 0 \), \( \beta > 0 \) and \( \gamma > 0 \).

Figure 3.5 shows the relationship between Eurocurrency and domestic rates. Domestic rates are determined by the demand for and supply of funds in the domestic market, whereas the Eurocurrency rates are determined by the demand for and supply of funds in the Eurocurrency market. In both cases, the bid rate is determined by the demand of market makers and the supply of others (including arbitragers), whereas the offer rate is determined by the demand of others and the supply of market makers. As we can see, the relationship between Eurocurrency and domestic rates is such that

\[ \text{FIGURE 3.5 Determination of domestic and Eurocurrency interest rates.} \]
We can also see that the bid–offer spread in the Eurocurrency market is narrower than the spread in the domestic market, which is normally interpreted to indicate that the Eurocurrency market is more efficient due to the absence of regulatory requirements.

What are the factors that prevent the equality of Eurocurrency and domestic interest rates despite the presence of arbitrage activity? These factors may include transaction costs, capital controls and political risk. Under full and effective capital controls, the domestic and Eurocurrency markets will be completely isolated from each other, preventing arbitrage activity from equating interest rates. Several factors can explain why the Eurocurrency offer rate (lending rate) is lower than the corresponding domestic offer rate:

1. The absence of regulatory requirements in the Eurocurrency market, which makes borrowing and lending cheaper than in the domestic market.
2. Borrowers are of higher quality and credit rating than in the domestic market, which makes the risk premium lower in the Eurocurrency market.
3. Eurocurrency transactions are much larger than domestic transactions, which allows the exploitation of the economies of scale.
4. Eurocurrency transactions can take place out of tax havens, which makes them cheaper to execute.

The Eurocurrency bid rate (deposit rate) is higher than the corresponding domestic bid rate, for the following reasons:

1. It has to be higher to attract domestic deposits.
2. Eurobanks can afford to pay higher deposit rates.
3. The absence of interest rate ceilings in the Eurocurrency market.
4. Eurobanks are more efficient than domestic banks, and they can lend out a larger percentage of deposits.

3.4 EUROCURRENCY–EUROBOND ARBITRAGE

Eurocurrency–Eurobond arbitrage depends on the level of short-term Eurocurrency interest rates and long-term Eurobond yields. As Eurobonds approach maturity, opportunities may arise to generate profit by borrowing short-term funds and using these funds to buy bonds. Suppose that there is a Eurobond denominated in currency $y$ with a coupon rate $i_y$, time to maturity of one year, a market price of $P$, and a face value of $V$. Let the spot and one-year forward rates be $S$ and $F$ respectively, and assume that the arbitrager can borrow $x$-denominated funds at $i_x$. The arbitrage operation consists of the following steps:

1. Borrowing $PS$ units of $x$, which is equivalent to $P$ units of $y$ (the market price of the bond).
2. Buying the bond and holding it until maturity, when it pays \( V(1 + i_y) \) units of \( y \) or \( V(1 + i_y)F \) units of \( x \) at the current forward rate.

3. The value of the loan to be repaid (principal plus interest) is equal to \( PS(1 + i_x) \).

Arbitrage profit would therefore be

\[
\pi = V(1 + i_y)F - PS(1 + i_x) \tag{3.19}
\]

which means that the no-arbitrage condition is

\[
V(1 + i_y)F = PS(1 + i_x) \tag{3.20}
\]

Equation (3.20) can be written as

\[
\frac{P}{V} = \left( \frac{1 + i_y}{1 + i_x} \right) \left( \frac{F}{S} \right) \tag{3.21}
\]

or

\[
\frac{P}{V} = \frac{(1 + i_y)(1 + f)}{1 + i_x} \tag{3.22}
\]

where \( f \) is the forward spread. If the bond is selling at par such that \( P = V \), then equation (3.22) reduces to the CIP no-arbitrage condition, \( i_x - i_y = f \).

### 3.5 Arbitrage Between Currency Futures and Forward Contracts

Currency futures and forward contracts represent transactions whereby the counterparties to the transaction are committed to the selling and purchase of a given amount of a particular currency some time in the future at an exchange rate determined at the present time. The difference between them is that futures contracts are standardised with respect to size and maturity date, whereas forward contracts are tailor-made, designed for specific needs.

Arbitrage between the forward and futures markets ensures that the exchange rates implicit in these contracts are equal or approximately so. Suppose that the exchange rate implicit in a futures contract for delivery in September was \( F_1(x/y) \), and the corresponding forward rate was \( F_2(x/y) \). Obviously, the no-arbitrage condition is \( F_1(x/y) = F_2(x/y) \). If \( F_1(x/y) < F_2(x/y) \), the arbitrager would buy the futures contract on \( y \) and sell an equivalent amount of \( y \) forward to earn the difference. Again, the no-arbitrage condition would be different from the strict equality of the two rates because futures contracts involve marking to market, whereas forward contracts do not.
3.6 REAL INTEREST ARBITRAGE

The term “real interest arbitrage” is not normally used in the literature. However, if covered interest arbitrage maintains covered interest parity and uncovered interest arbitrage maintains uncovered interest parity, then it is reasonable to put forward the idea that “real interest arbitrage” maintains real interest parity (RIP). This is so because RIP is derived either by combining other international parity conditions (UIP and \textit{ex ante} PPP) or by the movement of funds from financial centres with low real rates of return to those with high real rates of return.

Let us see how this works. The no-arbitrage real interest parity condition is

\[ r_{x,t} = r_{y,t} \]  \hspace{1cm} (3.23)

where the real interest rates are defined as

\[ r_{x,t} = i_{x,t} - \hat{P}_{x,t+1} \] \hspace{1cm} (3.24)
\[ r_{y,t} = i_{y,t} - \hat{P}_{y,t+1} \] \hspace{1cm} (3.25)

where \(\hat{P}\) is the inflation rate measured as the percentage change in the general price level. Hence the no-arbitrage condition can be written as

\[ i_{x,t} - i_{y,t} = \hat{P}_{x,t+1} - \hat{P}_{y,t+1} \] \hspace{1cm} (3.26)

which says that the nominal interest rate differential is equal to the expected or subsequent inflation differential. Obviously, changes in supply and demand as a result of arbitrage will only affect the nominal interest rates, changing them to an extent that will be sufficient to equate the real interest rates. This is at least the conventional wisdom.

However, one may ask the following legitimate question: why would investors whose base currency is \(x\), investing in \(y\) and wanting to repatriate the receipts, be concerned about inflation in country \(y\) ? Surely they should be concerned about inflation in country \(x\), because it determines the purchasing power (in country \(x\)) of their return on investment in \(y\). Moreover, they should also be more concerned about the return in terms of currency \(x\) than in terms of currency \(y\), which means that the exchange rate factor, which is not incorporated in the no-arbitrage relationship (3.26), is important and should be taken into account. This means that the no-arbitrage condition should be modified accordingly. Notice that the exact expression for the equality of real returns on both currencies is

\[ \frac{1 + i_{x,t}}{1 + \hat{P}_{x,t+1}} = \frac{1 + i_{y,t}}{1 + \hat{P}_{y,t+1}} \] \hspace{1cm} (3.27)

The suggested modification means that the condition will be written as follows:
\[
\frac{1 + i_{x,t}}{1 + \bar{P}_{x,t+1}} = \frac{(1 + i_{y,t})(1 + \hat{S}_{t+1})}{1 + \bar{P}_{x,t+1}}
\]  

(3.28)

where \( \hat{S}_{t+1} \) is the percentage change in the exchange rate between \( t \) and \( t + 1 \). The left-hand side of equation (3.28) is the real return on currency \( x \), whereas the right-hand side is the real return on \( y \) expressed in terms of currency \( x \). Equation (3.28) can be simplified to produce

\[
i_{x,t} - i_{y,t} = \hat{S}_{t+1}
\]  

(3.29)

which is uncovered interest parity. Now, notice that (ex ante) purchasing power parity tells us that

\[
\bar{P}_{x,t+1} - \bar{P}_{y,t+1} = \hat{S}_{t+1}
\]  

(3.30)

and so if we combine (3.29) and (3.30) we go back to the original condition. Thus it seems that what maintains RIP is not real interest arbitrage, in the sense that it is not a single operation that maintains the condition. Rather, the condition is maintained by two kinds of arbitrage: uncovered interest arbitrage and intertemporal commodity arbitrage. The first kind of arbitrage maintains uncovered interest parity, whereas the second kind maintains purchasing power parity.

### 3.7 Uncovered Arbitrage When the Cross Rates Are Stable

If the interest rate on currency \( x \) is lower than the interest rate on currency \( y \), then profitable uncovered arbitrage can be executed by going long on \( y \) and short on \( x \), provided that \( y \) does not depreciate by more than the interest rate differential before unwinding the positions. Recall that the uncovered margin in this case would be \( \pi = i_y - i_x + \hat{S} \), which means that if \( \hat{S} = 0 \) the uncovered margin will be equal to the interest rate differential. Of course, it is unlikely that \( \hat{S} = 0 \) under a system of flexible exchange rates, so we could reformulate the proposition to the following: \( \pi \approx i_y - i_x \) if \( \hat{S} \approx 0 \).

It may be possible to find a pair of currencies whose cross exchange rate is so stable that the condition \( \hat{S} \approx 0 \) may be satisfied. Since major floating currencies have a general tendency to move against the US dollar in the same direction, the cross exchange rate between two currencies tends to be stable if the two currencies move proportionately against the dollar. The proposition that is put forward here is that if the exchange rates of two currencies, \( x \) and \( y \), against a numeraire, \( z \), are highly correlated, then the cross exchange rate between \( x \) and \( y \) tends to be stable. The following is a proof of this proposition.

At points in time \( t \) and \( t + 1 \), we have
3.7 UNCOVERED ARBITRAGE WHEN CROSS RATES ARE STABLE

\[ S_t(x/y) = \frac{S_t(x/z)}{S_t(y/z)} \]  

(3.31)

\[ S_{t+1}(x/y) = \frac{S_{t+1}(x/z)}{S_{t+1}(y/z)} \]  

(3.32)

Therefore

\[ \dot{S}(x/y) = \frac{S_{t+1}(x/y)}{S_t(x/y)} - 1 = \frac{[S_{t+1}(x/z)]/[S_{t+1}(y/z)]}{[S_t(x/z)]/[S_t(y/z)]} - 1 \]  

(3.33)

which gives

\[ \dot{S}(x/y) = \frac{1 + \dot{S}(x/z)}{1 + \dot{S}(y/z)} - 1 = 0 \]  

(3.34)

because perfect correlation implies that \( \dot{S}(x/z) = \dot{S}(y/z) \). In a less extreme case, high correlation implies that \( \dot{S}(x/z) \approx \dot{S}(y/z) \), and hence \( \dot{S}(x/y) \approx 0 \).

An illustration

In this example, the numeraire currency is the US dollar, and there are nine currencies under consideration. Table 3.1 is a correlation matrix of the exchange rates against the US dollar, calculated from quarterly data covering the period 1990:1–2000:4. In general it mostly shows high and significant positive correlations among the exchange rates, except some negative correlations involving the Japanese yen. The indication here is that, with the exception of the yen, currencies have been moving against the dollar in the same direction. The highest correlation is found between the exchange rates of the Norwegian krone and the Danish kroner (which is not surprising). Other high correlations are found between the Australian dollar and each of the following

<table>
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<tr>
<th></th>
<th>AUD</th>
<th>CAD</th>
<th>CHF</th>
<th>DKK</th>
<th>GBP</th>
<th>JPY</th>
<th>NOK</th>
<th>NZD</th>
<th>SEK</th>
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</table>
currencies: Norwegian krone, New Zealand dollar, Swedish krona and Danish kroner.

High correlations are a necessary but not sufficient condition for executing a profitable uncovered arbitrage operation along the lines suggested in this section. The necessary condition is a wide interest rate differential. Table 3.2 is a matrix of (three-month) interest rate differentials as at the end of 2000.

By combining the information in the two tables we can pick some currency pairs that are suitable for this operation. These are shown in Table 3.3. For example, by going short on the Swiss franc and long on the Norwegian krone, an arbitrager can earn over four percentage points in interest rate differential. With a correlation coefficient of 0.81, it is unlikely that the Swiss franc would appreciate against the Norwegian currency by an amount that would wipe out this differential.

### Table 3.2 Interest rate differentials (end of 2000).a

<table>
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<tr>
<th></th>
<th>AUD</th>
<th>CAD</th>
<th>CHF</th>
<th>DKK</th>
<th>GBP</th>
<th>JPY</th>
<th>NOK</th>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SEK</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Each cell represents the interest rate on the currency in the row minus the interest rate on the currency in the column. For example, −5.62 is the interest rate on the yen minus the interest rate on the Australian dollar.

### Table 3.3 Some examples of uncovered arbitrage operations.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Short position</th>
<th>Long position</th>
<th>Correlation coefficient</th>
<th>Interest rate differential</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AUD</td>
<td>SEK</td>
<td>0.79</td>
<td>2.13</td>
</tr>
<tr>
<td>2</td>
<td>CHF</td>
<td>DKK</td>
<td>0.92</td>
<td>1.97</td>
</tr>
<tr>
<td>3</td>
<td>CHF</td>
<td>NOK</td>
<td>0.81</td>
<td>4.06</td>
</tr>
<tr>
<td>4</td>
<td>CHF</td>
<td>NZD</td>
<td>0.77</td>
<td>3.35</td>
</tr>
<tr>
<td>5</td>
<td>DKK</td>
<td>NOK</td>
<td>0.94</td>
<td>2.09</td>
</tr>
<tr>
<td>6</td>
<td>SEK</td>
<td>NOK</td>
<td>0.91</td>
<td>3.35</td>
</tr>
</tbody>
</table>
3.8 UNCOVERED INTEREST ARBITRAGE WHEN THE BASE CURRENCY IS PEGGED TO A BASKET

In this section we deal with uncovered interest arbitrage when the base currency is pegged to a basket such that its exchange rate against a numeraire (normally the US dollar) is determined by the exchange rates of the component currencies against the numeraire. Let \( x \) be the base (pegged) currency, \( y_0 \) the numeraire and \( y_j \) any other currency, whether or not it is included in the basket. Define \( S_0 = S(x/y_0) \), \( S_j = S(y_j/y_0) \) and \( E_j = S(x/y_j) \). If the basket contains \( n \) currencies, \( S_0 \) is calculated as

\[
S_0 = \alpha_0 + \sum_{j=1}^{n} \alpha_j S_j
\]  
(3.35)

where \( \alpha_0 \) reflects the weight of the numeraire and \( \alpha_j \) reflects the weight of currency \( j \) in the basket. For these coefficients to be exactly the weights, equation (3.35) must be written in a logarithmic form as

\[
\log S_0 = \alpha_0 + \sum_{j=1}^{n} \alpha_j \log S_j
\]  
(3.36)

We also have

\[
E_j = \frac{S_0}{S_j}
\]  
(3.37)

If a short or a long position is taken on a portfolio of currencies against a short or a long position on the base currency, the foreign exchange profit/loss will be a weighted average of the gains and losses resulting from changes in the exchange rates \( S_0 \) and \( E_j \). In continuous time, the foreign currency profit/loss resulting from changes in the exchange rate \( S_0 \) is defined as

\[
C(S_0) = \frac{1}{S_0} \frac{dS_0}{dt} = \hat{S_0}
\]  
(3.38)

where \( C(S_0) \) is the profit/loss, which we shall call the “currency factor”. From equations (3.36) and (3.37), we have

\[
\frac{1}{S_0} \frac{dS_0}{dt} = \sum_{j=1}^{n} \alpha_j \frac{1}{S_j} \frac{dS_j}{dt} = \sum_{j=1}^{n} \alpha_j \hat{S}_j
\]  
(3.39)

and

\[
\hat{E}_j = \hat{S}_0 - \hat{S}_j
\]  
(3.40)
Assume now that there is a foreign currency portfolio consisting of some currencies that are included in the basket and others that are not, such that \( j = 1, 2, ..., m, m + 1, ..., n, n + 1, n + 2, ..., k \). Currencies included in the basket and the portfolio are \( 1, 2, ..., m \); currencies included in the basket but not in the portfolio are \( m + 1, ..., n \); and those included in the portfolio only are \( n + 1, n + 2, ..., k \). Hence

\[
C = \beta_0 \hat{S}_0 + \beta_1 \hat{E}_1 + \beta_2 \hat{E}_2 + \ldots + \beta_m \hat{E}_m + \beta_{n+1} \hat{E}_{n+1} + \beta_{n+2} \hat{E}_{n+2} + \ldots + \beta_k \hat{E}_k
\]  

(3.41)

where \( \beta_0 \) is the weight assigned to currency \( y_0 \) and \( \beta_j \) is the weight assigned to currency \( y_j \) in the portfolio for \( j = 1, 2, ..., m \) and \( j = n, n + 1, n + 2, ..., k \). This gives

\[
C = \beta_0 \hat{S}_0 + \beta_1 (\hat{S}_0 - \hat{S}_1) + \beta_2 (\hat{S}_0 - \hat{S}_2) + \ldots + \beta_m (\hat{S}_0 - \hat{S}_m) + \beta_{n+1} (\hat{S}_0 - \hat{S}_{n+1}) + \beta_{n+2} (\hat{S}_0 - \hat{S}_{n+2}) + \ldots + \beta_k (\hat{S}_0 - \hat{S}_k)
\]  

(3.42)

Since

\[
\sum_{j=0}^{m} \beta_j + \sum_{j=n+1}^{k} \beta_j = 1
\]  

(3.43)

it follows that

\[
C = \hat{S}_0 - (\beta_1 \hat{S}_1 + \beta_2 \hat{S}_2 + \ldots + \beta_m \hat{S}_m + \beta_{n+1} \hat{S}_{n+1} + \ldots + \beta_k \hat{S}_k)
\]  

(3.44)

Since

\[ \hat{S}_0 = \sum_{j=1}^{n} \alpha_j \hat{S}_j = \sum_{j=1}^{m} \alpha_j \hat{S}_j + \sum_{j=m+1}^{n} \alpha_j \hat{S}_j \]  

(3.45)

then

\[
C = \sum_{j=1}^{m} \alpha_j \hat{S}_j + \sum_{j=m+1}^{n} \alpha_j \hat{S}_j - \sum_{j=1}^{m} \beta_j \hat{S}_j - \sum_{j=n+1}^{k} \beta_j \hat{S}_j
\]  

(3.46)

or

\[
C = \sum_{j=1}^{m} (\alpha_j - \beta_j) \hat{S}_j + \sum_{j=m+1}^{n} \alpha_j \hat{S}_j - \sum_{j=n+1}^{k} \beta_j \hat{S}_j
\]  

(3.47)

The objective of arbitrage is to utilise the interest rate differential while eliminating foreign exchange risk. For simplicity we will ignore the bid–offer spread on interest rates and assume that the borrowing and lending rates for each currency are equal. Let the interest rate on the base currency be \( i_x \), and on
foreign currency $j$ be $i_j$ ($j = 1, 2, ..., k$). Thus, the effective interest rate on the foreign currency portfolio is $\Sigma^k_{j=1} \beta_j i_j$. Let us also start with the general case where there are currencies that are included in the basket and in the portfolio, others that are included in the basket only, and those included in the portfolio only. If a short (long) position is taken on the base currency while a long (short) position is taken on the portfolio then the uncovered margin, $\pi$, is given by

$$\pi = \sum_{j=1}^{k} \beta_j i_j - i_x + \sum_{j=1}^{m} (\alpha_j - \beta_j) \hat{S}_j + \sum_{j=m+1}^{n} \alpha_j \hat{S}_j - \sum_{j=n+1}^{k} \beta_j \hat{S}_j$$  \hspace{1cm} (3.48)

If the objective is to eliminate foreign exchange risk while gaining the interest rate differential, the following conditions must be satisfied: $k = n$ and $\alpha_j = \beta_j$ for $j = 1, 2, ..., m$. These two conditions imply the following: (i) the currencies in the portfolio are equal in number and identical to those in the basket; and (ii) the weight assigned to a particular currency in the portfolio is equal to its weight in the basket. If these conditions are satisfied, equation (3.48) reduces to

$$\pi = \sum_{j=1}^{n} \beta_j i_j - i_x$$  \hspace{1cm} (3.49)

because

$$\sum_{j=1}^{m} (\alpha_j - \beta_j) \hat{S}_j + \sum_{j=m+1}^{n} \alpha_j \hat{S}_j = \sum_{j=1}^{n} (\alpha_j - \beta_j) \hat{S}_j = 0$$  \hspace{1cm} (3.50)

and

$$\sum_{j=n+1}^{k} \beta_j \hat{S}_j = 0$$  \hspace{1cm} (3.51)

Equation (3.49) obviously indicates the possibility of obtaining risk-free profit because all interest rates are known in advance. It also implies that there is no possibility for arbitrage if

$$i_x = \sum_{j=1}^{n} \beta_j i_j$$  \hspace{1cm} (3.52)

which means that if the interest rate on the base (pegged) currency is calculated as a weighted average of the interest rates on the components of the basket, arbitrage will not be profitable. If, however, condition (3.52) is not satisfied, then the following can be done. If $i_x < \Sigma \beta_j i_j$, a short position should be taken on the base currency and a long position on the portfolio. If, on the other hand, $i_x > \Sigma \beta_j i_j$, then the opposite positions should be taken.
An illustration
This kind of arbitrage operation does not work when the base currency is a composite currency like the former European currency unit (ECU) or the IMF’s special drawing rights (SDR) because the interest rates on composite currencies like these two are calculated as a weighted average as in equation (3.52). However, there are other pegged currencies (such as the Kuwaiti dinar, KWD) for which equation (3.52) is not satisfied, which makes arbitrage along the lines suggested earlier a profitable operation. Perhaps this is why the structure of the basket to which the Kuwaiti dinar is pegged is not publicly known, unlike the case for the ECU and the SDR. If the structure of the basket is unknown then the \( a_j \)s are not known, so the arbitrager would not know in advance whether or not the condition \( a_j = \beta_j \) is satisfied. However, the \( a_j \)s can be estimated empirically from equation (3.36) based on observed data.

In this illustration, we use the KWD as the base currency and monthly data covering the period March 1993–June 1999. Previous research has revealed that the basket to which the KWD is pegged contains the currencies of the top four exporters to Kuwait: the US, Japan, Germany and the UK. Moosa (2001a) has estimated the weights of the four currencies in the KWD basket to be 0.824, 0.05796, 0.05606 and 0.06554 respectively. Figure 3.6(a) shows that during the 1990s the interest rate on the KWD (\( i_x \) in the model) was higher than the weighted average interest rate on the component currencies. The differential was almost five percentage points in early 1993, as shown in Figure 3.6(b). Arbitrage would in this case require taking short positions on the four currencies in exactly the same percentages as the weights stated above, and an equivalent long position on the KWD. Figure 3.6(c) shows that the exchange rate factor, \( C \), resulting from such an operation seems to be negligible and zero towards the end of the sample period. This observation confirms the predictions of the model. Figure 3.6(d) shows the uncovered margin (net profit) that would have resulted from an operation like this in the 1990s, as calculated from equation (3.48). Obviously, an operation like this would have been rather lucrative in early 1990s. Towards the end of the 1990s, the profitability of the operation would have been lower owing to the convergence of the KWD interest rate on the weighted average interest rate of the basket component currencies.

3.9 MISCONCEPTIONS ABOUT ARBITRAGE

Arbitrage is commonly defined as “the act of simultaneously buying and selling the same or equivalent assets or commodities for the purpose of making certain, guaranteed profits” (Eun and Resnick, 1998, p 104). The Palgrave Dictionary defines an arbitrage opportunity as “an investment strategy that guarantees a positive payoff in some contingency with no possibility of a negative payoff and with no net investment” (Dybvic and Ross,
MISCONCEPTIONS ABOUT ARBITRAGE

(a) One month interest rates

(b) The interest rate differential (KWD-basket)

FIGURE 3.6 Uncovered arbitrage when the base currency is pegged to a basket (continued overleaf).

1993). These two definitions and several of their variants give arbitrage its perceived characteristics: (i) that it is a riskless operation, (ii) that it does not require net investment or the use of own capital, and (iii) the simultaneity of the buy and sell transactions. It is argued here that these assertions are questionable and that the definitions are misrepresentations that cannot be generalised.

Let us start by examining some types of arbitrage to see if they fit the generic definitions stated earlier. Consider first two-point arbitrage in the foreign exchange market. In this case arbitragers buy a currency in a financial centre where it is cheap and sell it simultaneously where it is more expensive. The simultaneity of the buy and sell transactions is obvious in the description of this operation, but it is not clear whether or not the condition of not using own capital is satisfied. Since the settlement of foreign exchange transactions takes
two business days, not using own capital requires the arbitrager to transfer the funds received from the selling of the currency to the counterparty from whom it was bought. Thus, not using own capital requires the arbitrager to make the payment for the buy transaction only after receiving the proceeds from the sell transaction. A problem could arise here when the two financial centres involved in the operation are in two different far-away time zones (for example, Tokyo and New York). This task will be even more difficult in the case of three-point arbitrage, which entails selling a currency and then buying it back by going through another two currencies and three separate sell-buy transactions.

To emphasise the point that risk is involved in this kind of operation, consider the origin of the concept of “Herstatt risk”. In 1974, a small bank in
Germany, Bankhaus Herstatt, was closed by the authorities in the middle of the day. The bank was insolvent, so it was unable to pay the dollars it owed on its foreign exchange deals. Even now banks continue to be exposed to Herstatt risk because of different time zones. This is because banks in New York do not receive dollars for the yens they paid in the morning or the euros they paid in the afternoon. Thus arbitrage operations are subject to Herstatt risk.

Consider now covered interest arbitrage, which consists of taking a short position on one currency and a long position on another currency while covering the long position in the forward market. Not using own funds is possible only if the arbitrager can borrow funds. However, the textbook exposition of covered interest arbitrage invariably starts by assuming that an agent (investor) has a certain amount of capital that may be invested in the domestic or foreign markets. This exposition eventually leads to the derivation of the covered interest parity (no-arbitrage) condition. Thus the derivation of this condition, which precludes arbitrage, is typically based on the assumption of using own funds. However, it remains true that the same condition can be derived by assuming that the arbitrager initially borrows funds, as is done in this book. But borrowing funds is not the same as not using own funds in the case of two-point arbitrage, where no borrowing is required.

The no-risk condition is satisfied in the case of covered interest arbitrage, since all of the decision variables (interest and exchange rates) are known in advance. This is not so for uncovered interest arbitrage, in which the long position is not covered in the forward market. Uncovered interest arbitrage is risky because one of the decision variables is the expected spot exchange rate. Hence uncovered arbitrage is a speculative activity that does not satisfy the no-risk condition. Moreover, a significant length of time elapses between the buy and sell transactions. Simultaneity is far from being the case here. The same arguments is valid for intertemporal commodity arbitrage.

Finally, consider arbitrage in the spot and forward commodity markets. In this case arbitrage is triggered by the violation of the cost of carry relationship (see, for example, Moosa and Al-Loughani (1995) and Moosa (2000b)). If the spot price of a commodity plus the cost of carry is lower than the forward selling price then arbitragers will make profit by buying spot and selling forward. Otherwise, they will make profit by buying forward and short selling spot. While simultaneity is obvious, and so is the absence of price risk, non-use of own capital does not seem to be the case.

Shleifer and Vishny (1997) present a strong critique of the textbook definition of arbitrage, particularly the propositions that it requires no capital and entails no risk. They argue that almost all arbitrage operations require capital and that it is typically risky. They further argue that arbitrage is invariably conducted by a relatively small number of highly specialised investors using other people’s capital. They present some examples to demonstrate the proposition that the textbook definition of arbitrage does not describe realistic trades and that the discrepancies (price anomalies) become particularly...
important when arbitragers manage other people’s funds. They further point out, by referring to the futures markets, that arbitragers can in reality incur losses because two futures contracts traded on two different exchanges have somewhat different trading hours, settlement dates and delivery terms. If the prices move rapidly, the value of assets an arbitrager delivers and the value of assets delivered to him may differ, exposing him to additional risk of losses.

So, is there after all a generic definition of arbitrage? Although the term “arbitrage” is used to describe a number of different operations, it may still be possible to come up with a generic definition. Such a definition would be based on two characteristics that are common to all of the operations described earlier. The first is that these operations aim at exploiting price anomalies in one or more markets. The second is that each operation is triggered by the violation of a pricing equilibrium condition. This condition is the equality of the exchange rates across financial centres in the case of two-point arbitrage; the consistency of cross rates in the case of three-point arbitrage; the equality of the forward spread and the interest rate differential (covered interest parity) in the case of covered interest arbitrage; and the equality of the expected change in the exchange rate and the interest rate differential (uncovered interest parity) in the case of uncovered interest arbitrage. In all cases, arbitrage restores the equilibrium condition by changing the forces of supply and demand in the underlying markets. Hence a plausible generic definition of arbitrage is the following. Arbitrage is a profit-seeking operation aimed at exploiting price anomalies arising from the violation of a pricing equilibrium condition. This definition says nothing about the absence of risk, the use of own capital or the simultaneity of buy and sell transactions. It is the closest thing to a valid generic definition of arbitrage.
4.1 DEFINITION AND MEASUREMENT OF FOREIGN EXCHANGE RISK

Foreign exchange risk arises because of uncertainty about the exchange rate prevailing in the future (after a decision involving exchange rate expectations has been taken, such that the outcome depends on the materialisation or otherwise of the expectations). It refers to the variability of the base currency value of assets, liabilities and cash flows (contractual or otherwise) resulting from the variability of the exchange rate. Therefore foreign exchange risk arises when a firm indulges in international operations involving currencies other than the base currency, including importing, exporting, investing and financing. As a result, the firm will be exposed to assets, liabilities and cash flows denominated in currencies other than the base currency. We have to remember that foreign exchange risk is associated with unanticipated changes in exchange rates, since anticipated changes are discounted and reflected in the value of the firm.

The concept of foreign exchange risk will be illustrated by referring to an investment decision. Assume that an investor with a base currency $x$ takes up an investment in a $y$-denominated asset at time $t$, maturing at time $t + 1$. If $V_x$ and $V_y$ are the $x$ and $y$ values of the asset respectively, then

$$V_x = S(x/y)V_y$$

(4.1)

In what follows, we will for simplicity drop $(x/y)$ from the symbol representing the spot exchange rate, but it is crucial to bear in mind that the exchange rate in this analysis is measured as the $x$ price of one unit of $y$. The rate of return on the asset between $t$ and $t + 1$ in terms of $x$ is given by
\[
(1 + R) = \frac{V_{x,t+1}}{V_{x,t}} = \frac{S_{t+1}V_{y,t+1}}{S_t V_{y,t}}
\]  

(4.2)

or

\[
(1 + R) = (1 + \hat{S})(1 + \hat{V}_y)
\]  

(4.3)

where \( \hat{S} \) and \( \hat{V}_y \) are respectively the percentage changes in the exchange rate and the \( y \)-denominated value of the asset between \( t \) and \( t + 1 \).

Now, the value at \( t + 1 \) of the asset in terms of currency \( y \) may or may not be known at time \( t \), which is the present time or the time at which a position in the asset is taken. For example, it will be known if the asset is a fixed-income security (the market value plus the value of the accumulated coupon payments), but it will not be known if the underlying asset is a share, an option or real estate. Let us, for the simplicity of the exposition, assume that the \( y \) value of the asset is known at time \( t \). Even if this is the case, the rate of return in terms of currency \( x \) will not be known at time \( t \) because the rate of change of the exchange rate between \( t \) and \( t + 1 \) is not known. In other words, \( V_{x,t+1} \) is not known at time \( t \) because \( S_{t+1} \) and therefore \( \hat{S} \) are not known. This is the source of foreign exchange risk. Of course, the same argument applies to liabilities and cash flows.

**Risk measurement**

How do we measure risk? In finance, risk is normally measured by the variance or the standard deviation of some variable, which in this case is the rate of change of the spot exchange rate or its level at \( t + 1 \) (\( S_{t+1} \) or \( \hat{S} \)). We will use the rate of change for the purpose of this exposition. The variance or the standard deviation is a measure of risk because it represents the dispersion of the rate of change around its mean value. The mean and standard deviation can be calculated in two ways: either from a probability distribution derived from some perceived scenarios or from historical data.

Let us first consider the scenario approach. Assume that the percentage change in the exchange rate is expected to assume \( n \) possible values, \( \hat{S}_i \), with a corresponding probability, \( p_i \), such that \( i = 1, 2, ..., n \) and \( \Sigma p_i = 1 \). In this case, the expected value of the rate of change of the exchange rate is calculated as

\[
E(\hat{S}) = \sum_{i=1}^{n} p_i(\hat{S}_i)
\]  

(4.4)

whereas the variance and standard deviation are calculated respectively as

\[
\sigma^2(\hat{S}) = \sum_{i=1}^{n} p_i [(\hat{S}_i - E(\hat{S}))^2]
\]  

(4.5)
\[ \sigma(S) = \sqrt{\frac{\sum_{i=1}^{n} p_i [\hat{S}_i - E(S)]^2}{n}} \]  

(4.6)

such that a higher variance or standard deviation implies a higher degree of risk.

The second approach is to utilise historical data. In this case the concept of the mean is used instead of the concept of expected value. Let us assume that we have a sample of historical observations on the percentage change in the exchange rate, \( S_t \), where \( t = 1, ..., n \). The mean value is calculated as

\[ \bar{S} = \frac{1}{n} \sum_{t=1}^{n} S_t \]  

(4.7)

whereas the variance and standard deviation are calculated respectively as

\[ \sigma^2 (\hat{S}) = \frac{1}{n-1} \sum_{t=1}^{n} (\hat{S}_t - \bar{S})^2 \]  

(4.8)

\[ \sigma(\hat{S}) = \sqrt{\frac{1}{n-1} \sum_{t=1}^{n} (\hat{S}_t - \bar{S})^2} \]  

(4.9)

Figure 4.1 shows the probability distribution or (in the case of historical data) the frequency distribution of the percentage change in the exchange rate under high and low foreign exchange risk. In the case of high foreign exchange risk, the percentage change in the exchange rate takes a wider range of values (extending between C and D) than in the case of low foreign exchange risk (extending between A and B). The total rate of return or the

**FIGURE 4.1** The probability (frequency) distribution of the percentage change in the exchange rate.
value of the asset will have a corresponding probability or frequency distribution. It should be observed that the mean or the expected value (the value on the vertical axis corresponding to the highest probability or frequency) is equal for both distributions. Table 4.1 displays the basic statistics, including the means and standard deviations, of the quarterly changes of the US dollar exchange rates of nine currencies over the period 1990–2000. It can be seen that the Canadian dollar is the most stable currency against the US dollar, whereas the yen is the most volatile (these exchange rates have the lowest and highest standard deviations respectively).

The standard deviation as a measure of risk has been criticised for the arbitrary manner in which deviations from the mean are squared and for treating positive and negative deviations in a similar manner, although negative deviations are naturally more detrimental. To meet these criticisms, one may use the mean absolute deviation (MAD) or the downside semi-variance (DSV), which are respectively given by

\[
MAD(\hat{S}) = \frac{1}{n} \sum_{i=1}^{n} |\hat{S}_i - \bar{S}|
\] (4.10)

\[
DSV(\hat{S}) = \frac{1}{n-1} \sum_{i=1}^{n} X_i^2
\] (4.11)

where \(X_i = \hat{S}_i - \bar{S}\) if \(\hat{S}_i < \bar{S}\) and \(X_i = 0\) otherwise.

A more general measure of dispersion is given by

\[
D = \int_{-\infty}^{\theta} (\theta - \hat{S})^\alpha f(\hat{S})d\hat{S}
\] (4.12)

where the parameter \(\alpha\) describes the attitude towards risk and \(\theta\) specifies the cut-off between the downside and the upside that the decision maker is and is not concerned about respectively. Many risk measures (including the downside semi-variance) are special cases of, or closely related to, this measure.
4.2 VALUE AT RISK

Value at risk (VAR) is a new approach to risk management that has been accepted by practitioners and regulators as the “right” way to measure risk. For example, the Bank for International Settlements (BIS) has allowed banks to use their own models of VAR to set the capital requirements for market risk. This measure of risk focuses on the tail of the distribution of the rate of return (or the profit/loss or the percentage change in the exchange rate), which means that the emphasis is placed on the worst given percentages of outcomes. In what follows, a brief (and perhaps superficial) treatment of VAR is presented. For more details and extensions, the reader is referred to KPMG-Risk (1997) and Dowd (2002).

Essentially, this approach is used to answer the question “over a given period of time with a given probability, how much money might be lost?”. The money lost pertains to the decline in the value of a portfolio, which may consist of a single asset or a large number of assets. The measurement of VAR requires the choice of: (i) a measurement unit, normally the base currency; (ii) a time horizon, which could be a day, a week or longer provided that the composition of the portfolio does not change during this period; and (iii) a probability, which normally ranges between 1% and 5%. Hence VAR is the maximum expected loss over a given holding period at a given level of confidence (that is, with a given probability).

VAR as a measure of risk is illustrated in Figure 4.2 for a position in currency $y$ when the base currency is $x$. This figure shows a probability distribution (or frequency distribution) for the percentage change in the exchange rate, $\delta$, over a chosen holding period. At a particular confidence level (say, $c$ per cent), VAR is calculated by identifying the point on the $x$-axis that cuts off the top $c$ per cent of observations from the bottom $1 - c$ per cent tail, which is $-a$. The maximum possible loss would, therefore, be obtained by applying this VAR factor to the size of the position ($VR = aK$, where $K$ is the size of the position). If the distribution is that of the size of profit/loss, then VAR can be read directly from the distribution (it would be $a$).

Implementation of VAR analysis

There are at least three approaches to the implementation of VAR analysis, all of which involve the estimation of the statistical distribution of asset returns: these are the parametric (or analytical) approach, the historical approach and the simulation approach. These approaches will be discussed briefly here. For more details, see Dowd (1998, 2002), Hendricks (1996) and KPMG-Risk (1997).

The main assumption of the parametric VAR is that the distribution of asset returns is normal. If the percentage change in the exchange rate is normally distributed, then 95% of the observations will fall within 1.96 standard deviations of the mean and 98% will fall within 2.33 standard deviations of the mean. By taking the latter figure, this means that 98% of all observations fall
between $\bar{S} - 2.33\sigma(\bar{S})$ and $\bar{S} + 2.33\sigma(\bar{S})$. Thus, VAR with a probability of 1% (a confidence level of 99%) is equal to $-K[(\bar{S} - 2.33\sigma(\bar{S})]$ or $2.33\sigma(\bar{S})K$ if $\bar{S} = 0$, where $K$ is the size of the position. Table 4.2 shows the quarterly VARs, with probabilities of 1% and 2.5%, on a position of USD1,000,000 from the perspectives of investors with nine different base currencies. It can be seen that the lowest VAR is found when the base currency is the Canadian dollar, because the CAD/USD rate is the most stable rate, and the highest when the base currency is the Japanese yen.

One problem with the parametric approach is the assumption of normally distributed returns. It has for a long time been established that this is not the case and that the distributions of asset returns have fat tails and tend to be skewed to the left (for example, Mandelbrot (1963) and Fama (1965)). Because of this deviation from the normal distribution, the historical method may be preferred. In this case, VAR (with a probability of 1%) can be calculated by identifying the lowest 1% of the percentage change in the exchange rate, and then applying this value to the size of the position.

The third approach is the simulation approach. Instead of calculating VAR on the basis of the historical rates of change in the exchange rate or by assuming that they are normally distributed, this approach is based on their simulated values.

**Pros and cons of the VAR methodology**

Value at risk has become a widely used method for measuring financial risk, and justifiably so. The attractiveness of the concept lies in its simplicity, as it represents the market risk of the entire portfolio by one number that is easy to comprehend. It thus conveys a simple message on the risk borne by a firm or
an individual. The concept is also suitable for risk limit setting and for measuring performance based on the correlation between the return earned and the risk assumed. Moreover, it can take account of complex movements such as non-parallel yield curve shifts. In general, it has two important characteristics: (i) it provides a common consistent measure of risk across different positions and risk factors; and (ii) it takes into account the correlations between different risk factors (for example, different currencies).

There are, however, several shortcomings associated with the VAR methodology. First, it can be misleading to the extent of giving rise to unwarranted complacency. And, as we have seen, the VAR is highly sensitive to the assumptions used to calculate it. Jorion (1996) argues that VAR is a number that itself is measured with some error or estimation risk. Thus, the VAR results must be interpreted with reference to the underlying statistical methodology. Moreover, this approach to risk management cannot cope with sudden and sharp changes in market conditions. It neglects the possibility of discrete, large jumps in financial prices (such as exchange rates), which occur quite often. Losses resulting from catastrophic occurrences are overlooked due to dependence on symmetric statistical measures that treat upside and downside risk in a similar manner.

VAR is useful, but it should be handled with care and should be used in conjunction with other measures of risk. For example, it can be complemented by a series of stress tests that account for extremely unfavourable market conditions. It is imperative, however, that VAR should not be viewed as a strict upper bound on the portfolio losses that can occur.

**Expected tail loss**
The expected tail loss (ETL) is a measure of risk that is also known as expected shortfall, conditional VAR, tail conditional expectation, and worst conditional expectation. The concept is very simple: ETL is the expected value of a loss that is in excess of VAR. It is defined formally as

<table>
<thead>
<tr>
<th>Base currency</th>
<th>2.5% VAR</th>
<th>1% VAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD</td>
<td>82,868</td>
<td>68,216</td>
</tr>
<tr>
<td>CAD</td>
<td>36,105</td>
<td>29,260</td>
</tr>
<tr>
<td>CHF</td>
<td>127,106</td>
<td>105,572</td>
</tr>
<tr>
<td>DKK</td>
<td>104,835</td>
<td>86,520</td>
</tr>
<tr>
<td>GBP</td>
<td>101,144</td>
<td>83,828</td>
</tr>
<tr>
<td>JPY</td>
<td>142,866</td>
<td>120,592</td>
</tr>
<tr>
<td>NOK</td>
<td>98,076</td>
<td>80,612</td>
</tr>
<tr>
<td>NZD</td>
<td>86,297</td>
<td>71,164</td>
</tr>
<tr>
<td>SEK</td>
<td>137,581</td>
<td>113,272</td>
</tr>
</tbody>
</table>
ETL = E(L | L > VAR) \hspace{1cm} (4.13)

While the VAR tells us the most that can be expected to be lost if a bad event does occur, the ETL tells us what we can expect to lose if a bad event does occur.

Kritzman and Rich (2002) argue that viewing risk in terms of the probability of a given loss or the amount that can be lost with a given probability at the end of the investment horizon is wrong. This view of risk, according to them, considers only the final result, arguing that investors should perceive risk differently because they are affected by risk and exposed to loss throughout the investment period. They suggest that investors consider risk and the possibility of loss throughout the investment horizon; otherwise, their wealth may not survive to the end of the investment horizon. As a result of this way of thinking, Kritzman and Rich suggest two new measures of risk: within-horizon probability of loss and continuous VAR. These new risk measures are then used to demonstrate that the possibility of making loss is substantially greater than what investors normally assume.

**Is VAR used in practice?**

VAR is widely used by major companies in real life. Microsoft, for example, uses VAR as a management tool to estimate its exposure to market risk, reporting the estimated VAR figures in its annual reports. For the purpose of calculating VAR, Microsoft uses a time horizon of 20 days, which is longer than what is typically used by banks. Another difference is that Microsoft uses a confidence level of 97.5% rather than 99%, which is what is used by banks.

In 1999, Moosa and Knight (2002) conducted a survey of the practices of Australian public shareholding companies with respect to the use of value at risk analysis. The results of the survey reveal significant unfamiliarity with VAR analysis, as half of the respondents indicated that they were unaware of the existence of this technique. Financial institutions, those involved in international operations and those using derivatives tend to be more familiar with the technique. Moreover, not all of the companies with VAR awareness actually use the technique for measuring risk. The results of the survey produced several findings, including the following: (i) companies that do not use VAR mostly employ scenario analysis; (ii) those not using VAR claim that it is not relevant to their operations; (iii) those using VAR predominantly employ the parametric approach; and (iv) the majority of users employ back testing and stress testing. The results also revealed that financial institutions, companies involved in international operations and those using derivatives are more aware of the existence of VAR analysis and more inclined to use it, and that companies not using VAR but intending to use it are predominantly those involved in international operations and those using derivatives.

According to the Reserve Bank of Australia (2000), the VAR of major Australian banks was 0.02% of their capital base, much lower than the corresponding
4.3 MEASUREMENT OF EXPOSURE TO FOREIGN EXCHANGE RISK

Foreign exchange exposure is a measure of the sensitivity of the base currency values of assets, liabilities and cash flows to changes in the exchange rate. For example, we consider asset exposure and operating exposure when respectively assets and operating cash flows are sensitive to changes in exchange rates. Also, we refer to a long exposure to assets and cash inflows, and to a short exposure to liabilities and cash outflows. Exposure may also refer to what is at risk (the foreign currency amount exposed). Whether exposure refers to what is at risk or the sensitivity of the base currency value depends on whether changes in this value and in the exchange rate are measured in absolute or percentage terms.

Let us go back to equation (4.1), which tells us that for each $V_y$ there is a corresponding $V_x$, given the value of the exchange rate, $S$. So, as $S$ rises (currency $y$ appreciates), $V_x$ will also rise. However, the underlying assumption here is that changes in the exchange rate do not affect $V_y$, and this is not necessarily true. Changes in the exchange rate tend to affect $V_y$, and this effect may be conspicuous or less so. For example, if the underlying asset is a bond, then $V_y$ may be affected by changes in the exchange rate if these changes affect interest rates (via central bank intervention, for example). The same would be true if the underlying asset is a share, as it is plausible to imagine that changes in the exchange rate affect the stock market. In any case, since $V_x$ depends on the exchange rate, exposure is a measure of the sensitivity of $V_x$ with respect to the exchange rate. Hence we may define exposure as

### TABLE 4.3 VAR of major banks.

<table>
<thead>
<tr>
<th>Country</th>
<th>Foreign exchange risk (% of capital)</th>
<th>Total market risk (% of capital)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.02</td>
<td>0.08</td>
</tr>
<tr>
<td>Canada</td>
<td>0.02</td>
<td>0.12</td>
</tr>
<tr>
<td>Germany</td>
<td>0.15</td>
<td>0.71</td>
</tr>
<tr>
<td>Japan</td>
<td>0.07</td>
<td>0.31</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.02</td>
<td>0.20</td>
</tr>
<tr>
<td>UK</td>
<td>0.03</td>
<td>0.16</td>
</tr>
<tr>
<td>USA</td>
<td>0.03</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Source: Reserve Bank of Australia (2000)
\[ E = \frac{dV_x}{dS} \]  

(4.14)

and in discrete form as

\[ E = \frac{\Delta V_x}{\Delta S} \]  

(4.15)

in which case the exposure is measured in terms of currency \( y \) (hence, the amount exposed, or at risk). Alternatively, the expression can be written in terms of the (percentage) rates of change, which gives

\[ E = \frac{\dot{V}_x}{S} \]  

(4.16)

It is obvious from equation (4.16) that exposure in this case is measured without units, more or less like an elasticity. But in all cases, the exposure is the slope of the line (or curve) representing the relationship between \( V_x \) and \( S \). Notice, in general, that

\[ V_y = f_1(S) \]  

(4.17)

and

\[ V_x = f_2(S) \]  

(4.18)

Therefore

\[ V_x = SV_y = Sf_1(S) \]  

(4.19)

If, for example, \( V_x = \alpha + ES + \epsilon \), then the exposure coefficient, \( E \), is calculated as

\[ E = \frac{\sigma(V_x, S)}{\sigma^2(S)} \]  

(4.20)

such that

\[ \sigma^2(V_x) = E^2 \sigma^2(S) + \sigma^2(\epsilon) \]  

(4.21)

where \( \sigma^2(\cdot) \) and \( \sigma(\cdot, \cdot) \) are the variance and covariance respectively, and \( \sigma^2(\epsilon) \) captures the residual variability that is independent of exchange rate movements. In what follows, we consider some possibilities for \( f_1(S) \) and \( f_2(S) \).

**The effect of the exchange rate on the \( y \)-denominated value**

We will consider four cases involving linear relationships between \( V_y \) and \( S \), some of which produce nonlinear relationships between \( V_x \) and \( S \). The list of possibilities presented here is not exhaustive. Because the relationship between \( V_y \) and \( S \) can take several shapes and forms, foreign exchange exposure can be zero, constant or variable. We will now consider the four cases by looking at an asset (that is, a long exposure), but the same description applies to a liability (that is, a short exposure).
**Case 1: \( V_y \) is independent of the exchange rate**

If \( V_y \) is constant, assuming the same value at any level of the exchange rate, then

\[
V_y = K \tag{4.22}
\]

Hence

\[
V_x = KS \tag{4.23}
\]

and

\[
\frac{dV_x}{dS} = K \tag{4.24}
\]

which means that the exposure does not change with the level of the exchange rate. This case is represented diagrammatically in Figure 4.3. The upper part of the diagram shows that \( V_y \) is independent of \( S \), and this is why the relationship is represented by a horizontal line. The bottom part of the diagram shows the derived relationship between \( V_x \) and \( S \) (equation 4.23). As the exchange rate rises, \( V_y \) is unaffected but \( V_x \) rises. Thus, the relationship between \( V_x \) and \( S \) is represented by the line \( V_x = KS \), and the exposure is represented by the slope of this line, which is constant at \( K \). Notice that \( V_x \) is represented in the upper part of the diagram by the area under the line \( V_y = K \), whereas in the bottom part it is measured on the vertical axis.

**Case 2: \( V_y \) is inversely proportional to the exchange rate**

In this case \( V_y \) falls (rises) when the exchange rate rises (falls) such that the change is proportional (equal percentage changes in opposite directions). This means that \( V_y \) and \( S \) are related as

\[
V_y = \frac{K}{S} \tag{4.25}
\]

which is a rectangular hyperbola indicating that the product of \( V_y \) and \( S \) (which is the area under the curve) is constant. Hence

\[
V_x = K \tag{4.26}
\]

and

\[
\frac{dV_x}{dS} = 0 \tag{4.27}
\]

which means that the exposure is zero (there is nothing at risk or that \( V_x \) is insensitive to changes in \( S \)). This is because any change in the exchange rate is completely offset by a change (in the opposite direction) in \( V_y \), leaving \( V_x \) unchanged. This case is represented by Figure 4.4, where a rise in the exchange rate leads to a proportional fall in \( V_y \), leaving \( V_x \) unchanged. Hence the exposure is zero because the slope of the line \( V_x = K \) is zero.
Case 3: $V_y$ is negatively and linearly related to the exchange rate
If $V_y$ is a decreasing linear function of the exchange rate then

$$V_y = a - bS$$  \hspace{1cm} (4.28)

where $b > 0$. Equation (4.28) is represented by a downward-sloping line in the upper part of Figure 4.5. In this case $V_x$ is a nonlinear function of $S$ that can be written as
4.3 MEASUREMENT OF EXPOSURE TO FOREIGN EXCHANGE RISK

\[ V_x = aS - bS^2 \]  \hspace{1cm} (4.29)

as shown in the bottom part of Figure 4.5 (the lowest value on the vertical axis on which \( V_x \) is measured is \( a \), not 0). Starting from a low level, as the exchange rate rises \( V_x \) also rises, as represented by a larger area under the line \( V_y = a - bS \). But as the exchange rate rises further, \( V_x \) reaches a maximum and starts to decline. This is because as \( S \) rises, the product of \( S \) and \( V_y \) may rise or fall, depending on its initial value.

**FIGURE 4.4** Exposure when \( V_y \) is Inversely proportional to the exchange rate.
Case 4: $V_y$ is positively and linearly related to the exchange rate
If $V_y$ is an increasing linear function of the exchange rate then

$$V_y = a + bS$$

which is represented by an upward-sloping line in the upper part of Figure 4.6. In this case $V_x$ is a nonlinear function of $S$, which is written as

$$V_x = aS - bS^2$$
as shown in the bottom part of Figure 4.6 (the lowest value on the vertical axis on which $V_x$ and $V_y$ are measured is $a$, not 0). As the exchange rate rises, $V_x$ also rises, as represented by a larger area under the line $V_y = a + bS$. It is important to observe that as the exchange rate rises, $V_x$ rises more proportionately. The curve in the bottom part of Figure 4.6 has an increasing positive slope, implying that exposure increases as the exchange rate rises.
The exposure line
Let us now concentrate on the case when exposure is constant, as in Figure 4.3, expressing the relationship in terms of the changes in $V_x$ and $S$ ($\Delta V_x$ and $\Delta S$) rather than their levels. This relationship is represented diagrammatically in Figure 4.7, where changes in the exchange rate are measured on the horizontal axis and changes in the base currency value of the asset are measured on the vertical axis. The line representing this relationship is called the exposure line. In this case, the exposure line has a positive slope to indicate a positive relationship between changes in the exchange rate and changes in the base currency value of the asset. The equation of the exposure line is

$$\Delta V_x = E\Delta S$$

(4.32)

where the slope of the line, $E$, is the exposure. In the case of a long exposure, as shown in Figure 4.7, $E > 0$. Notice that there are two lines: a steep line representing high exposure and a shallow line representing low exposure. Hence zero exposure would be represented by a horizontal line, whereas an infinite exposure would be represented by a vertical line.

We now consider the case of exposure to foreign liabilities (short exposure), as shown in Figure 4.8. In this case, a rise in the exchange rate induces a rise in

![Figure 4.7](image-url)
the base currency value of liabilities, which entails a loss. This is why the exposure line in this case is downward-sloping. It has the same equation as (4.32) except that \( E < 0 \).

The relationship between risk and exposure can be determined from equation (4.32), which gives

\[ \sigma^2(\Delta V_x) = E^2 \sigma^2(\Delta S) \]  

Equation (4.33) tells us that the variance of changes in the base currency value of foreign assets and liabilities is related to the variance of changes in the exchange rate by a factor that reflects exposure, \( E^2 \).

**Multiple exposure**

So far we have dealt with exposure to a single currency, \( y \). In practice, exposure to several currencies is normally the case, as international business firms diversify their investment and financing portfolios. Hence a multiple exposure model may be written as

\[ V_x = a_1 \hat{S}(x/y_1) + a_2 \hat{S}(x/y_2) + \ldots + a_n \hat{S}(x/y_n) \]  

where \( \hat{S}(x/y_i) \) is the percentage change in the exchange rate between the base currency, \( x \), and currency \( y_i \) for \( i = 1, 2, \ldots, n \). The coefficient \( a_i \) (\( i = 1, 2, \ldots, n \))
measures the exposure to currency \( y_i \). In the case of assets, the coefficients are positive, whereas in the case of liabilities they are negative.

There are at least two problems with the empirical model of multiple exposure that is represented by equation (4.34). The first problem is that the model assumes a linear exposure, which may not be the case. A nonlinear exposure may arise because of nonlinearity in the firm’s price elasticity of demand. The second problem is that exposure may not be constant over time, whereas this model would produce constant exposures if it is estimated by a conventional estimation method, such as OLS. This problem can be circumvented by resorting to an estimation method that allows the estimated coefficients to vary over time. This can be accomplished by specifying a state space model that can be estimated by utilising the recursive method of the Kalman filter (see Chapter 6).

The model represented by equation (4.34) can be used to formulate general multicurrency exposure relationships. If we measure value in terms of currency \( y_1 \), then we have

\[
\hat{V}_{y_1} = b_1 \hat{S}(y_1/x) + b_2 \hat{S}(y_1/y_2) + \ldots + b_n \hat{S}(y_1/y_n) \] (4.35)

\[
\hat{V}_{y_2} = c_1 \hat{S}(y_2/x) + c_2 \hat{S}(y_2/y_1) + \ldots + c_n \hat{S}(y_2/y_n) \] (4.36)

If, for example, \( x \) is the US dollar and \( y_1 \) is the pound, then equation (4.34) relates changes in the US dollar value of the (US) company to changes in various exchange rates. If this US company has British shareholders, whose base currency is the pound, then equation (4.35) relates the pound value of the company (which is what matters for British shareholders) to changes in the exchange rates of the other currencies against the pound. The same interpretation can be given to equation (4.36) if \( y_2 \) is, for example, the Japanese yen. Adler and Jorion (1992) have shown that the exposure coefficients of equation (4.34) are related to those of equation (4.35) as follows

\[ (b_2, b_3, \ldots, b_n) = (a_2, a_3, \ldots, a_n) \] (4.37)

and

\[ b_1 = 1 - (a_2 + a_3 + \ldots + a_n) \] (4.38)

### 4.4 TRANSACTION EXPOSURE

Transaction exposure to foreign exchange risk arises if payables and receivables (cash inflows and outflows) are denominated in a currency that is different from the base currency. It measures the sensitivity of the base currency value of contractual cash flows to changes in the exchange rate, which means that it can be determined from accounting statements. It is, therefore, a cash flow exposure that may be associated with trade flows (resulting from exports and imports) and capital flows (for example, dividends
and interest payments). This kind of exposure arises, for example, from (i) a foreign currency asset or a liability that is already recorded on the balance sheet; and (ii) a contract or an agreement involving a future foreign currency cash flow. Thus, transaction exposure is specific to transactions where the risk arises from the future value of the exchange rate. It therefore corresponds to Case 1 in the previous section, where the \( y \)-value of the cash flow is known precisely but its \( x \)-value is not known in advance because of changes in the exchange rate.

Measurement of the transaction exposure of a multinational firm with subsidiaries requires a calculation of the consolidated net amount in currency inflows and outflows for all subsidiaries. For example, suppose that there are a long position and a short position on currency \( y \) amounting to \( K_1 \) and \( K_2 \) respectively. The exposure in this case is a measure of the sensitivity of the net position to changes in the exchange rate. Hence

\[
E = \frac{d}{dS} [S(K_1 - K_2)]
\]

(4.39)

which obviously shows that if \( K_1 = K_2 \), then \( E = 0 \).

Two points are worthy of consideration here. The first is the degree of variability of each exchange rate. An exposure to a foreign currency that fluctuates sharply against the base currency is a source of more concern than an exposure to a currency that is relatively stable. Second, attention should be paid to the correlation coefficients of the underlying exchange rates. If the exchange rates between the base currency and other currencies are strongly and positively correlated, then the foreign currencies will all depreciate or appreciate against the base currency more or less proportionately. If they are positively but weakly correlated then these currencies will tend to move in the same direction but in different proportions. Negative correlation implies that other currencies move against the base currency in different directions, thus providing some sort of natural hedge.

Perfectly positive correlation leads to a perfect hedge when there are a short position on one currency and an equivalent long position on another currency. Consider a long position on currency \( y \) and a short position on currency \( z \). The \( x \)-currency value of the two positions are

\[
V_{x,t}(y) = V_{y,t}S_t(x/y) \quad (4.40)
\]

\[
V_{x,t}(z) = V_{z,t}S_t(x/z) \quad (4.41)
\]

where \( V_{x,t}(y) \) is the \( x \)-currency value of the \( y \) position at time \( t \) and \( V_{x,t}(z) \) is the same for the \( z \) position. Suppose that \( V_y \) and \( V_z \) are not affected by changes in exchange rates, which gives

\[
V_{x,t+1}(y) = V_{y,t}S_t(x/y)[1 + S(x/y)]
\]

(4.42)
\[ V_{x,t+1}(z) = V_{z,t}S_I(x/z)[1 + \bar{S}(x/z)] \]  

(4.43)

The percentage change in the net position, \( \bar{V}_x \), is given by 

\[ \bar{V}_x = \frac{V_{y,t}S_I(x/y)[1 + \bar{S}(x/y)]}{V_{y,t}S_I(x/y)} - \frac{V_{z,t}S_I(x/z)[1 + \bar{S}(x/z)]}{V_{z,t}S_I(x/z)} \]  

(4.44)

If \( S(x/y) \) and \( S(x/z) \) are perfectly correlated, it follows that \( \bar{S}(x/y) = \bar{S}(x/z) \), which gives \( \bar{V}_x = 0 \). Thus, any profit (loss) on the long position will be exactly offset by the loss (profit) on the short position, so that the change in the net base currency value of the position would be zero.

### 4.5 ECONOMIC AND OPERATING EXPOSURE

Economic exposure arises because changes in exchange rates affect the firm’s domestic and foreign cash flows. This may sound like transaction exposure, but the difference lies in the fact that we are in this case concerned with non-contractual or unplanned future cash flows. These cash flows pertain to sales in foreign and domestic markets, as well as input costs, whether these inputs are domestic or foreign. While transaction exposure arises from transactions that are planned, currently in progress or have already been completed, economic exposure refers to changes in future earning power as a result of changes in exchange rates.

Economic exposure arises because of the effect of changes in exchange rates on a firm’s cash flows (revenues and costs) and its equity value. If the reference is to the firm’s operating cash flows, then the exposure is called operating exposure. If, on the other hand, we are concerned with the firm’s equity value, then it is equity exposure. We will deal with equity exposure in a subsequent chapter, but in this section we concentrate on operating exposure.

Economic exposure is sometimes portrayed as consisting of transaction exposure and operating exposure. Operating exposure can be defined as the extent to which the firm’s operating cash flows would be affected by random changes in exchange rates. Lessard and Lightstone (1986) argue that business executives are more familiar with transaction exposure than with operating exposure. They also argue that operating exposure has become increasingly important because exchange rates have become more volatile and business has become more international.

There are key questions to ask when considering economic exposure. These questions pertain to where the firm produces, where it sells its products, and where it sources its output. They also pertain to whether competition is domestic or foreign and to the currency of pricing. Table 4.4 presents these questions and their implications. Generally speaking, foreign components are associated with higher exposure. This should not be taken to mean that a firm
4.5 Economic and Operating Exposure

<table>
<thead>
<tr>
<th>Question</th>
<th>Implication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Where does the firm sell its products?</td>
<td>Higher exposure from sales in foreign markets</td>
</tr>
<tr>
<td>Who are the competitors?</td>
<td>Higher exposure results if main competitors are foreign</td>
</tr>
<tr>
<td>How sensitive is demand to price?</td>
<td>The higher the elasticity of demand the higher is exposure</td>
</tr>
<tr>
<td>Where are the production facilities located?</td>
<td>Foreign production facilities result in higher exposure</td>
</tr>
<tr>
<td>What are the sources of input?</td>
<td>Sourcing inputs from abroad implies higher exposure</td>
</tr>
<tr>
<td>How are the inputs or outputs priced?</td>
<td>Foreign currency pricing leads to higher exposure</td>
</tr>
</tbody>
</table>

that produces, sells its products and sources its inputs domestically is immune from exposure. Exposure may result from foreign competitors entering the domestic market for exchange rate-related reasons, and from the exposure of other (domestic) firms that the purely domestic firm deals with. Thus, even purely domestic firms that have no cross-border operations may be subject to operating exposure because changes in exchange rates are likely to change its competitive position in the domestic market, affecting its market share, revenue and profit.

Unlike transaction exposure, operating exposure is determined by (i) the structure of the markets in which the firm sources its inputs and sells its products, and (ii) the firm’s ability to mitigate the effects of exchange rate changes by adjusting its markets, product mix and sourcing. Generally speaking, a firm is subject to a high degree of operating exposure when either its costs or its revenues are sensitive to exchange rate changes. On the other hand, when both costs and revenues are sensitive or insensitive to exchange rate changes, the firm has no major operating exposure (zero exposure if they are equally sensitive). Given the market structure, the extent to which a firm is subject to operating exposure depends on its ability to stabilise cash flows in the face of exchange rate changes. One has to remember that financial costs, such as interest on debt, are not relevant in determining operating cash flows. However, we will find out later that interest payments are a determining factor of net cash flows and hence net cash flow exposure.

Consider Figure 4.9, which shows four different possibilities for the effect of changes in the exchange rate on operating cash flows in terms of the base currency, $x$. Specifically, this figure shows the effect of the exchange rate on revenues, $R_x$, costs, $C_x$, and profit, $\pi_x$. In Figure 4.9(a), revenues and costs are equally sensitive to changes in the exchange rate, so profit is the same at any

---

**TABLE 4.4 Questions revealing the extent of operating exposure.**

<table>
<thead>
<tr>
<th>Question</th>
<th>Implication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Where does the firm sell its products?</td>
<td>Higher exposure from sales in foreign markets</td>
</tr>
<tr>
<td>Who are the competitors?</td>
<td>Higher exposure results if main competitors are foreign</td>
</tr>
<tr>
<td>How sensitive is demand to price?</td>
<td>The higher the elasticity of demand the higher is exposure</td>
</tr>
<tr>
<td>Where are the production facilities located?</td>
<td>Foreign production facilities result in higher exposure</td>
</tr>
<tr>
<td>What are the sources of input?</td>
<td>Sourcing inputs from abroad implies higher exposure</td>
</tr>
<tr>
<td>How are the inputs or outputs priced?</td>
<td>Foreign currency pricing leads to higher exposure</td>
</tr>
</tbody>
</table>
level of the exchange rate. In this case, there is no operating exposure, as (net) operating cash flows are not sensitive to changes in the exchange rate. In Figure 4.9(b) costs are not affected by changes in the exchange rate, but revenues rise as the exchange rate rises. This would be the case if the firm sources its inputs domestically and sells its products abroad. Thus, as the exchange rate rises profit rises. Figure 4.9(c) shows the case when both costs and revenues rise with the level of the exchange rate, but revenues rise faster. In this case there is a positive operating exposure, as profit rises with the exchange rate. Finally, Figure 4.9(d) shows the case when costs are more sensitive than revenues. In this case profit declines as the exchange rate rises, reaching zero at a certain level of the exchange rate and turning negative if it rises further.

**Measurement of operating exposure**

Operating exposure can be calculated as the percentage change in the level of the base currency operating cash flows for a given percentage change in the
exchange rate. For the purpose of the following discussion, the level of profit is taken to be the net operating cash flow. If this level changes from \( \pi_{x,t} \) to \( \pi_{x,t+1} \) as the exchange rate changes from \( S_t \) to \( S_{t+1} \), we get

\[
E_{\pi} = \frac{\pi_x}{S} = \frac{(\pi_{x,t+1}/\pi_{x,t}) - 1}{(S_{t+1}/S_t) - 1}
\]  
(4.45)

Equation (4.45) can be rearranged to obtain

\[
\pi_{x,t+1} = \pi_{x,t} \left[ 1 + E_{\pi} \left( \frac{S_{t+1}}{S_t} - 1 \right) \right] \]  
(4.46)

or

\[
\frac{\pi_{x,t+1}}{\pi_{x,t}} - 1 = E_{\pi} \left( \frac{S_{t+1}}{S_t} - 1 \right)
\]  
(4.47)

which means that the percentage change in the base currency cash flow is equal to the operating exposure times the percentage change in the exchange rate.

It is obvious from equation (4.45) that the operating exposure may assume any value: positive, negative or zero. If, for example, \( \pi_x = 0 \) when \( \dot{S} \neq 0 \), then \( E_{\pi} = 0 \). And if \( \pi_x = \dot{S} \), then \( E_{\pi} = 1 \). Finally, if \( \pi_x < 0 \) when \( \dot{S} > 0 \) then \( E_{\pi} < 1 \). Figure 4.10 shows these possibilities in terms of the slope of the operating exposure line.

**Components of the operating exposure**

Consider the following relationships for an exporter whose base currency is \( x \):

\[
\pi_x = R_x - C_x
\]  
(4.48)

\[
R_x = P_x Q
\]  
(4.49)

\[
Q = f(P_y), \quad f' < 0
\]  
(4.50)

\[
P_y = \frac{P_x}{S}
\]  
(4.51)

\[
C_x = F(S), \quad F' > 0
\]  
(4.52)

where \( Q \) is the quantity sold. Equation (4.48) defines profit, or net operating cash flows, in terms of currency \( x \); equation (4.49) defines revenues in terms of currency \( x \); equation (4.50) is a demand function; equation (4.51) relates the \( y \) price to the \( x \) price via the exchange rate; and equation (4.52) is a cost function. Changes in \( S \), therefore, affect \( \pi_x \) via several channels, and this is why there are many components of operating exposure. These components will be discussed in turn.
**Recovery exposure and cost exposure**

Revenue exposure and cost exposure can be defined respectively as:

\[
E_R = \frac{\dot{R}_x}{S} = \frac{(R_{x,t+1}/R_{x,t}) - 1}{(S_{t+1}/S_t) - 1}
\]  \hspace{1cm} (4.53)

\[
E_C = \frac{\dot{C}_x}{S} = \frac{(C_{x,t+1}/C_{x,t}) - 1}{(S_{t+1}/S_t) - 1}
\]  \hspace{1cm} (4.54)

If we define the cash flow margin as \( m = \pi_{x,t}/R_{x,t} \), we can show that the operating exposure is determined by the revenue exposure, cost exposure and the cash flow margin. Since \( R_{x,t} = \pi_{x,t} + C_{x,t} \), it follows that

\[
\dot{R}_x = \pi_x \left[ \frac{\pi_{x,t}}{R_{x,t}} \right] + C_x \left[ \frac{C_{x,t}}{R_{x,t}} \right]
\]  \hspace{1cm} (4.55)
Multiplying both sides of equation (4.55) by $R_{x,t}$ and rearranging, we obtain

$$\dot{\pi}_x \pi_{x,t} = \dot{R}_x R_{x,t} - \dot{C}_x C_{x,t}$$  \hspace{1cm} (4.56)

Dividing by $\dot{S}\pi_{x,t}$, we obtain

$$\frac{\dot{\pi}_x}{\dot{S}} = \frac{\dot{R}_x}{\dot{S}} \left( \frac{R_{x,t}}{\pi_{x,t}} \right) - \frac{\dot{C}_x}{\dot{S}} \left( \frac{C_{x,t}}{\pi_{x,t}} \right)$$  \hspace{1cm} (4.57)

Since $C_{x,t}/\pi_{x,t} = (R_{x,t}/\pi_{x,t})^{-1}$ and $(R_{x,t}/\pi_{x,t}) = 1/m$, it follows that

$$E_{\pi} = \frac{E_R}{m} - \frac{E_C}{m-1}$$  \hspace{1cm} (4.58)

Again, $E_R$ and $E_C$ may assume various values. For example, $E_R = 1$ if the $y$-operating revenues are not affected by changes in the exchange rate, such that the $x$-operating revenues change by the same percentage change as the exchange rate. $E_C$, on the other hand, depends on the geographical distribution of the operations and suppliers. Thus $E_C$ may be zero for a purely domestic firm with no imports of inputs such as raw materials. We say “may be” because $E_C \neq 0$ if the domestic suppliers have operating cost exposures (this is indirect exposure, as we are going to see later). On the other hand, $E_C = 1$ for a firm with a subsidiary producing entirely abroad with inputs that are not imported, and neither are their prices affected by exchange rate changes. In general, cost exposure faced by an importing firm is the opposite of the exporter’s operating revenue exposure (if $E_R = 1$ for an exporter, then $E_C = 0$ for the importer).

**Conversion exposure**

When cash flows in terms of $y$ are not affected by changes in the exchange rate, we have on our hands a pure conversion exposure, which is obtained when $E_R = 1$ and $E_C = 1$. A pure conversion exposure arises if changes in the exchange rate do not affect $P_y$, but affect $P_x$ via the equation $P_x = SP_y$. This may seem like transaction exposure, but it is not. Under transaction exposure, the cash flows are contractual and the conversion takes place. Under operating exposure the cash flows are not contractual and are unknown in advance, and conversion may or may not take place.

**Price exposure**

In general, $\dot{P}_x/\dot{S}$ measures a combination of conversion and price exposures. Let us for this purpose consider the following cases for an exporter to examine the effect of changes in the exchange rate on operating revenues in base currency, $R_x$.

1. The selling price is set in terms of currency $y$, so that $P_y$ is unaffected by changes in the exchange rate.
2. The selling price is set in terms of currency \( x \), so that \( P_x \) is unaffected by changes in the exchange rate.

3. The selling price is set in terms of currency \( x \), but it is changed to offset changes in \( S \) partially.

We will consider what happens when the exchange rate declines (currency \( y \) depreciates) under the three possibilities. These are shown in Figures 4.11, 4.12 and 4.13 respectively. Each of these figures consists of four related parts. The upper right-hand part shows the relationship between \( P_x \) and \( P_y \) expressed as \( P_y = (1/S)P_x \). The slope of the line is \( 1/S \), which means that a decline in the exchange rate is represented by a movement of the line to the left (and vice versa). This is because as \( S \) declines the line becomes steeper. The upper left-hand part is a demand function relating the quantity sold, \( Q \), to \( P_y \). The lower right-hand part is a 45° line (\( P_x = P_y \)) used to rotate the axes, so that \( P_x \) would appear on the vertical axis. Finally, the lower left-hand part shows combinations of \( P_x \) and \( Q \), so that total revenue is represented by the area of the rectangle defined by \( Q \) and \( P_x \), because \( R_x = P_xQ \). As \( S \) changes, \( R_x \) will change as a result of changes in \( P_x \) and/or \( Q \).

Consider Figure 4.11 first, which represents the first case. As the exchange rate declines, the exporter reacts by reducing \( P_x \) proportionately so that \( P_y \) would remain unchanged. Since there is no change in \( P_y \), \( Q \) will not change. Revenue in terms of \( x \) will decline only because of the decline in \( P_x \). The decline in \( R_x \), \( \Delta R_x \), is represented by the shaded area in the lower left-hand part of the diagram.

Consider now Figure 4.12 representing the second case in which the exporter sets the selling price in terms of \( x \). As the exchange rate declines, without a change in \( P_x \), \( P_y \) will rise, leading to a fall in \( Q \). Revenue in terms of \( x \) will decline only because of the decline in \( P_y \). The decline in \( R_x \), \( \Delta R_x \), is represented by the shaded area in the lower left-hand part of the diagram.

The third case is shown in Figure 4.13. As the exchange rate declines, \( P_y \) rises. The exporter reacts by reducing \( P_x \) less than proportionately, so that we have on our hands a change in both \( P_x \) and \( P_y \). Revenue in terms of \( x \) will decline because of the decline in both \( Q \) and \( P_x \), and this is why there are two shaded areas in Figure 4.13: \( \Delta R_x(Q) \) is the decline in revenue resulting from lower \( Q \), whereas \( \Delta R_x P_x \) results from lower \( P_x \). Whether \( \Delta R_x(Q) > \Delta R_x(P_x) \) or \( \Delta R_x(Q) < \Delta R_x(P_x) \) depends on the change in \( P_x \) relative to \( P_y \) and also on the elasticity of demand (the slope of the demand function).

**Demand exposure**

Changes in the exchange rate lead to changes in demand and hence revenue in terms of \( x \), given that \( R_x = P_xQ \), \( Q = f(P_y) \) and \( P_y = (1/S)P_x \). As the exchange rate changes \( P_y \) changes, leading to a change in \( Q \) and hence a change in \( R_x \). Even if \( P_y \) is kept unchanged by changing \( P_x \), changes in \( S \) may lead to changes in other factors that may affect demand.
Demand exposure, measured as $\dot{Q}/\dot{S}$, is determined by the elasticity of demand, as shown in Figure 4.14. The figure shows the possibility of elastic and inelastic demand. As the exchange rate falls, base currency revenue falls as a result of a decline in demand (we are assuming no change in $P_x$). But the extent of the decline in revenue depends on the elasticity of demand. It is shown that the decline is greater under elastic demand, since a rise in $P_y$ brings about a greater decline in quantity under elastic than under inelastic demand. If changes in the exchange rate affect demand via factors other than $P_y$, this effect will be represented by a shift in the demand curve.

**Competitive exposure**

Demand and price exposures may be the result of a firm’s competitive position. If currency $y$ appreciates (the exchange rate rises), more companies with a base currency $x$ would be inclined to compete for business in the country whose currency is $y$. Thus, the demand for existing firms’ products in this
FIGURE 4.12 The effect of a decline in the exchange rate on the revenue of an exporter when the price is set in terms of $x$.

country will decline, as represented by a downward shift in the demand curve, and this will affect revenues, as shown in Figure 4.15. Notice that without a competitive exposure, the rise in the exchange rate leads to a decline in $P_y$ and a rise in base currency revenue for the existing firm.

**Indirect exposure**
A firm that has no direct foreign exchange exposure due to conversion, price, demand or competitive effects could still have an indirect exposure if it is a supplier to firms that have direct exposures. If changes in the exchange rate lead to a fall in the profitability of these firms, the supplier will suffer a reduction in the demand for its products. Again, this is represented by an inward shift in the demand for its products, as shown in Figure 4.15.

**Multimarket exposure**
A firm that derives its revenue by selling in various markets is said to have a multimarket exposure. If this firm operates in a foreign market and the
domestic market, it will have revenue exposure only to the extent of the revenue derived from the foreign market.

### 4.6 A FORMAL TREATMENT OF OPERATING EXPOSURE

We will now examine in a more formal manner the effect of changes in the exchange rate, $S(x/y)$, on the revenues, costs and profits of exporters and importers with a base currency $x$, exporting to and importing from a country with a currency $y$. The analysis will be done both in terms of currency $x$ and currency $y$, although we are primarily interested in the analysis in terms of currency $x$. 

![Diagram](image-url)
Case 1: The exporter in terms of currency $x$
Assume that the firm sells all of its output in the country whose currency is $y$.
Total revenue in terms of $x$, $R_x$, is
\[ R_x = SP_yQ \]  \hspace{1cm} (4.59)
If the unit cost is $c_x$, the total cost of production is
\[ C_x = c_xQ \]  \hspace{1cm} (4.60)
For profit maximisation, marginal cost must be equal to marginal revenue, which gives
\[ \frac{dR_x}{dQ} = \frac{dC_x}{dQ} \]  \hspace{1cm} (4.61)
or

\[ SP_y + SQ \frac{dP_y}{dQ} = SP_y \left(1 + \frac{Q}{P_y} \frac{dP_y}{dQ}\right) = c_x \]  

(4.62)

because \(dS/dQ = 0\). Hence

\[ SP_y \left(1 - \frac{1}{\eta}\right) = c_x \]  

(4.63)

where

\[ \eta = -\frac{dQ}{dP_y} \frac{P_y}{Q} \]  

(4.64)
is the elasticity of demand. Therefore, the profit maximising price in terms of $y$ is

$$p_y = \frac{c_x}{S[1-(1/\eta)]}$$  \hspace{1cm} (4.65)

which says that the exporter should set the price in terms of $y$ according to the unit cost of production (in terms of $x$), the exchange rate, and the elasticity of demand. Assuming that the unit cost and the elasticity of demand are constant, and differentiating equation (4.65) with respect to $S$ gives us

$$\frac{dp_y}{dS} = -\frac{c_x}{S^2 [1-(1/\eta)]} = -\frac{p_y}{S} < 0$$  \hspace{1cm} (4.66)

which follows by substituting the value of $c_x$ from equation (4.63). Equation (4.66) means that an appreciation of $y$ (a rise in $S$) lowers the profit-maximising price in that currency. What happens to total revenue can be gleaned from the equation

$$\frac{dR_x}{dS} = p_y Q + SQ_p \frac{dQ}{dP_y} \frac{dp_y}{dS} + SQ \frac{dp_y}{dS} = p_y Q + SQ(1-\eta) \frac{dp_y}{dS}$$  \hspace{1cm} (4.67)

By substituting the value of $dp_y/dS$ from equation (4.66), we obtain

$$\frac{dR_x}{dS} = p_y Q - SQ(1-\eta) \frac{p_y}{S} = \eta p_y Q > 0$$  \hspace{1cm} (4.68)

which means that an increase in $S$ leads to an increase in sales revenue in terms of currency $x$.

The effect on cost in terms of currency $x$ is also calculated by differentiating $C_x$ with respect to $S$, which gives

$$\frac{dC_x}{dS} = c_x \frac{dQ}{dS} = c_x \frac{dQ}{dp_y} \frac{dp_y}{dS} = -c_x \frac{dQ}{dp_y} p_y S$$  \hspace{1cm} (4.69)

or

$$\frac{dC_x}{dS} = \frac{\eta c_x Q}{S} > 0$$  \hspace{1cm} (4.70)

Now, consider the effect on profit, $\pi_x$, measured as the difference between revenue and cost. Since

$$\pi_x = R_x - C_x$$  \hspace{1cm} (4.71)

it follows that

$$\frac{d\pi_x}{dS} = \frac{dR_x}{dS} - \frac{dC_x}{dS} = \eta Q \left( p_y - \frac{c_x}{S} \right) > 0$$  \hspace{1cm} (4.72)
where the factor in parentheses is the mark-up per unit in currency $y$. If this is positive, then a rise in $S$ would lead to a rise in profit in terms of $x$.

**Case 2: The exporter in terms of currency $y$**

In this case, revenue and costs are given by

$$R_y = P_y Q$$

$$C_y = \frac{c_x Q}{S}$$

For profit maximisation, $dR_y/dQ = dC_y/dQ$, which gives

$$P_y = \frac{c_x}{S[1-(1/\eta)]}$$

Now,

$$\frac{dR_y}{dS} = P_y \frac{dQ}{dS} + Q \frac{dP_y}{dS} = Q(1-\eta) \frac{dP_y}{dS}$$

From equation (4.66), we have $dP_y/dS = -P_y/S$. Hence

$$\frac{dR_y}{dS} = \frac{c_x Q \eta}{S^2} > 0$$

which means that a rise in the exchange rate raises revenue in terms of $y$. We also have

$$\frac{dC_y}{dS} = c_x \frac{d(Q/S)}{dS} = \frac{c_x Q}{S} \frac{dQ}{dS} - \frac{c_x Q}{S^2}$$

Since

$$\frac{dQ}{dS} = \frac{dQ}{dP_y} \frac{dP_y}{dS} = - \frac{dQ}{dP_y} \frac{P_y}{S} = \frac{Q \eta}{S}$$

it follows that

$$\frac{dC_y}{dS} = \frac{c_x Q \eta}{S^2} - \frac{c_x Q}{S^2} = \frac{c_x (\eta-1) Q}{S^2} > 0$$

which means that a rise in the exchange rate raises costs in terms of $y$. Hence, the effect on profit in terms of $y$ is given by

$$\frac{d\tau_y}{dS} = \frac{dR_y}{dS} - \frac{dC_y}{dS} = \frac{c_x Q \eta}{S^2} - \frac{c_x (\eta-1) Q}{S^2} = \frac{c_x Q}{S^2} > 0$$

Equation (4.81) implies that profit in terms of $y$ increases as a result of the depreciation of $x$. 

**Case 3: The importer in terms of currency x**

For the importer, revenue and cost are given by

\[
R_x = P_x Q \quad (4.82)
\]

\[
C_x = Sc_y Q \quad (4.83)
\]

where \( Q \) is now the quantity imported. Hence

\[
\frac{dR_x}{dQ} = P_x + Q \frac{dP_x}{dQ} = P_x \left(1 - \frac{1}{\eta}\right) \quad (4.84)
\]

\[
\frac{dC_x}{dQ} = Sc_y \quad (4.85)
\]

For profit maximisation, \( \frac{dR_x}{dQ} = \frac{dC_x}{dQ} \), which gives

\[
P_x = \frac{Sc_y}{1 - (1/\eta)} \quad (4.86)
\]

Hence

\[
\frac{dP_x}{dS} = \frac{c_y}{1 - (1/\eta)} = \frac{P_x}{S} > 0 \quad (4.87)
\]

which means that a depreciation of \( x \) raises the \( x \)-price of imported goods.

The effect of changes in the exchange rate on revenue is given by

\[
\frac{dR_x}{dS} = P_x \frac{dQ}{dS} + Q \frac{dP_x}{dS} = Q(1 - \eta) \frac{dP_x}{dS} \quad (4.88)
\]

which gives

\[
\frac{dR_x}{dS} = \frac{P_x Q}{S} (1 - \eta) < 0 \quad (4.89)
\]

because \( \eta > 1 \) for any profit maximising firm. This means that a depreciation of \( x \) leads to an a decline in revenue in terms of \( x \).

The effect of a change in the exchange rate on costs in terms of \( x \) is given by

\[
\frac{dC_x}{dS} = c_y Q + Sc_y \frac{dQ}{dS} = c_y Q + Sc_y \frac{dQ}{dP_x} \frac{dP_x}{dS} \quad (4.90)
\]

which can be simplified to

\[
\frac{dC_x}{dS} = c_y Q - c_y Q\eta = c_y Q(1 - \eta) < 0 \quad (4.91)
\]

This means that a rise in the exchange rate reduces the cost of imported goods, in which case the effect on profit is given by
\[
\frac{d\pi_x}{dS} = \frac{dR_x}{dS} - \frac{dC_x}{dS} = Q(1-\eta)\left[\frac{P_x}{S} - c_y\right] < 0
\] (4.92)

because \( \eta > 1 \) and \( \frac{P_y}{S} - c_y > 0 \) for a profit maximiser, which means that a rise in the exchange rate reduces profit.

**Case 4: The importer in terms of currency \( y \)**

In this case, revenue and costs are given by

\[
R_y = \frac{P_xQ}{S} \quad \text{(4.93)}
\]

\[
C_y = c_yQ \quad \text{(4.94)}
\]

For profit maximisation, we have

\[
P_x = \frac{Sc_y}{1-(1/\eta)} \quad \text{(4.95)}
\]

which gives

\[
\frac{dP_x}{dS} = \frac{c_y}{1-(1/\eta)} = \frac{P_x}{S} \quad \text{(4.96)}
\]

We know that

\[
\frac{dR_y}{dS} = S\left(\frac{d(P_xQ)}{dS}\right) - \frac{P_xQ}{S^2} = P_xQ(1-\eta) - \frac{P_xQ}{S^2} \quad \text{(4.97)}
\]

which gives

\[
\frac{dR_y}{dS} = -\frac{P_xQ\eta}{S^2} < 0 \quad \text{(4.98)}
\]

implying that a rise in the exchange rate reduces revenue in terms of \( y \). Likewise

\[
\frac{dC_y}{dS} = c_y \quad \frac{dQ}{dP_x} \quad \frac{dP_x}{dS} = -\frac{c_yQ\eta}{S} < 0 \quad \text{(4.99)}
\]

which again means that a rise in the exchange rate reduces costs in terms of \( y \). Finally, the effect on profit is given by

\[
\frac{d\pi_y}{dS} = \frac{dR_y}{dS} - \frac{dC_y}{dS} = -\frac{P_xQ\eta}{S^2} + \frac{c_yQ\eta}{S} = \frac{Q\eta}{S}\left(c_y - \frac{P_x}{S}\right) < 0 \quad \text{(4.100)}
\]

which means that profit in terms of \( y \) decreases as a result of a rise in the exchange rate. All of the results are summarised in Table 4.5.
TABLE 4.5 The effect of changes in exchange rates on importers and exporters.

<table>
<thead>
<tr>
<th>Item</th>
<th>Effect</th>
<th>An increase in S leads to</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exporters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_x$</td>
<td>$dR_x/dS = \eta P_y Q &gt; 0$</td>
<td>Increase in $R_x$</td>
</tr>
<tr>
<td>$C_x$</td>
<td>$dC_x/dS = \eta c_y Q/S &gt; 0$</td>
<td>Increase in $C_x$</td>
</tr>
<tr>
<td>$\pi_x$</td>
<td>$d\pi_x/dS = \eta (P_y - c_x/S) &gt; 0$</td>
<td>Increase in $\pi_x$</td>
</tr>
<tr>
<td>$R_y$</td>
<td>$dR_y/dS = c_x Q/\eta S^2 &gt; 0$</td>
<td>Increase in $R_y$</td>
</tr>
<tr>
<td>$C_y$</td>
<td>$dC_y/dS = c_x (\eta - \eta) Q/S^2 &gt; 0$</td>
<td>Increase in $C_y$</td>
</tr>
<tr>
<td>$\pi_y$</td>
<td>$d\pi_y/dS = c_x Q/S^2 &gt; 0$</td>
<td>Increase in $\pi_y$</td>
</tr>
<tr>
<td>Importers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_x$</td>
<td>$dR_x/dS = P_x Q(1-\eta)/S &lt; 0$</td>
<td>Decrease in $R_x$</td>
</tr>
<tr>
<td>$C_x$</td>
<td>$dC_x/dS = c_y Q(1-\eta) &lt; 0$</td>
<td>Decrease in $C_x$</td>
</tr>
<tr>
<td>$\pi_x$</td>
<td>$d\pi_x/dS = Q(1-\eta)(P_x/S - c_y) &lt; 0$</td>
<td>Decrease in $\pi_x$</td>
</tr>
<tr>
<td>$R_y$</td>
<td>$dR_y/dS = -P_x Q/\eta S^2 &lt; 0$</td>
<td>Decrease in $R_y$</td>
</tr>
<tr>
<td>$C_y$</td>
<td>$dC_y/dS = -c_y Q/\eta S &lt; 0$</td>
<td>Decrease in $C_y$</td>
</tr>
<tr>
<td>$\pi_y$</td>
<td>$d\pi_y/dS = (Q/\eta S)(c_y - P_x/S) &lt; 0$</td>
<td>Decrease in $\pi_y$</td>
</tr>
</tbody>
</table>

4.7 TRANSLATION EXPOSURE

Translation exposure arises from the consolidation of foreign currency assets, liabilities, net income and other items in the process of preparing base currency consolidated financial statements (balance sheet and income statement). Also called accounting exposure, it may be defined as the potential that a firm’s consolidated financial statements can be adversely affected (showing more inferior figures than otherwise) by changes in exchange rates. Consolidation involves the translation of subsidiaries’ financial statements to the base currency. When exchange rates change, the value of the assets and liabilities of a subsidiary, whose base currency is different from that of the parent firm, may change when viewed from the perspective of the parent firm. In this respect a distinction is sometimes made between the functional currency, which is the currency of the primary economic environment in which the firm (or subsidiary) operates, and the reporting currency, which is the currency in which the firm prepares its consolidated financial statements. In the case of a multinational firm, the functional and reporting currencies correspond to the base currencies of the subsidiary and the parent firm respectively.

Translation exposure gives rise to the possibility that the conversion of foreign currency-denominated items into the base currency for the purpose of consolidation may show a loss or gain. It is, therefore, a function of the
accounting system and may have little to do with the true value in an economic sense. Firms with identical balance sheet and income statement items may show different consolidated results, depending on the translation method used. It is important to bear in mind that the difference between translation and operating exposure is that the measurement of translation exposure is retrospective (as it is based on activities that occurred in the past), whereas the measurement of operating exposure is prospective (as it is based on future activities and hence future cash flows). The measurement of transaction exposure is both retrospective and prospective, because it is based on activities that occurred in the past but will be settled in the future.

The importance of translation exposure lies in the distinction between the economic value and the book value of a firm, which are based on the historical value and future cash flows respectively. The change in accounting net worth produced by a movement in exchange rates often has little relevance to the change in the market value of the firm, because economic exposure is a measure of the extent to which a change in exchange rates affects the present value of future cash flows. Although all items on a firm’s balance sheet represent future cash flows, not all cash flows appear there. Investors may see behind accounting conventions and understand the firm’s true economic situation, even though translation exposure affects the reported financial statements. A problem arises when investors rely on financial statements as a source of fundamental information to the extent that they are unable to discern when financial statements reflect the true economic value and when not. Thus, translation exposure tends to confuse investors to the extent of perceiving it as a real problem itself.

The management of the firm may be concerned, particularly if compensation and performance evaluation are based on reported financial statements. Indeed, managers may not aim at maximising risk-adjusted cash flows, because they are preoccupied with accounting-based foreign exchange gains. Apart from the compensation motive, they may behave in this way because they believe that the stock market evaluates a firm on the basis of its reported earnings or changes in accounting net worth, regardless of the underlying cash flows. It remains true, however, that managers can make serious errors of judgement by failing to distinguish between the accounting description of foreign exchange risk and the effect of exchange rate movements on the economic value of the firm. However, the distortions associated with translation exposure do not mean that accounting statements are irrelevant. A large body of research on financial markets suggests that investors are relatively sophisticated in responding to publicly available information. They appear to understand detailed financial statements and properly interpret various accounting conventions.

Dufey (1978) presents a good example on this issue. Because of an expected devaluation of the French currency, the French subsidiary of a US multinational was instructed to reduce its working capital and, therefore, curtail
operations. Given that the subsidiary was selling all of its output to subsid-
riaries in Germany and Belgium, devaluation would lead to an increase in prof-
itability (the value of output would remain constant, whereas costs would
decline in dollar terms). As a result, the French manager argued correctly for
expanding operations.

Translation methods
Translation methods refer to the choice of the exchange rate used for converting
(translating) the values of foreign currency items into the base currency. The
balance sheet contains the values of assets and liabilities as at the end of the
accounting period (which may be a year, a quarter or a month). The income
statement reports items such as revenues, costs and net income realised over
the accounting period. The following three rates can be used for conversion:

1. The closing (or current) rate, which is the rate prevailing at the end of the
   accounting period (coinciding with the balance sheet date).
2. The average rate, which reflects the average value of the exchange rate over
   the accounting period. The simplest procedure is to take a simple average of
   the closing rate and the rate prevailing at the beginning of the period.
   Otherwise a time-weighted average may be used.
3. The historical rate, which is the rate prevailing on the date when an asset is
   acquired or a liability is committed. The historical rate may, therefore, fall
   outside the current accounting period. In fact, this is invariably the case for
   long-term assets and liabilities.

In translating the income statement items, either the closing rate or the
average rate are used, which means that the amount exposed is net income.
The possibility of using historical rates in translating balance sheet items
makes the matter more complicated. For the purpose of translating balance
sheet items, the following methods are used.

The current/non-current method
The current/non-current method is based on the traditional accounting
distinction between current items (for example, short-term deposits and
inventory) and long-term items (for example, real estate and long-term debt).
According to this method, current items are translated at the closing rate,
whereas long-term items are translated at the historical rate. Obviously, the
use of the historical rate precludes foreign exchange risk, whereas the use of
the closing rate does not. Hence, if this method is used, the amount exposed to
foreign exchange risk is net current assets. A foreign subsidiary with current
assets in excess of current liabilities will cause a translation gain (loss) if its
functional currency appreciates (depreciates). There is an obvious problem
with this method, which is that items such as long-term loans are portrayed as
not being subject to foreign exchange risk, which does not make sense. This is
why there has been a move away from this method.
The current rate method
The current rate method is the most widely used worldwide for its simplicity. All items are translated at the current exchange rate prevailing at the end of the accounting period (the closing rate). When this method is used, the amount exposed is shareholders’ equity. If a firm’s foreign currency-denominated assets exceed its foreign currency liabilities, a depreciation of the foreign currency will result in a loss and vice versa.

The monetary/non-monetary method
Monetary items are those items whose values are fixed in terms of the number of units of the currency of denomination. For example, a bond is a monetary item since its par (or face) value (the value received by the bondholder on maturity) is fixed by contract and displayed on the face of the bond. Real estate, on the other hand, is a non-monetary item, since its value in the currency of denomination may rise or fall. According to this method, the monetary items are translated at the closing rate, whereas non-monetary items are translated at the historical rate. The amount exposed in this case is the value of net monetary items.

The temporal method
According to the temporal method, the use of the closing rate or the historical rate is determined by the valuation of the underlying item. The closing rate is used for items stated at replacement cost, realisable value, market value or expected future value. The historical rate is used for all items stated at historical cost. The rationale for this method is that the translation rate should preserve the accounting principles used to value assets and liabilities in the original (foreign or functional currency) financial statements.

The temporal method appears to be a modified version of the monetary/non-monetary method. The only difference is that under the monetary/non-monetary method inventory is always translated at the historical rate. Under the temporal method, inventory is normally translated at the historical rate, although it can be translated at the current rate if the inventory is shown on the balance sheet at market value. The choice of the translation exchange rate is based on the type of assets and liabilities in the monetary/non-monetary method, but in the temporal method it is based on the underlying approach to evaluating cost (historical versus market).

What is used in practice?
In general, the following principles are observed in practice:

1. The translation of the balance sheet items is based on the closing rate.
2. Transactions gains and losses are accounted for in the income statement.
3. Non-transaction gains and losses are recorded on the balance sheet as reflected by changes in reserves.
4. If a transaction profit or loss arises from foreign currency borrowing designed as a hedge for a net investment in the same foreign currency, then the gain or loss (if less than that on the investment) will be accounted for by movements in reserves. Otherwise, the excess will be reported on the income statement.
CHAPTER 5

Financial and Operational Hedging of Exposure to Foreign Exchange Risk

5.1 WHY DO FIRMS HEDGE EXPOSURE TO FOREIGN EXCHANGE RISK?

Management of exposure to foreign exchange risk centres on the concept of hedging, which is a process whereby a firm can be protected from unanticipated changes in exchange rates. As business becomes global, firms get increasingly engaged in international activities such as exports, cross-border sourcing, joint venture with foreign partners, and establishing production and sales affiliates abroad. As a result, firms find it necessary to pay careful attention to the exposure to foreign exchange risk and to the design and implementation of appropriate hedging strategies. This is because changes in exchange rates affect the values of cash flows (costs and revenues), assets, liabilities, market share and the competitive position of the firm.

As we have seen, not even purely domestic firms can insulate themselves from the ramifications of exchange rate fluctuations. Indicative of the importance of foreign exchange exposure is the documented evidence on a significant relationship between stock returns and exchange rate movements (for example, Jorion, 1990; Choi and Prasad, 1995; Simkins and Laux, 1996). Simkins and Laux (1996) distinguish between market betas and Forex betas, which are measured as the sensitivities of an industry or a portfolio to the market index and the effective exchange rate respectively. However, Dominguez and Tesar (2001) examined the relationship between stock prices and exchange rates, arguing that the evidence should be stronger if it was not for “the restrictions imposed on empirical specifications used in previous studies”.

At the outset, a question should be answered concerning the motivation for hedging. It is often assumed that the motivation to hedge is risk reduction or
finding an optimal balance between risk and return. Hoffman (1932, p. 382) argued that “hedging is shifting risk”. Smith (1922, p. 81) suggested that “hedging enables hedgers to insure against the risk of price fluctuations”. Marshall (1919, p. 260) confirmed this view by stating that “the hedger does not speculate: he insure”. Keynes (1930), Hicks (1939) and Kaldor (1939) discussed hedging in terms of risk avoidance. According to this view, any loss made by the hedger on the hedged transaction represents an insurance premium paid on the risk-assuming speculator. Blau (1944) defines hedging as the shifting of risk arising from unknown future changes in prices, which cannot be covered by means of ordinary insurance.

Working (1953a) challenged the idea of risk insurance by arguing that hedging is motivated by the desire to make market by expecting the prospective movement of the prices of the assets to be hedged and those of the spot and hedging instruments. According to this view, hedging is some sort of arbitrage to be engaged in only when the hedger perceives a promising opportunity for profit. On the other hand, the portfolio theory stresses the risk–return trade-off. Stein (1961) and Johnson (1960) used the foundations of the portfolio theory as put forward by Markowitz (1959) to explain hedging. The hedger is viewed as maximising the expected utility derived from a portfolio of the asset and the hedging instrument. Williams (1986) challenged the portfolio theory by arguing that it is risk in operations that motivates hedging. Kamara (1982, p. 263) argues that hedging is motivated in part by the desire to stabilise income and in part by the desire to increase expected profits.

5.2 TO HEDGE OR NOT TO HEDGE?

In general, existing work suggests that firms hedge to reduce (i) the agency problem (Bessembrinder, 1991); (ii) effective corporate taxes (Smith and Stulz, 1985); (iii) risk aversion among managers and other contracting parties (Stulz, 1984); (iv) the probability of financial distress (Smith and Stulz, 1985); and (v) the adverse information content of earnings (De Marzo and Duffie, 1995). The theoretical explanations identified by these incentives for hedging are likely to benefit the contracting parties. However, hedging might not benefit all parties equally and, therefore, the hedging strategies of firms vary.

As documented by Rawls and Smithson (1990), hedging is considered by financial managers to be one of their primary objectives. However, there hardly exists a consensus view on whether a firm should or should not hedge. This debate involves two issues: (i) whether or not hedging is necessary, and (ii) whether hedging should be undertaken by the firm or by individual shareholders. We start with the first issue of whether or not hedging is indeed worthwhile.

First of all, there is the idea stipulating an inverse relationship between risk and shareholder value (for example, Bishop, 1996), implying that hedging can boost the value of the firm. However, this proposition is not universally
accepted. In the 1950s Modigliani and Miller (1958) demonstrated, by coming up with the Modigliani–Miller Theorem, that the mode of financing does not determine the value of the firm. This conclusion implies that managing risk does not add to the value of the firm, and could even lower this value because the use of hedging instruments and techniques is not free of charge. A similar argument is that what matters for the valuation of the firm is systematic risk, whereas hedging may only reduce total risk. It follows once more that hedging may not add to the value of a firm. But some economists do not agree with this proposition. Froot et al. (1994) argue that firms should hedge to ensure that they always have cash flows to fund their planned investment programmes. Otherwise, some potentially profitable investments may be missed because of inefficiencies in the bond and equity markets, which are the alternative means of raising funds (as opposed to internal financing). Lewent and Kearney (1993) explained the strategy of Merck (an American pharmaceutical firm), using derivatives to ensure that R&D plans can always be financed. They argue that cash flows and earnings uncertainty caused by exchange rate volatility lead to a reduction in research spending. However, it is arguable that factors causing cash flows to fall below expectations may also cut the number of profitable investment opportunities, which reduces the need to hedge anyway.

Stulz (1995) argues that there are two reasons why a firm should hedge: the desire to cut its tax bill and because it is unable to get cash when it is needed. Thus, a firm with little or highly rated debt has no need to hedge, as the probability of getting into financial trouble is low. However, it can be argued that even firms with little debt can reduce their riskiness by hedging, and this enables them to borrow more and rely less on equity, which is more expensive.

An elaboration on Stulz’s argument about tax is warranted. Under a progressive corporate tax system, stable before-tax earnings lead to lower taxes than volatile earnings with the same average value. This is because under a progressive tax system the firm pays more in high earning periods than it saves in low earning periods, which can be explained as follows. Suppose that the corporate income tax rate is \( \tau_1 \) for earnings up to a certain level, \( y_1 \), and \( \tau_2 \) for anything higher than \( y_1 \). Consider two firms, A and B. A does not hedge, and because of exchange rate fluctuations its expected income ranges between \( y_1 \) and \( y_2 \) (\( y_2 > y_1 \)) with equal probabilities, such that \( y_1 \) falls within the first tax bracket and \( y_2 \) falls within the second bracket. Thus, A’s expected tax payment is \( \tau_1 y_1 + \tau_2 y_2 / 2 \). On the other hand, B always hedges its exposure and so its income is more certain at \( y_3 \), which is equal to the expected value of A’s income (\( y_3 = (y_1 + y_2) / 2 \)) and falls within the first bracket. This means that the tax payment of B is \( \tau_1 y_3 \) or \( \tau_1 (y_1 + y_2) / 2 \). Thus, the firm’s average tax bill can be reduced through hedging so that profits are reduced in good years and increased in bad years.
Another argument as to why hedging is unnecessary is that the effects of exchange rate fluctuations on a firm average out over the long run. Moosa (2002c) used historical data on three exchange rates (yen, mark and pound against the dollar) to demonstrate that if the exposure is recurrent, then over a long period of time hedging the exposure by using a forward contract will not produce results that are superior to those obtained by leaving the exposure unhedged. The explanation put forward for this result is that the unbiased efficiency hypothesis holds over a long period of time. Thus, although the forward rate may overestimate or underestimate the future spot rate in the short run, it gets it right on average in the long run. As we will find out later, forward hedging of the exposure means that foreign currency payables and receivables are converted at the forward rate, whereas leaving it unhedged means that payables and receivables are converted at the spot rate prevailing on the date when they become due. If, as the unbiased efficiency hypothesis stipulates, the current forward rate and the future spot rate are equal over a long period of time, it follows that the results will be similar with and without hedging. Indeed, given that the bid–offer spread in the forward market is wider than the spread in the spot market, leaving the exposure uncovered may be more profitable. However, Moosa warns of the hazard of interpreting this result to mean that exposure to foreign exchange risk should never be hedged. This result, it is stressed, holds only over a long period of time or on average. The story may be completely different in the short run or when the operation is not repeated frequently, thus not allowing the “on average” qualification to materialise. If, for example, a large exposure arises at a particular point in time, a significantly adverse exchange rate movement could wipe out the whole business, and there would no long run to count on.

This brings us to the general argument that if international parity conditions hold, there is no need to hedge exposure to foreign exchange risk. We have already considered the unbiased efficiency hypothesis, so we turn to uncovered interest parity (UIP) and \textit{ex ante} purchasing power parity (PPP). If UIP holds then the (uncovered) foreign currency return will be equal to the base currency return (or the cost of borrowing), and hence the former is known with certainty. Any change in the exchange rate will be counterbalanced by a change in the interest rate differential in such a way as to keep returns at the same level. But the available empirical evidence indicates that deviations from UIP are significant, and hence the base currency value of the foreign return will not be known with certainty. Consequently, risk would arise.

If \textit{ex ante} PPP holds, then real currency depreciations and appreciations will not occur. This is because changes in the nominal exchange rate will be counterbalanced by equivalent changes in prices. Again, the empirical evidence shows that deviations from PPP in the short run are large and persistent. But even if that was the case, it is unusual for the firm’s individual costs and revenues to move proportionately with inflation. Hence economic exposure would arise.
In general, international parity conditions hold, at best, in the long run if at all, and the long run can be very long indeed, as in the case of PPP. In the short run, deviations from the conditions are significant and persistent. The empirical failure of international parity conditions is that foreign exchange risk is alive and kicking.

The other argument why hedging may not be necessary is that if it is possible to forecast exchange rates then there is no need to hedge exposure to foreign exchange risk. However, it has for a long time been established (by both academics and practitioners) that forecasting exchange rates is a rather difficult task (see, for example, Moosa, 2000a). Some views even point to the near impossibility of forecasting exchange rates because they are driven by “news”, which is unpredictable by definition. The empirical failure of unbiased efficiency and UIP implies that foreign exchange risk cannot be controlled by using the forward rate or the interest differential as forecasters of the future spot rate. Similarly, we cannot control foreign exchange risk by using any other forecaster, since the accuracy of the forecasts is questionable, to say the least.

An argument why hedging is important is that it results in a more stable income stream (as we have seen before), which has several benefits. It may be conducive to sales in the case of consumer durables and capital goods. This is because a stable income may give the impression that the firm will last for long enough to provide after-sale services. On the other hand, volatile earnings may cause a high degree of employee turnover or demands for higher wages if they are interpreted to imply lack of job security.

Now we turn to the second issue involving the argument that hedging at the firm level is redundant because shareholders are naturally hedged through diversification. Some of the arguments for hedging that we have discussed so far are relevant to this issue, but the following are more relevant. These include: (i) information asymmetry, (ii) differential transaction costs and (iii) default costs. Hedging should be carried out at the firm level based on arguments pertaining to (i), (ii) and (iii). First, the management knows better about the firm’s exposure position than shareholders. Second, the firm is in a position to acquire low-cost hedges, whereas individual shareholders can hedge only at a substantial cost, and some investors may not be willing or able to hold diversified portfolios. Third, if default costs are significant, corporate hedging would be justifiable, because it would reduce the probability of default.

Having gone through the arguments for and against hedging, it is time to find out whether or not firms hedge in practice. Culp and Miller (1995) argue that most value-maximising firms do not hedge. But surveys of big US non-financial firms conducted by the Wharton School and Chase Manhattan Bank found that 75% of the firms that use derivatives do so to hedge their commitments. Forty per cent of the respondents said they sometimes took a view on the direction of markets, but only 8% admitted that they did that frequently. Dolde (1993) found that some firms may not hedge simply because they have
no exposure, whereas others may not hedge or partially hedge depending on their perception about the exchange rate behaviour. Joseph (2000) shows in a survey of 109 British firms that all of them hedge foreign exchange risk.

5.3 THE DESIGN OF A HEDGING STRATEGY

Evans and Folks (1979) identify the following elements in an effective hedging strategy:

1. Determining the types of exposure to be monitored: transaction, economic and accounting.
2. Formulating corporate objectives, including guidance in resolving potential conflicts in objectives: hedging may be in conflict with other corporate objectives.
3. Ensuring that the objectives are consistent with maximising shareholder value and that they are implementable.
4. Assigning responsibilities for each exposure, and determining the criteria whereby each manager is to be judged.
5. Making explicit any constraints on the use of hedging techniques, such as limitations on entering into forward contracts and other derivatives.
6. Identifying the channels through which exchange rate considerations are incorporated into operating decisions that affect exposure to foreign exchange risk.
7. Developing a system for monitoring and evaluating hedging operations.

A hedging strategy should have an objective, and this objective should be compatible with the overall corporate objectives. Table 5.1 presents a set of possible objectives (Zenoff, 1978).

One question that arises here is whether hedging should be centralised or decentralised. Centralisation is favoured for the following reasons: (i) the fear that local managers want to optimise their exposure positions irrespective of the overall corporate objective; (ii) the ability to take advantage through exposure netting of the portfolio effects, which is not possible under decentralised hedging; (iii) it enables the choice of the cheapest means of hedging worldwide; and (iv) international tax considerations. Arguments against centralisation include the loss of local knowledge and the lack of incentive for local managers to take advantage of particular situations with which they are familiar.

5.4 THE BASIC PRINCIPLES OF HEDGING

Hedging is of two kinds: operational and financial. As the name implies, operational hedging involves some operational measures that aim at reducing
5.4 The Basic Principles of Hedging

TABLE 5.1 Objectives of hedging strategies.

<table>
<thead>
<tr>
<th>Objective</th>
<th>Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimising translation exposure</td>
<td>Protecting foreign currency denominated assets and liabilities from exchange rate fluctuations</td>
</tr>
<tr>
<td>Minimising fluctuations in earnings</td>
<td>Considering both transaction and translation exposure</td>
</tr>
<tr>
<td>Minimising transaction exposure</td>
<td>Managing a subset of the firm’s cash flow exposure</td>
</tr>
<tr>
<td>Minimising economic exposure</td>
<td>Ignoring accounting earnings while concentrating on reducing cash flow fluctuations resulting from changes in exchange rates</td>
</tr>
<tr>
<td>Minimising hedging costs</td>
<td>Balancing the costs and benefits of hedging</td>
</tr>
<tr>
<td>Avoiding surprises</td>
<td>Preventing large foreign exchange losses</td>
</tr>
</tbody>
</table>

exposure to foreign exchange risk, and this is why it is sometimes described as involving the use of “internal” hedging techniques (for example, Joseph, 2000). It is mainly, but not exclusively, used to hedge economic exposure. Financial hedging of a currency exposure involves entering an offsetting position so that whatever is lost or gained on the original exposure is offset by a corresponding foreign exchange gain or loss on the hedge. It typically involves the use of a financial hedging instrument (such as forwards and options), and this is why it is sometimes described as involving “external” hedging techniques (for example, Joseph, 2000). Regardless of what happens to the future exchange rate, hedging locks in the base currency value of the exposure. In this section, we concentrate on financial hedging, but the techniques of operational hedging will be described later in this chapter.

Financial hedging consists of five steps:

1. Exchange rate forecasting, which involves reviewing the likelihood of adverse exchange rate movements. More will be said about hedging and exchange rate forecasting later.
2. Assessing strategic plan impact. Once the future exchange rate changes are estimated, cash flows and earnings are projected and compared under alternative scenarios.
3. Deciding whether or not to hedge. A company will decide to hedge if, for example, it has a large portion of earnings generated abroad, while a disproportionate share is denominated in the base currency. The decision to hedge or not to hedge may be affected by a host of other factors, such as those discussed earlier, and the outlook for changes in exchange rates, as we will find out later.
4. Selecting the hedging instrument, which is the most cost-effective hedging tool that accommodates the firm’s risk preferences. We will show later how
the choice of the instrument can be determined by the base currency value of the payables and receivables.

5. Constructing a hedging programme, including the time horizon and the hedge ratio. The estimation of the hedge ratio will be dealt with in Chapter 6.

**Illustrating financial hedging**

In the remainder of this section the principles of financial hedging are illustrated by assuming that the \( y \) value of an asset (long exposure) or a liability (short exposure) is known. Consider the following possibilities, which are shown in Table 5.2. First, two decisions are involved: the hedge decision and the no-hedge decision. Second, the currency of denomination, \( y \), may appreciate, depreciate or stay unchanged against the base currency, \( x \), between two points in time. Now, consider the case of a long exposure under the appreciation of the currency of denomination. If \( y \) appreciates, the base currency value of the asset rises, which makes the no-hedge decision the right decision, in the sense that profit would materialise as compared with what would be obtained under the hedge decision. This (relative) profit is equal to the difference between the base currency value of the asset under the no-hedge and hedge decisions. In Table 5.2, this is indicated by a plus sign. If there is a short exposure, then under the same conditions the no-hedge decision would be the wrong decision, as a loss would be incurred compared with the outcome under the hedge decision. Remember that the appreciation of the currency of denomination is favourable if the exposure is long, and unfavourable if the exposure is short.

There are two equivalent ways of looking at the hedging operation, both of which involve a hedging instrument or a hedge. The first is to look at it as an operation whereby the base currency value of assets, liabilities and cash flows is locked in, in the sense that this value would be independent of movements in the exchange rate. This is applicable to cash flows. The second is to view the operation as taking an opposite position on a hedging instrument so that any deterioration in the base currency value of the asset is offset by gains on the

<table>
<thead>
<tr>
<th>Decision/currency change</th>
<th>Long exposure</th>
<th>Short exposure</th>
</tr>
</thead>
<tbody>
<tr>
<td>No-hedge decision</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Appreciation of ( y )</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Depreciation of ( y )</td>
<td>–</td>
<td>+</td>
</tr>
<tr>
<td>Hedge decision</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Appreciation of ( y )</td>
<td>–</td>
<td>+</td>
</tr>
<tr>
<td>Depreciation of ( y )</td>
<td>+</td>
<td>–</td>
</tr>
</tbody>
</table>

**TABLE 5.2** Outcomes under the hedge and no-hedge decisions.
hedging instrument. In this case, the base currency value of the combined position (the asset and the hedging instrument) is unaffected.

Consider Figure 5.1, which shows the outcome of hedging a long exposure by locking in the base currency value of an asset. Panel (a) shows the base currency value of the asset under the no-hedge and the hedge decisions ($V_{x,n}$ and $V_{x,h}$ respectively). As we can see, the value of the asset under the hedge decision is unaffected by changes in the exchange rate, but the value of the asset under the no-hedge decision varies, increasing with the exchange rate. They are equal at one level of the exchange rate only. Panel (b) shows the profile of the profit/loss realised from the hedge decision relative to the no-hedge decision ($V_{x,h} - V_{x,n}$). At low exchange rates, profit would be made, but at high exchange rates loss would be incurred. The opposite is true if we measure profit/loss under the no-hedge decision relative to the outcome under the hedge decision, as shown in panel (c). Figure 5.2 shows exactly the same thing when exposure is short (and this is why the value is shown to be negative). As we can see, the profiles under long and short exposures are reversed.

Now, examine Figure 5.3, which is derived from Figures 5.1 and 5.2. Under a long exposure, the upper part of the diagram shows the payoff on the asset (the unhedged position) relative to the hedging instrument, as well as the payoff on the hedging instrument relative to the hedged position. These payoffs are exactly equivalent to the profiles shown in Panels (c) and (b) of Figure 5.1. As we can see, the two payoffs offset each other exactly, so that the combined position (shown in the bottom part of the diagram) has a zero payoff. This is because $V_{x,n} - V_{x,h} + V_{x,h} - V_{x,n} = 0$. In this case any loss incurred on the asset (the unhedged position) will be offset by profit on the hedging instrument, and vice versa. The same applies to the hedging of a short exposure, which is shown in the left part of Figure 5.3 as derived from Figure 5.2.

If the profit/loss on the asset (the unhedged position) can be offset exactly by equivalent loss/profit on the hedging instrument, then we have what is called a “perfect hedge”. A perfect hedge may not always be obtained because it requires certain conditions to be satisfied: (i) the unhedged position and the hedging instrument must have the same maturity or liquidation date; (ii) the total value of the unhedged position must be hedged (that is, the hedge ratio should be one), which means that the y currency value must be known precisely; and (iii) the values or prices of the unhedged position and the hedging instrument must be perfectly correlated. Only under these conditions will the relative payoff of the combined position be zero, as in Figure 5.3. If one or more of these conditions are not satisfied, then we have a less than perfect hedge, as we can see in Figure 5.4.

**Hedging and exchange rate forecasting**
The decision to hedge or not to hedge an uncovered or open foreign currency position is basically a speculative decision. It all depends on the expected spot
rate or the movement of the exchange rate between the point in time when the decision is taken and when its effect materialises. Remember that if the decision to hedge is taken, then some costs may be incurred up front, such as the premium paid to acquire an option. If the decision to hedge the position is taken, and the exchange rate moves in a favourable direction (for example the currency denominated receivables appreciates against the base currency) then some potential gain would be lost. Some gain would be made by leaving the position unhedged. On the other hand, if the decision not to hedge is taken and the exchange rate moves in an unfavourable direction (for example,
the currency denominating payables appreciates against the base currency) then some losses would be incurred. These losses can be avoided by taking a decision to hedge. This is why exchange rate forecasting is Step 1 in financial hedging.

We will now put forward the proposition that what matters for the hedge/no-hedge decision is not the absolute forecasting accuracy but rather the accuracy of forecasting the level of the spot exchange rate, \( S_{t+1} \), relative to the guaranteed exchange rate implied by the hedge, \( S_t \). As we shall find out later, \( S_t \) is equivalent to the actual forward rate in forward hedging and the interest
parity forward rate in money market hedging. The base currency value of a \( y \)-denominated asset depends on which of the following three policies are adopted by the hedger: (i) always hedge; (ii) never hedge; and (iii) hedge or no-hedge, depending on exchange rate forecasting (the hedge/no-hedge strategy). Under the three strategies, the base currency value of a \( y \)-denominated asset is given by

\[
V_x(H) = V_y \overline{S}_t 
\]  
(5.1)

\[
V_x(N) = V_y S_{t+1} 
\]  
(5.2)

\[
V_x(H/N) = V_y \left[ \max(\overline{S}_t, E_t(S_{t+1})) \right] 
\]  
(5.3)
Equation (5.1) tells us that under the hedge decision \((H)\), conversion is made at the exchange rate implicit in the hedging instrument, whereas equation (5.2) says that under the no-hedge decision conversion takes place at the actual exchange rate prevailing in the future. Equation (5.3) says that if the hedging decision is based on forecasting, conversion takes place at the higher of the exchange rate implicit in the hedging instrument and the forecast exchange rate, \(E_t(S_{t+1})\). If forecasts are perfectly accurate, then

\[
\begin{align*}
V_x(H/N) &= V_y S_t \\
V_x(H/N) &= V_y S_{t+1} \\
&\text{if } S_t > S_{t+1} \text{ and } S_t < S_{t+1}
\end{align*}
\]

which gives
\[ V_y [\mathbb{S}_t - \max(\mathbb{S}_t, S_{t+1})] \leq 0 \] (5.5)
and
\[ V_y [\max(\mathbb{S}_t, S_{t+1}) - S_{t+1}] \geq 0 \] (5.6)

Equation (5.5) tells us that the difference between the \( x \) values of the asset under the hedge decision and the hedge/no-hedge decision (based on the forecast) is negative or zero, implying that the hedge/no-hedge strategy is better. Equation (5.6) says that the difference between the \( x \) values of the asset under the hedge/no-hedge decision and the hedge decision is positive or zero, implying the superiority of the former.

Suppose that there are two forecasts, \( E_t^A(S_{t+1}) \) and \( E_t^B(S_{t+1}) \), such that the latter would turn out to be more accurate (that is, \( E_t^A(S_{t+1}) - S_{t+1} > E_t^B(S_{t+1}) - S_{t+1} \)). If \( E_t^A(S_{t+1}) > E_t^B(S_{t+1}) > \mathbb{S}_t \), then both forecasts would indicate that the no-hedge decision should be taken, irrespective of the size of the forecasting error. Likewise, if it turns out that \( \mathbb{S}_t > E_t^A(S_{t+1}) > E_t^B(S_{t+1}) \), then the hedge decision should be taken irrespective of the forecasting error. Now, consider the situation when \( E_t^A(S_{t+1}) > \mathbb{S}_t \) and \( E_t^B(S_{t+1}) < \mathbb{S}_t \). In this case, the first forecast tells us that the no-hedge decision is better, whereas the second tells us that the hedge decision is better. If it turns out that \( S_{t+1} > \mathbb{S}_t \), then the less accurate forecast leads us to take the right decision and vice versa.

Moreover, the condition \( E_t(S_{t+1}) = S_{t+1} \) (accurate forecasting) is not necessary for (5.6) to hold. If \( E_t(S_{t+1}) < \mathbb{S}_t \), the decision to hedge will be taken. If this forecast is correct then \( \mathbb{S}_t = \max(\mathbb{S}_t, S_{t+1}) \) and condition (5.6) will hold irrespective. Similarly, if \( E_t(S_{t+1}) > \mathbb{S}_t \), the decision not to hedge will be taken. If the forecast is correct then \( S_{t+1} = \max(\mathbb{S}_t, S_{t+1}) \), and again the condition is satisfied. In short, what is important for the hedging decision is not the condition \( E_t(S_{t+1}) = S_{t+1} \), but rather the condition \( E_t(S_{t+1}) = \mathbb{S}_t \). Hence, strict forecasting accuracy is irrelevant for the hedging decision.

### 5.5 Money Market Hedging of Short-Term Transaction Exposure

In this section we explain how to hedge foreign currency payables (outflows) and receivables (inflows) in the money market. We will make the assumption that the hedge ratio is one (that is, the full exposure is hedged). We will also assume that the decision to hedge is not always taken, but it is rather considered in view of exchange rate expectations. For the purpose of illustration, we will consider a two-period model in which \( t \) is the present time (when the decision to hedge or not to hedge is taken) and \( t + 1 \) is the point in time when the payables or receivables are due.

A money market hedge involves taking a money market position to cover expected payables or receivables. Taking these positions means borrowing
and lending the base currency \((x)\) and the currency of denomination \((y)\). We start by illustrating how to hedge payables.

**Money market hedging of payables**
Assume the amount due at \(t + 1\) is \(K\) units of \(y\). Obviously, a decision not to hedge at time \(t\) means waiting until the amount becomes due at \(t + 1\) and converting an amount of the base currency at the spot exchange rate prevailing then to obtain \(K\) units of \(y\). A money market hedge of payables involves borrowing and lending such that the final product is \(K\) units of currency \(y\). A money market hedge consists of the following steps:

1. At time \(t\) an amount equal to \(KS_t/(1 + i_x)\) of the base currency is borrowed, where \(K\) is the \(y\) currency value of the payables.
2. This amount is converted into currency \(y\) at the prevailing exchange rate, \(S_t\), to obtain \(K/(1 + i_y)\) units of \(y\). This amount is the present value of the payables.
3. This amount is invested at the interest rate on \(y\) \((i_y)\) to obtain \(K/(1 + i_y)\) units of currency \(y\) when the payables are due at time \(t + 1\). This amount is then paid out.
4. At time \(t + 1\), the base currency loan becomes due, so the amount of the principal and interest should be repaid. This amount is equal to \(KS_t(1 + i_x)/(1 + i_y)\).

This operation does not involve the exchange rate prevailing at time \(t + 1\). The domestic currency amount required to meet the payables is \(KS_t/(1 + i_y)\), which is known in advance. The base currency amount needed to meet the payables if the position is not hedged is \(KS_{t+1}\), which is uncertain because \(S_{t+1}\) is not known at time \(t\). The decision whether or not to hedge payables depends on a comparison between the base currency amounts required to meet payables under the hedge and no-hedge decisions. A decision to hedge payables will be taken if

\[
KS_t \left( \frac{1 + i_x}{1 + i_y} \right) < KS_{t+1} \tag{5.7}
\]

or if

\[
S_t \left( \frac{1 + i_x}{1 + i_y} \right) < S_{t+1} \tag{5.8}
\]

The left-hand side of the inequality given by (5.8) is the interest parity forward rate, \(F_t\), which is the forward rate consistent with covered interest parity (the spot rate adjusted for a factor reflecting the interest rate differential). Hence, decision to hedge payables will be taken if
\[ F_t < S_{t+1} \] (5.9)

which represents a violation of uncovered interest parity \( (F_t = S_{t+1}) \). Notice that \( F_t \) is an implicit forward rate resulting from the hedging process. It can be calculated directly from the base currency amount required to meet the payables and the foreign currency value of the payables. Thus

\[
F_t = \frac{KS_t[(1+i_x)/(1+i_y)]}{K} = S_t \left( \frac{1+i_x}{1+i_y} \right) \] (5.10)

Let us now examine the conditions under which a decision to hedge or not to hedge is taken. If the condition represented by (5.7) holds, then a hedge decision should be taken. This is because the base currency amount required to meet the payables is smaller under the hedge decision than under the no-hedge decision. The hedge decision would, therefore, involve some sort of gain relative to the alternative course of action, which is the difference between the two amounts. This gain is given by

\[
\pi = KS_{t+1} - KS_t \left( \frac{1+i_x}{1+i_y} \right) \] (5.11)

or

\[
\pi = K(S_{t+1} - F_t) \] (5.12)

The size of this gain depends on the value of \( S_{t+1} \) (since \( K \) and \( F_t \) are known at time \( t \)). The higher \( S_{t+1} \), the greater is the gain made. If, on the other hand, a no-hedge decision is taken, then a loss equivalent to \( \pi \) would be incurred, in the sense that more money is paid to meet the payables than under the hedge decision.

**Money market hedging of receivables**

Hedging receivables works the other way round. This time, a \( y \) amount is expected to be received, but how much this amount is worth in terms of \( x \) when it is received is unknown. The following steps are involved:

1. At time \( t \) an amount of \( y \) equal to \( K/(1+i_y) \), which is the present value of the receivables, is borrowed.
2. This amount is then converted into currency \( x \) at the prevailing exchange rate, \( S_t \), to obtain \( KS_t/(1+i_y) \) units.
3. This amount is invested at the base currency interest rate to obtain \( KS_t(1+i_x)/(1+i_y) \) units of \( x \) when the receivables are due at time \( t + 1 \).
4. At time \( t + 1 \), the foreign currency loan becomes due, and the amount of the principal and interest should be repaid. This amount is equal to \( K \), which is covered by the receivables.
5.5 Hedging of Short-Term Transaction Exposure

<table>
<thead>
<tr>
<th>Payables</th>
<th>Receivables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{F}<em>t &lt; S</em>{t+1}$</td>
<td>Hedge</td>
</tr>
<tr>
<td>$\bar{F}<em>t &gt; S</em>{t+1}$</td>
<td>No hedge</td>
</tr>
<tr>
<td>$\bar{F}<em>t = S</em>{t+1}$</td>
<td>Indifference</td>
</tr>
</tbody>
</table>

Again, this operation does not involve the exchange rate prevailing at time $t + 1$. The $x$ amount received is $KS_t(1 + i_x)/(1 + i_y)$, which is known at time $t$. The decision to hedge receivables will be taken if

$$KS_t \left(\frac{1 + i_x}{1 + i_y}\right) > KS_{t+1} \quad (5.13)$$

or if

$$\bar{F}_t > S_{t+1} \quad (5.14)$$

If this is the case then a hedge decision will produce a gain that is given by

$$\pi = K(\bar{F}_t - S_{t+1}) \quad (5.15)$$

whereas the no-hedge decision produces an equivalent loss. Table 5.3 lists all of the possibilities. Notice that if $\bar{F}_t = S_{t+1}$, then the gain as given by equations (5.12) and (5.15) will be zero. In this case, the hedge and no-hedge decisions produce similar results.

Introducing the bid–offer spread

Now, we examine money market hedging by allowing for the bid–offer spreads in interest and exchange rates. The process of hedging payables in the money market changes to the following:

1. At time $t$ an amount of $x$ equal to $KS_{a,t}/(1 + i_{y,b})$ is borrowed.
2. This amount is converted into currency $y$ at the prevailing offer exchange rate, $S_{a,t}$, to obtain $K/(1 + i_{y,b})$ foreign currency units. This amount is the present value of the payables.
3. This amount is invested at the bid interest rate on $y$, $i_{y,b}$, to obtain $K$ units of $y$ when the payables are due at time $t + 1$. This amount is then paid out.
4. At time $t + 1$, the base currency loan becomes due, so the amount of the principal and interest should be repaid. This amount is equal to $KS_{a,t}(1 + i_{x,a})/(1 + i_{y,b})$.

A decision to hedge payables will be taken if

$$KS_{a,t} \left(\frac{1 + i_{x,a}}{1 + i_{y,b}}\right) < KS_{a,t+1} \quad (5.16)$$
or if
\[ \bar{F}_{a,t} < S_{a,t+1} \]  

(5.17)

where

\[ \bar{F}_{a,t} = S_{a,t} \left( \frac{1 + i_{x,a}}{1 + i_{y,b}} \right) \]  

(5.18)

Notice that \( \bar{F}_{a,t} \) can also be calculated directly from the base currency amount required to meet the payables and the foreign currency value of the payables, that is

\[ \bar{F}_{a,t} = \frac{K S_{a,t}}{(1 + i_{x,a})/(1 + i_{y,b})} = S_{a,t} \left( \frac{1 + i_{x,a}}{1 + i_{y,b}} \right) \]  

(5.19)

The hedge decision would, in this case, produce some gain relative to the no-hedge decision, which is given by

\[ \pi = K S_{a,t+1} - K S_{a,t} \left( \frac{1 + i_{x,a}}{1 + i_{y,b}} \right) \]  

(5.20)

or

\[ \pi = K(S_{a,t+1} - \bar{F}_{a,t}) \]  

(5.21)

Now we consider hedging receivables. The following steps are involved in the process:

1. At time \( t \) an amount of \( y \) equal to \( K/(1 + i_{y,a}) \), which is the present value of the receivables, is borrowed.
2. This amount is converted into \( x \) at the prevailing bid exchange rate, \( S_{b,t} \), to obtain \( K S_{y,t}/(1 + i_{y,a}) \) units of \( x \).
3. This amount is invested at the interest rate on \( x \) to obtain \( K S_{b,t} (1 + i_{x,b})/(1 + i_{y,a}) \) units of \( x \) when the receivables are due at time \( t + 1 \).
4. At \( t + 1 \), the foreign currency loan becomes due, and the amount of the principal and interest should be repaid. This amount is equal to \( K \) units of \( y \), which is covered by the receivables.

The decision to hedge receivables will be taken if

\[ K S_{b,t} \left( \frac{1 + i_{x,b}}{1 + i_{y,a}} \right) > K S_{b,t+1} \]  

(5.22)

or if

\[ \bar{F}_{b,t} > S_{b,t+1} \]  

(5.23)
where

\[ \bar{F}_{b,t} = S_{b,t} \left( \frac{1 + i_{x,b}}{1 + i_{y,a}} \right) \]  

(5.24)

Notice that \( \bar{F}_{b,t} \) can also be calculated directly from the domestic currency amount received and the foreign currency value of the receivables, that is

\[ \bar{F}_{b,t} = \frac{KS_{b,t}[(1 + i_{x,b})/(1 + i_{y,a})]}{K} = S_{b,t} \left( \frac{1 + i_{x,b}}{1 + i_{y,a}} \right) \]  

(5.25)

The hedge decision would, in this case, produce a gain relative to the no-hedge decision, which is given by

\[ \pi = KS_{b,t} \left( \frac{1 + i_{x,b}}{1 + i_{y,a}} \right) - KS_{b,t+1} \]  

(5.26)

or

\[ \pi = K(\bar{F}_{a,t} - S_{b,t+1}) \]  

(5.27)

Table 5.4 lists all of the possibilities in the presence of a bid–offer spread.

Remember that the bid–offer spreads are transaction costs. Hence, allowing for these spreads implies that, under both hedge and no-hedge decisions, the amount of payables will be greater and the amount of receivables will be smaller than if the operations are conducted at the basis of the mid interest and exchange rates. Consider, for example, the amount paid under a hedge decision. Since \( S_{a} > S_{b} \), \( i_{x,a} > i_{x} \) and \( i_{y,b} < i_{y} \), it follows that

\[ KS_{a,t+1} \left( \frac{1 + i_{x,a}}{1 + i_{y,b}} \right) > KS_{t} \left( \frac{1 + i_{x}}{1 + i_{y}} \right) \]  

(5.28)

Consider now the amount received under the hedge decision. Since \( S_{b} < S \), \( i_{x,b} < i_{x} \) and \( i_{y,a} > i_{y} \), it follows that

| TABLE 5.4 The hedge/no-hedge decision in the presence of bid–offer spreads. |
|---------------------------------|---------------------------------|
| \( \bar{F}_{a,t} < S_{a,t+1} \) | Payables: Hedge                 |
| \( \bar{F}_{a,t} > S_{a,t+1} \) | Payables: No hedge              |
| \( \bar{F}_{a,t} = S_{a,t+1} \) | Payables: Indifference          |
| \( \bar{F}_{b,t} > S_{b,t+1} \) | Receivables: Hedge             |
| \( \bar{F}_{b,t} < S_{b,t+1} \) | Receivables: No hedge           |
| \( \bar{F}_{b,t} = S_{b,t+1} \) | Receivables: Indifference       |
TABLE 5.5 The effect of bid–offer spreads on the hedging process.

<table>
<thead>
<tr>
<th></th>
<th>With bid–offer spread</th>
<th>Without bid–offer spread</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Payables</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Amount paid under hedge</td>
<td>$K_S_{a,t} (1+i_{x,a}) / (1+i_{y,b})$</td>
<td>$K_S_t (1+i_{x}) / (1+i_{y})$</td>
</tr>
<tr>
<td>Amount paid under no hedge</td>
<td>$K_S_{a,t+1}$</td>
<td>$K_S_{t+1}$</td>
</tr>
<tr>
<td><strong>Receivables</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Amount received under hedge</td>
<td>$K_S_{b,t} (1+i_{x,b}) / (1+i_{y,a})$</td>
<td>$K_S_t (1+i_{x}) / (1+i_{y,b})$</td>
</tr>
<tr>
<td>Amount received under no hedge</td>
<td>$K_S_{b,t+1}$</td>
<td>$K_S_{t+1}$</td>
</tr>
</tbody>
</table>

\[ K_S_{b,t+1} \frac{1+i_{x,b}}{1+i_{y,a}} \times K_S_t \frac{1+i_{x}}{1+i_{y}} < K_S_t \frac{1+i_{x}}{1+i_{y}} \]  \( (5.29) \)

Table 5.5 shows the amounts paid and received under the hedge and no-hedge decisions with and without the bid–offer spreads.

5.6 FORWARD AND FUTURES HEDGING OF SHORT-TERM TRANSACTION EXPOSURE

Forward hedging of foreign currency payables and receivables entails locking in the rate at which the payables and receivables are converted from a foreign currency, $y$, into the base currency, $x$. This is achieved by buying $y$ forward in the case of payables and selling it forward in the case of receivables.

**Forward hedging of payables**
Forward hedging of payables denominated in currency $y$ amounts to buying the currency forward. Thus, the amount of payables, $K$ units of $y$, is bought forward at time $t$ at a cost of $KF_t$ units of $x$. Since this amount is known with certainty at time $t$, the exposure is covered. The decision to hedge payables is taken if

$$ KF_t < K_S_{t+1} \quad (5.30) $$

If this is the case, some gain will be made out of the hedge decision. This gain is given by

$$ \pi = K(S_{t+1} - F_t) \quad (5.31) $$

If a decision to hedge is not taken, then a loss of a similar amount would be incurred. Notice that if covered interest parity holds then $\bar{F}_t = F_t$, so the gain will be equal to that made under a money market hedge. In fact, if
covered interest parity holds, then money market hedging and forward hedging will produce identical results in terms of the base currency value of the payables. Otherwise, forward hedging would be preferred to money market hedging if

\[ K_F t < K_F^t \]  

(5.32)

The two conditions that trigger the hedge decision and the one under which forward hedging is preferred can be combined to produce the following. Forward hedging of payables would be preferred to money market hedging and to the no-hedge alternative if

\[ K_F t < K_F^t < K_S t+1 \]  

(5.33)

If the bid–offer spreads are allowed for, then the conditions change as follows. If the decision to hedge is taken, then the foreign currency is bought forward at the offer forward rate. Thus the decision to hedge is taken if

\[ K_F a, t < K_S a, t+1 \]  

(5.34)

and the gain resulting from the hedge decision is

\[ \pi = K(S_{a, t+1} - F_{a, t}) \]  

(5.35)

Forward hedging is preferred to money market hedging if

\[ K_F a, t < K_F^a, t \]  

(5.36)

The general condition for preferring forward hedging to money market hedging and the no-hedge alternative is

\[ K_F a, t < K_F^a, t < K_S a, t+1 \]  

(5.37)

Hedging receivables works the other way round. In this case currency \( y \) is sold forward at the bid forward rate. Table 5.6 summarises all of the possibilities.

Let us examine the diagrammatic representation of forward hedging as shown in Figure 5.5. Consider the hedging of payables first. If the position is hedged, then the base currency value of the payables will be unchanged at \( K_F t \). If, on the other hand, the position is not hedged, then the base currency value of the payables rises as the spot exchange rate rises. The top part of the diagram shows the profit/loss on the unhedged position relative to the alternative of hedging, which is \( K(F_t - S_{t+1}) \). As \( S_{t+1} \) rises losses will be incurred on the unhedged position. In the second part of the diagram, we plot the profit/loss on the hedging instrument (long forward) relative to the alternative of no-hedging, which is \( K(S_{t+1} - F_t) \). As \( S_{t+1} \) rises, profit will be made on the long forward position. When the two positions are combined we get the payoff on the hedged position, which is zero, as shown in the lower part of the diagram. The right-hand panel shows the hedging of receivables, which works in exactly the opposite way to that of the payables.
### Table 5.6 General conditions and the hedge/no-hedge decision.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Payables</th>
<th>Receivables</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Without bid–offer spreads</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_1 &lt; \bar{F}<em>1 &lt; S</em>{t+1}$</td>
<td>Hedge (forward)</td>
<td>No hedge</td>
</tr>
<tr>
<td>$F_1 &gt; \bar{F}<em>1 &gt; S</em>{t+1}$</td>
<td>No hedge</td>
<td>Hedge (forward)</td>
</tr>
<tr>
<td>$F_1 = \bar{F}<em>1 = S</em>{t+1}$</td>
<td>No hedge</td>
<td>Hedge (money market)</td>
</tr>
<tr>
<td><strong>With bid–offer spreads</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_{a,t} &lt; \bar{F}<em>{a,t} &lt; S</em>{a,t+1}$</td>
<td>Hedge (forward)</td>
<td></td>
</tr>
<tr>
<td>$F_{a,t} &lt; \bar{F}<em>{a,t} &lt; S</em>{a,t+1}$</td>
<td>Hedge (money market)</td>
<td></td>
</tr>
<tr>
<td>$F_{a,t} &gt; \bar{F}<em>{a,t} &gt; S</em>{a,t+1}$</td>
<td>No hedge</td>
<td></td>
</tr>
<tr>
<td>$F_{a,t} = \bar{F}<em>{a,t} = S</em>{a,t+1}$</td>
<td>No hedge</td>
<td></td>
</tr>
<tr>
<td>$F_{b,t} &lt; \bar{F}<em>{b,t} &lt; S</em>{b,t+1}$</td>
<td>No hedge</td>
<td></td>
</tr>
<tr>
<td>$F_{b,t} &lt; \bar{F}<em>{b,t} &lt; S</em>{b,t+1}$</td>
<td>No hedge</td>
<td></td>
</tr>
<tr>
<td>$F_{b,t} &gt; \bar{F}<em>{b,t} &gt; S</em>{b,t+1}$</td>
<td>Hedge (forward)</td>
<td></td>
</tr>
<tr>
<td>$F_{b,t} &gt; \bar{F}<em>{b,t} &gt; S</em>{b,t+1}$</td>
<td>Hedge (money market)</td>
<td></td>
</tr>
<tr>
<td>$F_{b,t} = \bar{F}<em>{b,t} = S</em>{b,t+1}$</td>
<td>Indifference</td>
<td></td>
</tr>
</tbody>
</table>

### Futures hedging

The consequences of using futures contracts to hedge transaction exposure are the same as those of using forward contracts. However, because of the standardisation of the futures contracts and because they involve marking-to-market, some quantitative rather than qualitative differences may arise. First, it may not be possible to hedge the amount of payables or receivables exactly because futures contracts are standardised with respect to size. For example, if the amount of the payables is $K$ units of $y$ and the size of the futures contract on $y$ is $2K/5$ then buying two contracts leaves the amount $K/5$ uncovered, in which case it has to be bought in the spot market at $S_{t+1}$. The currency value of the payables will thus be $(4KF_{t}/5) + (KS_{t+1}/5)$ or $(K/5)(4F_{t} + S_{t+1})$, where $F_{t}$ in this case is the futures rate. Alternatively, if three contracts are bought then the excess amount of $y$ ($K/5$ units) has to be sold spot, in which case the currency value of the payables will be $(6KF_{t}/5) - (KS_{t+1}/5)$ or $(K/5)(6F_{t} - S_{t+1})$. Second, it is more likely the case that the date on which the receivables are due does not coincide with a settlement date because futures contracts are standardised with respect to the settlement date. Even if the size and the settlement dates are the same as what is required, marking-to-market risk will introduce some variation *vis-à-vis* the forward market. Third, some variation results from changes in the margin account associated with any futures position.
The standardisation of futures contracts makes forward hedging more appealing than futures hedging. However, there are compelling reasons why futures contracts are used for the purpose of hedging. Telser (1981) argued that organised futures markets exist because they are superior to informal forward markets. An organised futures market has elaborate written rules, standing committees for adjudicating disputes, and a limited membership. In contrast to futures contracts, forward contracts rely on the good faith of individual parties. Because forward contracts are tailor-made they cannot be
offset by identical contracts, and there is no scope for the advantages of clearing houses and settlement by the payment of the difference. Through their rules and standardisation, futures provide liquidity and eliminate counterparty risk. Penings and Leuthold (2000) argue that futures contracts can provide jointly preferred contracting arrangement, enhancing relationships between firms. What motivates the use of futures contracts is then contract preference, level of power and conflicts in contractual relationships of firms.

**CIP and the relative effectiveness of forward and money market hedging**

We have seen that money market hedging entails the conversion of payables and receivables at the interest parity forward rate, whereas forward hedging amounts to conversion at the actual forward rate. Under covered interest parity these rates are equal, which means that money market hedging and forward hedging will produce similar results. Al-Loughani and Moosa (2000) use this idea to devise an indirect test of CIP. The idea is that if money market hedging and forward hedging produce similar results then CIP must be valid. They tested this hypothesis using five exchange rates and found that CIP actually holds.

### 5.7 OPTIONS HEDGING OF SHORT-TERM TRANSACTION EXPOSURE

Unlike the case of money market hedging, forward hedging and futures hedging, the outcome of options hedging is not known with certainty because it depends on whether or not the option is exercised. This in turn depends on whether the actual exchange rate when the payables or receivables are due, $S_{t+1}$, is higher or lower than the exercise exchange rate. However, options can be used to ensure that the domestic currency value of payables does not rise above a certain value, and that the domestic currency value of receivables does not fall below a certain value. It is also possible, by using over the counter non-standardised options, to hedge the exact amount of payables and receivables.

**Hedging payables with a call option**

A call option gives the hedger the right to buy $K$ units of a currency at the exercise exchange rate. For simplicity let us assume that the option is a European option with an expiry date that coincides with the date when the payables are due, $t + 1$. Let us also assume that the exercise exchange rate is equal to the current exchange rate, $S_t$ (the option is at the money). The exchange rate prevailing on the expiry date, $S_{t+1}$, will assume any value, and depending on this value we have the following possibilities:
1. If it assumes the value $S_{t+1,1}$, such that $S_{t+1,1} < S_t$, then the option will not be exercised, and the foreign currency will be bought on the spot market. The total cost of covering the payables is the base currency value, $K S_{t+1,1}$ and the option premium lost, which is equal to $K m$, where $m$ is the premium per unit of the underlying currency. The total cost will thus be $K (S_{t+1,1} + m)$.

2. If it assumes the value $S_{t+1,2}$, such that $S_{t+1,2} > S_t$, then the option will be exercised, and the foreign currency will be bought at the exercise exchange rate, $S_t$. The total cost of covering the payables is the base currency value, $K S_t$, and the option premium paid up front, which is equal to $K m$. The total cost will thus be $K (S_t + m)$.

**Hedging receivables with a put option**

We now consider the case of hedging receivables worth $K$ units of currency $y$ via a put option. In this case the put option used as a hedging instrument gives the hedger the right to sell $K$ units of currency $y$ at the exercise exchange rate. Making the same assumptions as before, we have the following possibilities:

1. If $S_{t+1,1} < S_t$, then the option will be exercised, and the underlying amount of currency $y$ will be sold to the option writer. The value obtained will be equal to the domestic currency value, $K S_t$, minus the option premium paid up front, which is equal to $K m$. Thus, the net amount is equal to $K (S_t - m)$.

2. If $S_{t+1,2} > S_t$, then the option will not be exercised, and the foreign currency will be sold on the spot market at $S_{t+1,2}$. The net amount received is therefore $K (S_{t+1,2} - m)$.

**A summary of the decision rules**

If the exchange rates $S_{t+1,1}$ and $S_{t+1,2}$ are expected with probabilities of $p_1$ and $p_2$ respectively, then the decision to hedge payables and receivables will depend on the expected domestic currency values under the hedge and no-hedge decisions. Table 5.7 lists all of the possibilities.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Payables</th>
<th>Receivables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(S_{t+1,1} + m)p_1 + (S_t + m)p_2 &lt; (S_{t+1,1} + S_{t+1,2}p_2)$</td>
<td>Hedge</td>
<td></td>
</tr>
<tr>
<td>$(S_{t+1,1} + m)p_1 + (S_t + m)p_2 &gt; (S_{t+1,1} + S_{t+1,2}p_2)$</td>
<td>No hedge</td>
<td></td>
</tr>
<tr>
<td>$(S_{t+1,1} + m)p_1 + (S_t + m)p_2 = (S_{t+1,1} + S_{t+1,2}p_2)$</td>
<td>Indifference</td>
<td></td>
</tr>
<tr>
<td>$(S_t - m)p_1 + (S_{t+1,2} - m)p_2 &gt; (S_{t+1,1} + S_{t+1,2}p_2)$</td>
<td>No hedge</td>
<td></td>
</tr>
<tr>
<td>$(S_t - m)p_1 + (S_{t+1,2} - m)p_2 &lt; (S_{t+1,1} + S_{t+1,2}p_2)$</td>
<td>Hedge</td>
<td></td>
</tr>
<tr>
<td>$(S_t - m)p_1 + (S_{t+1,2} - m)p_2 = (S_{t+1,1} + S_{t+1,2}p_2)$</td>
<td>Indifference</td>
<td></td>
</tr>
</tbody>
</table>
Figure 5.6 illustrates options hedging of payables and receivables assuming an exercise exchange rate of $S_t$, which implies that the option is at the money. Consider the hedging of payables, which is shown in the left-hand panel of Figure 5.6. The top part of the diagram shows the payoff on the unhedged position measured as the difference between the value of the payables at $S_t$ and $S_{t+1}$. The middle part of the diagram shows the payoff on a long call position at an exercise exchange rate of $S_t$. The bottom part of the diagram shows the payoff on the combined position. Unlike the unhedged position, the combined position shows an upper limit on the possible loss arising from
adverse exchange rate movements. To hedge receivables, a long put position is taken as the hedging instrument. The hedged position shows that while there is a lower limit on the loss arising from adverse exchange rate movements, there is no limit on the potential gain arising from favourable exchange rate movements.

**Hedging a contingent exposure**

A contingent exposure means that the exposure arises only if a certain outcome materialises (for example, when a contract is awarded to the underlying firm). In this case, an option hedge is preferable to a forward hedge because the latter can cover all possible eventualities. Let us assume that a firm bids for a contract valued at $K$ units of currency $y$. Exposure to currency $y$ would arise only if the contract is awarded to the firm, in which case the firm would have receivables worth the value of the contract. If, in the mean time, currency $y$ depreciates against the base currency, the firm would incur some losses.

Let us see what happens when the firm uses forward contracts and options to hedge this contingent exposure. To use a forward contract, the firm takes a short forward position on currency $y$ by selling $K$ units of the currency forward. In any case (that is, whether or not the contract is awarded) the firm is committed to come up with $K$ units of currency $y$ on the maturity of the contract. There are then two outcomes:

1. The contract is awarded, in which case the amount of the receivables covers the forward contract. Hence there is no problem, as the hedge works in the sense that the firm manages to lock in the base currency value of the receivables.
2. The contract is not awarded, in which case the amount of $y$ has to be provided at the pre-specified forward rate. If the spot rate at that time happens to be higher than the forward rate, the firm would incur unlimited loss, proportional to the difference between the spot and forward rates.

Now, let us consider what happens if a put option is used to hedge this contingent exposure. In this case there are four possible outcomes, because the option may or may not be exercised. The following outcomes are possible:

1. The contract is awarded and the actual exchange rate turns out to be less than the exercise exchange rate ($S_{t+1} < S_t$). The firm exercises the option, converting the proceeds of the contract to the base currency at the exercise exchange rate, obtaining $K S_t$ units of $x$.
2. The contract is awarded but the exchange rate turns out to be greater than the exercise exchange rate ($S_{t+1} > S_t$). The firm does not exercise, losing the premium on the option but the proceeds from the contract are converted at the higher rate ($K S_{t+1}$).
3. The contract is not awarded and the actual exchange rate turns out to be less than the exercise exchange rate \((S_{t+1} < S_t)\). The firm exercises the option, making profit of \(K(S_t - S_{t+1})\).

4. The contract is awarded but the exchange rate turns out to be greater than the exercise exchange rate \((S_{t+1} > S_t)\). The firm does not exercise, losing the premium on the option.

All of these outcomes are illustrated in Figure 5.7. It is obvious that if a forward contract is used to hedge a contingent exposure, then the loss will be unlimited if the exchange rate rises and the contract is not awarded. If, on the other hand, a put option is used to hedge this exposure, then the maximum loss would be the premium on the option, whether or not the contract is awarded.

Writing an option in itself creates a contingent exposure that can be hedged by taking an opposite option position. Figure 5.8 shows the payoff on a short call, which arises when a firm writes a call, giving the holder the right to buy the currency at an exercise exchange rate of \(S_t\). As long as the actual exchange rate does not turn out to be higher than the exercise exchange rate \((S_{t+1} < S_t)\), the option will not be exercised and the firm writing the option will make profit equal to the option premium paid by the holder \((m)\). But if the actual exchange rate turns out to be higher than the exercise exchange rate \((S_{t+1} > S_t)\), the option will be exercised and the loss will be unlimited, increasing with the level of \(S_{t+1}\). Suppose now that the firm decided to hedge this exposure
with a forward contract, assuming for simplicity that the forward rate is equal to the exercise exchange rate. The payoff on the forward contract is shown by the middle part of the diagram. Profit will be made on this position as long as the actual exchange rate turns out to be higher than the exercise exchange rate and the forward rate. The bottom part of the diagram shows the payoff on the combined (hedged) position. If the actual exchange rate turns out to be higher than the exercise exchange rate, profit on the forward contract will offset the loss on the option, and the firm will gain the premium on the put option. If, on
the other hand, the actual exchange rate turns out to be lower than the exercise exchange rate, losses on the forward contract will be unlimited.

Consider now Figure 5.9, which shows the situation when the exposure is hedged by a long call position with the same exercise exchange rate but at a lower premium, $m'$. If the actual exchange rate is lower than the exercise rate, neither of the two options will be exercised, and the firm will make profit that is equal to the difference between the two premiums ($m - m'$). At higher exchange rates both of the options will be exercised, and the losses on the unhedged
position will be offset by the gains on the hedge (the long call). Net profit remains at \( m - m' \).

Finally, Figure 5.10 shows the situation when the hedging instrument is an option with a lower premium and a higher exercise exchange rate \( (S'_t) \). In this case, the firm will incur a maximum loss of \( (m - m') - (S'_t - S_t) \), if the market exchange rate turns out to be higher than \( S'_t \).
Hedging against exchange rate volatility
A trading firm engaged in exporting and importing may find it desirable to work with stable exchange rates. This firm can hedge the risk arising from exchange rate volatility by taking an option position that compensates it if the underlying exchange rate rises above or falls below a certain level. This position is called a long straddle, which consists of a long call and a long put at the same exercise exchange rate.

A long straddle is shown in Figure 5.11. The top part of the diagram shows the payoff on a long call with a premium of $m_c$. The middle part shows the payoff on a long put with a premium of $m_p$. The total cost of the long straddle,
whose payoff is shown in the bottom part of the diagram, is \( m_c + m_p \). If the exchange rate at time \( t + 1 \) goes above or below the exercise exchange rate, \( S_t \), by more than \( m_c + m_p \), the firm will be compensated for the difference by exercising the call (above) or the put (below).

### 5.8 Financial Hedging of Long-Term Transaction Exposure

Long-term transaction exposure involves payables and receivables materialising over a long time in the future (say five years). If it is possible to estimate this exposure, it can be hedged using three alternative techniques. These techniques are discussed in turn.

#### Long-term Forward Contracts

Commercial banks do provide forward contracts in major currencies with long maturities (for example, five or ten years). However, because of the risk involved, banks only offer these contracts to the most creditworthy customers.

#### Currency Swaps

Swaps are suitable for hedging a recurrent exposure that consists of a series of payables or receivables. A currency swap resembles a portfolio of forward contracts, whereby two parties agree to exchange two cash flows denominated in two different currencies at a predetermined exchange rate on a sequence of future dates.

Suppose that a firm whose base currency is \( x \) anticipates a series of receivables (\( K \) units of \( y \)) arising at points in time 1, 2, 3, ..., \( n \). To hedge this exposure, the firm arranges a swap with a counterparty whereby the firm receives payments in terms of \( x \) equal to \( K \) converted at the contract rate, \( S_t \), while the counterparty receives \( K \) units of \( y \). Table 5.8 shows the payments received by the firm and the counterparty at each payment period in both currencies.

**TABLE 5.8** Payments involved in a currency swap.

<table>
<thead>
<tr>
<th>Payment date</th>
<th>The firm receives</th>
<th>The counterparty receives</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Units of ( x )</td>
<td>Units of ( y )</td>
</tr>
<tr>
<td>1</td>
<td>( K S_t )</td>
<td>( K )</td>
</tr>
<tr>
<td>2</td>
<td>( K S_t )</td>
<td>( K )</td>
</tr>
<tr>
<td>3</td>
<td>( K S_t )</td>
<td>( K )</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( n )</td>
<td>( K S_t )</td>
<td>( K )</td>
</tr>
</tbody>
</table>
Parallel loans
Parallel loans (or back-to-back loans) can be used to hedge long-term payables and receivables in exactly the same way as swaps. In this case, however, the exchanged payments are based on the forward rates.

5.9 OTHER FINANCIAL AND OPERATIONAL TECHNIQUES OF HEDGING TRANSACTION EXPOSURE

It may not be possible to implement the hedging techniques discussed so far. This may be due, for example, to the inability to forecast the sales resulting from an advertised reduction in prices over the following six months. Another reason is that the hedging costs may be prohibitively high. A third reason is the unavailability of forward, futures or options contracts for the underlying currency. When this is the case the following techniques may be considered.

Leading and lagging
Leading and lagging represent an operational hedging technique that involves an adjustment in the timing of the realisation of foreign currency payables or receivables. If the foreign currency is expected to appreciate it would be a good idea to pay the foreign currency dues sooner than later. This is called leading. If, on the other hand, the base currency is expected to depreciate, it would be a good idea to meet the payables later than sooner. This is lagging.

There are some problems with the implementation of leading and lagging. Suppose that a firm requires a prepayment because there are concerns about a depreciating currency. This firm would face the following problems: (i) the payer may not agree to pay unless there is some incentive such as discounts; (ii) pressing for prepayments may hamper future sales efforts; and (iii) to the extent that the original invoice price incorporates the expected depreciation of the foreign currency, the receiving firm is already partially protected. The technique of leading and lagging is more appropriate for intra-firm trade.

Currency diversification
Currency diversification is again an operational hedging technique. The depreciation and appreciation of foreign currencies against the base currency will not be as harmful if a large number of currencies are involved, provided that the exchange rates of these currencies against the base currency are not highly correlated. The base currency value of foreign currency payables rises when the foreign currencies appreciate against the base currency. If the exchange rates are not highly and positively correlated, then the adverse effect will be smaller, because some of these currencies will appreciate only slightly, while others may even depreciate.
Consider a firm with a base currency \( x \) and long positions of \( K_y \) and \( K_z \) on currencies \( y \) and \( z \) respectively. The base currency value of the \( y \) position at \( t \) and \( t + 1 \) is
\[
V_{x,t}(y) = K_y S_t(x/y) \tag{5.38}
\]
\[
V_{x,t+1}(y) = K_y S_{t+1}(x/y) \tag{5.39}
\]
Thus
\[
\dot{V}_x(y) = \frac{K_y S_{t+1}(x/y)}{K_y S_t(x/y)} - 1 = \dot{S}(x/y) \tag{5.40}
\]
Similarly, the percentage change in the \( x \) currency value of the \( z \) position is
\[
\dot{V}_x(z) = \dot{S}(x/z) \tag{5.41}
\]
If \( S(x/y) \) and \( S(x/z) \) are perfectly negatively correlated, it follows that
\[
\dot{S}(x/z) = -\dot{S}(x/y) \tag{5.42}
\]
which gives
\[
\dot{V}_x(y) = -\dot{V}_x(z) \tag{5.43}
\]
implying that the profit/loss on one position is completely offset by the loss/profit on the other. If \( S(x/y) \) and \( S(x/z) \) are perfectly positively correlated, it follows that \( \dot{V}_x(y) = \dot{V}_x(z) \). In this case, taking a long position on \( y \) and a short position on \( z \), or vice versa, produces the same result.

**Cross hedging**

Cross hedging is used when it is not possible to hedge exposure to a foreign currency because of the unavailability of hedging instruments, such as forward contracts and options, on this currency. In this case we look for another foreign currency that is highly correlated with the currency to be hedged, and then take a forward, futures or an options position on this currency.

Let us now assume that a forward contract is not available on currency \( y \) but a contract on another foreign currency, \( z \), is available. Cross forward hedging boils down to buying forward an amount of \( z \) that is equivalent to the payables at the current spot rate between \( y \) and \( z \). At this exchange rate, the \( z \) amount equivalent to \( K \) units of \( y \) is \( K/[S_t(y/z)] \). The domestic currency value of this amount when it is bought forward is
\[
V_C = \frac{K F_t(x/z)}{S_t(y/z)} \tag{5.44}
\]
where \( V_C \) is the domestic currency value of the payables under cross hedging. At time \( t + 1 \), the amount \( z \) bought forward, \( K/[S_t(y/z)] \), is converted spot to \( y \).
and the proceeds are used to meet the payables in $y$. The amount of $y$ obtained is

$$V_C = \frac{KF_t(x/z)}{S_t(y/z)} - (A - K)S_{t+1}(x/y)$$  \hspace{1cm} (5.45)

Notice, however, that this amount may or may not be equal to the amount of the payables, $K$. The two amounts are equal (that is, $A = K$) only if the spot exchange rate between $y$ and $z$ is stable such that $S_{t+1}(y/z) = S_t(y/z)$. This will be the case if $S(x/y)$ and $S(x/z)$ are perfectly correlated. If this is not the case then the deficit is met by buying currency $y$ against $x$ spot at $t + 1$, whereas the surplus can be converted back to $x$ at the same rate. Hence, the base currency value of the payables under cross hedging is

$$V_C = \frac{KF_t(x/z)}{S_t(y/z)} - (A - K)S_{t+1}(x/y)$$  \hspace{1cm} (5.46)

Notice that if $A - K = 0$, then equation (5.46) will be identical to equation (5.44). By following this approach to cross currency hedging, Moosa (2001b) demonstrated, by using historical data on four currencies, that cross and direct hedging of recurrent exposure produced similar results.

Another approach to cross currency hedging does not involve the forward market. In this case a long (short) position on one currency can be hedged by taking a short (long) spot position on another currency. If $S(x/y)$ and $S(x/z)$ are highly correlated, then a firm with a base currency $x$ can hedge payables in $y$ by buying currency $z$. If $y$ appreciate against $x$, then $z$ will also appreciate, in which case the loss incurred on the short position in $y$ will be offset by the profit on the long position in $z$. Formally, if $S(x/y)$ and $S(x/z)$ are related by the equation

$$S(x/z) = a + bS(x/y)$$  \hspace{1cm} (5.47)

then

$$\dot{S}(x/z) = b\dot{S}(x/y)$$  \hspace{1cm} (5.48)

and thus perfect correlation implies a perfect hedge because

$$\dot{S}(x/z) - b\dot{S}(x/y) = 0$$  \hspace{1cm} (5.49)

which means that any loss on one position will be completely offset by gains on the other and vice versa. Brooks and Chong (2001) found that cross currency hedging reduces volatility by about 15%. Siegel (1997) used a similar approach to hedging currency risk.

Table 5.9 shows the correlation coefficients of the percentage changes in the exchange rates of a number of currencies when the exchange rates are measured against the US dollar and when they are measured against the Swedish krona (in parentheses). It can be seen that there are some really high
TABLE 5.9 Correlations of percentage changes in exchange rates against the USD and SEK.

<table>
<thead>
<tr>
<th></th>
<th>AUD</th>
<th>CAD</th>
<th>CHF</th>
<th>DKK</th>
<th>GBP</th>
<th>JPY</th>
<th>NOK</th>
<th>NZD</th>
<th>SEK</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD</td>
<td>1.00</td>
<td>0.43</td>
<td>-0.13</td>
<td>0.01</td>
<td>0.08</td>
<td>0.04</td>
<td>0.06</td>
<td>0.70</td>
<td>0.12</td>
</tr>
<tr>
<td>(1.00)</td>
<td>(0.82)</td>
<td>(0.18)</td>
<td>(0.41)</td>
<td>(0.50)</td>
<td>(0.50)</td>
<td>(0.50)</td>
<td>(0.89)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CAD</td>
<td>1.00</td>
<td>-0.24</td>
<td>-0.17</td>
<td>0.05</td>
<td>-0.04</td>
<td>-0.01</td>
<td>0.16</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>(1.00)</td>
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<td>(0.66)</td>
<td>(0.57)</td>
<td>(0.60)</td>
<td>(0.79)</td>
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</tr>
<tr>
<td>(1.00)</td>
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<td>(0.64)</td>
<td>(0.59)</td>
<td>(0.71)</td>
<td>(0.38)</td>
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correlations that would make spot cross currency hedging successful. For example, a firm whose base currency is the Swedish krona can hedge a short exposure on the Canadian dollar by going long on the New Zealand dollar or the Canadian dollar. Likewise, a US dollar based firm can hedge a long exposure on the Danish kroner by going short on the Swiss franc.

**Exposure netting**

A natural hedge would arise when a firm has both payables and receivables in the same currency. In this case only the net exposure should be covered. Jorion (2001, p. 475) argued that, until recently, hedging systems typically consisted of focusing on and hedging each source of risk separately, which is an inefficient approach.

Exposure netting may involve the same currency or currencies with highly correlated exchange rates. Consider the case of the same currency first by assuming that a firm has both payables and receivables in currency $y$. If the positions are of equal sizes, then the net position will be zero, and there is no need to do anything about it because a natural perfect hedge is in place. If the positions are of different sizes then only the difference should be hedged. For example, if payables are $K_1$ and receivables are $K_2$ such that $K_1 > K_2$, then a long forward position of $K_1 - K_2$ should be taken on currency $y$. 

Consider now a position of payables in currency $y$ and receivables in currency $z$, such that $S(x/y)$ and $S(x/z)$ are highly, but not perfectly, correlated. A gain on one position will be partially offset by a loss on the other. In this case only the residual risk that cannot be eliminated by combining the two positions should be hedged via a forward or a futures contract.

**Currency of invoicing**

A firm can shift, share or diversify exposure by choosing an appropriate currency of invoicing. Price shifting is implemented by setting the price completely in base currency terms. In this case, the base currency price will be fixed but the foreign currency price will change with the exchange rate, rising as the exchange rate falls, as shown in Figure 5.12. From the perspective of the exporting firm this method eliminates foreign currency exposure, but it may not be possible to implement if, for example, foreign prices are fixed by contract. Another way to reduce foreign exchange exposure by shifting is to invoice receivables in the same currency used to invoice payables, be it the base or a foreign currency. Diversifying means using currency baskets or composite currencies, such as the SDR, as the currency of invoicing. On several occasions it has been mentioned that OPEC may invoice oil in SDR rather than in US dollar.

Risk sharing means invoicing part of the shipment in base currency terms. Sometimes, risk sharing is implemented by using a customised hedge contract embedded in the underlying trade contract. The hedge agreement typically

**FIGURE 5.12** Increasing the foreign currency price to counterbalance foreign currency depreciation.
takes the form of a price adjustment clause whereby a base price is adjusted to reflect certain exchange rate changes. Formally, risk sharing works as follows. Given the $y$ currency value of the contract, the $x$ currency value is obtained by converting at a range of exchange rates. First, a base rate is set, say $\bar{S}$. Then a neutral zone is set around this rate, say between $\bar{S}(1-\theta)$ and $\bar{S}(1+\theta)$, where $0<\theta<1$. Within the neutral zone, the payables are converted at $\bar{S}$, which means that the $x$ currency value of the payables is $KS$. Formally, if $\bar{S}(1-\theta)<S_{t+1}<\bar{S}(1+\theta)$, then $V_x = KS$ and $dV_y/dS_{t+1} = 0$. If the exchange rate falls below the lower limit of the neutral zone such that $S_{t+1}<\bar{S}(1-\theta)$, then the payables are converted at a rate that is equal to the base rate less half the difference between the lower limit and the actual exchange rate. This gives

$$V_x = K \left[ \frac{\bar{S}(1-\theta)-S_{t+1}}{2} \right]$$  \hspace{1cm} (5.50)

which can be simplified to produce

$$V_x = K \left[ \frac{\bar{S}(1+\theta)+S_{t+1}}{2} \right] > KS_{t+1}$$  \hspace{1cm} (5.51)

which means that the $x$ currency value of the payables is greater than what it would be if there was no hedge. The benefit of a depreciation in $y$ is shared between the payer and the payee in the sense that the payer does not enjoy the full extent of the depreciation of $y$, whereas the payee does not suffer to the full extent. The benefit accruing to the payee, compared with the no-hedge decision, is given by

$$V_x - KS_{t+1} = K \left[ \frac{\bar{S}(1+\theta)-S_{t+1}}{2} \right] > 0$$  \hspace{1cm} (5.52)

On the other hand, if the exchange rate rises above the upper limit of the neutral zone, such that $S_{t+1} > \bar{S}(1+\theta)$, then the payables are converted at a rate that is equal to the base rate and half the difference between the actual exchange rate and the upper limit of the neutral zone. This gives

$$V_x = K \left[ \frac{\bar{S}+S_{t+1}-\bar{S}(1+\theta)}{2} \right]$$  \hspace{1cm} (5.53)

which can be simplified to produce

$$V_x = K \left[ \frac{\bar{S}(1-\theta)+S_{t+1}}{2} \right] < KS_{t+1}$$  \hspace{1cm} (5.54)

which means that the $x$ currency value of the payables is lower than what it would be if there was no hedge. The result of an appreciation of $y$ is shared between the payer and the payee in the sense that the payer pays less than
under the no-hedge decision, while the payee receives less than otherwise. The benefit accruing to the payer, as compared with the no-hedge decision is given by

\[ V_x - KS_{t+1} = K \left[ \frac{S(1-\theta) - S_{t+1}}{2} \right] < 0 \]  

(5.55)

Notice that under the no-hedge decision \( dV_x/dS_{t+1} = K \), but under a risk-sharing scheme, \( dV_x/dS_{t+1} = K/2 \), which means that the base currency value of the payables changes at half the rate than under the no-hedge decision.

A foreign exchange risk sharing scheme is represented diagrammatically in Figure 5.13. The value of the payables without this arrangement is represented by an upward sloping straight line with the equation \( V_x = KS_{t+1} \). The value of the payables under foreign exchange risk sharing follows a staggered path. If the exchange rate falls below the lower limit of the neutral zone, it is higher than under no-hedge decision, although it rises at half the rate. If the exchange rate rises above the upper limit, it falls below that under the no-hedge decision, and rises half as fast. In between, the value is constant, as represented by a horizontal line with the equation \( V_x = KS \).

**Currency collars**

We will now illustrate the use of currency collars, also called range forward, in hedging receivables. A currency collar is used to set a minimum value for the base
currency receivables at the expense of setting a maximum value. Thus, it involves a trade off between potential loss and potential gain. A currency collar contains a certain range for the exchange rate ranging between a lower limit, $S_L$, and an upper limit, $S_U$. If the exchange rate falls below the lower limit, the rate used to convert receivables into the base currency is the lower limit itself, and this is how the minimum value is obtained. If the exchange rate falls within the range, the conversion rate is the current exchange rate, which means that the base currency value of the receivables rises with the exchange rate within this range. Finally, if the exchange rate rises above the upper limit, the conversion rate is the upper limit, and this is how the maximum value is obtained. These possibilities are displayed in Table 5.10, while a diagrammatic representation can be found in Figure 5.14.

A currency collar can be created by using a cylinder consisting of a short call and a long put with the same price and exercise exchange rates of $S_U$ and $S_L$ respectively. Figure 5.15 shows how this can be made possible. The upper two parts of the diagram show the payoff on these two positions. By combining

<table>
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<tr>
<th>$S_{t+1}$</th>
<th>$V_x$</th>
<th>$dV_x/dS_{t+1}$</th>
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<tbody>
<tr>
<td>$S_{t+1} &lt; S_L$</td>
<td>$KS_L$</td>
<td>0</td>
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<tr>
<td>$S_L \leq S_{t+1} \leq S_U$</td>
<td>$KS_{t+1}$</td>
<td>$K$</td>
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<tr>
<td>$S_{t+1} &gt; S_U$</td>
<td>$KS_U$</td>
<td>0</td>
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**TABLE 5.10** The value of base currency receivables under a currency collar.

![Diagram](image.png)

**FIGURE 5.14** Hedging receivables by using a currency collar.
these payoffs we obtain a cylinder, and by combining the payoff on the cylinder with the payoff on the unhedged position (the receivables), we get the payoff on the hedged position as shown in the lower part of the diagram. This part exactly resembles the currency collar displayed in Figure 5.14.

FIGURE 5.15 Creating a currency collar by using an option cylinder.
5.10 HEDGING OPERATING EXPOSURE

Hedging operating exposure is not a short-run tactical issue, but is rather a strategic issue that invariably takes the form of applying techniques involving the restructuring of operations. In general, these operational techniques can be classified into marketing techniques and production techniques. Financial hedging may also be used. These will now be discussed in turn.

Marketing techniques
Marketing techniques include the following:

1. Diversification of markets. Reduced sales in one country can be counterbalanced by increased sales in another, provided that exchange rates are not perfectly correlated. It is also possible to diversify across business lines if the demands have different elasticities.
2. Pricing strategy. This strategy boils down to the question of whether to emphasise market share or profit margin. The greater the elasticity of demand the greater the incentive to hold prices down.
3. Product strategy. Firms can respond to foreign exchange risk by altering their product strategy, which encompasses such areas as new product introduction, product line decisions and product innovation. One way to cope with exchange rate fluctuations is to change the timing of the introduction of new products. For example, the period after base currency depreciation may be the best time to develop a brand franchise. Exchange rate fluctuations also affect product line decisions. Following depreciation of the base currency, a firm may find it useful to expand product line.
4. Advertising. The effect of advertising is to reduce the elasticity of demand. Lower elasticity means that demand and revenue will not be significantly affected by exchange rate-induced price hikes.

Production techniques
Production techniques include the following:

1. Sources of inputs. A flexible sourcing policy means that even if production facilities are located only at home, input cost can be reduced by sourcing from low-cost countries.
2. Selecting low-cost production sites. When the domestic currency is strong or expected to be so, production facilities must be located in a country where the currency is undervalued.
3. Shifting production among plants. Firms with worldwide production facilities can allocate production among their plants in line with the changing base currency value of inputs.
4. Plant location. Erecting new plants as dictated by changes in exchange rates.
5. Raising productivity, which can be accomplished by closing down inefficient plants, automating heavily, negotiating wages and revising product offerings.

6. Research and Development (R&D). This activity results in lower costs, higher productivity and new products with no close substitutes.

**Financial hedging**

Marketing and production techniques constitute operational hedging, which is different from financial hedging, as we have seen before. The latter involves lending and borrowing on a long-term basis, swaps, long-term currency options, and forwards. The problem is that financial contracts are designed for hedging against changes in the nominal exchange rates, whereas operating exposure results from changes in the real exchange rate.

Indeed, there is some controversy about the most appropriate approach to hedging operating exposure. One group of researchers, including Srinivasulu (1981), Aggarwal and Soenen (1989), Lessard and Lightstone (1986) and Cornell and Shapiro (1983) emphasise the role of production techniques. Specifically, they argue that firms facing operating exposure should establish their own production facilities in the same country or countries as their competitors and shift production to these countries when their currencies appreciate in real terms. The second group, including Ware and Winter (1988) and Sercu (1992), examined the risk profile of economic exposure and concluded that this is a nonlinear function of real exchange rates. On the basis of the shape of the risk profile of the exposure, they argued that currency options should be used for hedging economic exposure, since the profile of currency options is nonlinear.

Kanas (2001) makes a contribution to this issue by developing a simple model of the production management approach to show that a change in the production location, as a means of managing exposure, is equivalent to the exercise of a real call option on the overseas production facility. In this sense, a project or an operation is said to involve a real option when the firm can choose between mutually exclusive strategic policies, depending on the behaviour of the underlying stochastic variable. Examples of real options include options to alter the scale of projects and the input mix, and abandoning the project altogether (Kulatilaka and Marcus, 1988). On the basis of this model, Kanas shows that the exercise of a real call option for hedging economic exposure is not linked with real exchange rate changes that cause the exposure. He also shows that exercising the option of changing the location of production may yield a negative net present value, which makes production management inconsistent with capital budgeting considerations. The most important result of this work is that a currency option can be efficiently used to hedge the exposure by adjusting the exercise exchange rate by a factor that reflects the inflation differential expected to prevail during the hedge period. Kanas regards this result to be a generalisation of the findings of Ware and Winter (1988) and those of Sercu (1992).
Allayannis *et al.* (2001) investigated the importance of financial and operational hedging strategies. They used four proxies for a firm’s operational hedging: (i) the number of countries in which it operates, (ii) the number of broad regions in which it is located, (iii) the geographical dispersion of its subsidiaries across countries, and (iv) the geographical dispersion of subsidiaries across regions. Using a sample of US non-financial multinationals, they found that operational hedging is not an effective substitute for financial hedging. They also found that more geographically dispersed firms are more likely to use financial hedging. The end result on the value of the firm is that operational hedging benefits shareholders only when used in combination with financial hedging strategies. They conclude that firms that rely exclusively on operational hedging may not maximise shareholder value.

**An example**

Hedging operating exposure invariably involves the restructuring of operations. Consider the risk of declining net income resulting from real foreign currency appreciation. Operating exposure could be reduced by increasing the sensitivity of revenues and reducing the sensitivity of expenses to exchange rate movements. The first task requires making demand more elastic, so that any small change in the relative prices of foreign and domestic goods resulting from foreign currency appreciation could bring about a greater increase in both domestic and foreign demand for the firm’s products. This can be accomplished, for example, by increasing expenditure on advertising. The second task of reducing the sensitivity of expenses can be accomplished by changing the source of imported raw materials, preferably to a domestic source, or even negotiating an agreement whereby the invoices are billed in base currency terms.

The case of Laker Airways, a British company that collapsed in the 1980s, is a classic example of a company going into bankruptcy because of both transaction exposure and operating exposure. When the pound depreciated against the dollar in early 1982, the economic exposure had its toll on the company’s operations as demand for transatlantic flights by British holiday makers fell, adversely affecting sales revenues. The transaction exposure had its toll when Laker’s US dollar-denominated debt became due. Operating exposure could have been reduced by reducing dollar-denominated expenses and increasing dollar-denominated revenues. Having failed to do that, the company went bankrupt.

### 5.11 Hedging Translation Exposure

As we have seen, translation exposure is a source of concern because different translation methods have different impacts on the reported earnings per share and other vital indicators. For the prosperity of the firm, how the investment community interprets the published financial statements is important.
There are three methods for hedging translation exposure: adjusting fund flows, entering into forward contracts, exposure netting, and balance sheet hedging.

**Fund adjustment**
Fund adjustment involves altering the amounts and/or the currencies of the planned cash flows of the firm or its subsidiaries to reduce exposure to the currency of the subsidiary. If the currency of the subsidiary is expected to depreciate, direct fund adjustment methods include: (i) pricing exports in hard currencies and imports in the base currency, (ii) investing in hard currency securities, and (iii) replacing hard currency borrowings with base currency loans. Indirect methods include (i) adjusting transfer prices; (ii) speeding up the payment of dividends, fees and royalties; and (iii) adjusting the leads and lags of the intersubsidiary accounts.

**Entering forward contracts**
Translation exposure can be hedged via forward contracts. If the base currency of a foreign subsidiary is expected to depreciate against the parent firm’s base currency, then translation exposure exists. Even if the foreign currency earnings of the subsidiary are not actually converted into the parent’s base currency, foreign currency depreciation results in a translation loss on the consolidated financial statements. The parent firm may hedge this exposure by selling forward an amount of the foreign currency that is equal to the subsidiary’s expected net income. If the expectation materialises and the foreign currency depreciates, the company will make profit on the short forward position that will compensate for the translation loss. A major problem with this operation is that the forward position results in increasing transaction exposure.

**Exposure netting and balance sheet hedging**
Exposure netting can be used by multinationals with offsetting positions in more than one foreign currency. A balance sheet hedge eliminates the mismatch between assets and liabilities in the same currency. If they are equal then a change in the exchange rate will not matter.

### 5.12 WHAT DO FIRMS DO IN PRACTICE?

Three questions pertaining to the behaviour of firms in reality are addressed in this section:

1. Do firms hedge?
2. If they do, which exposure do they hedge?
3. If they do, what hedging instruments and techniques do they use?
The questions can only be answered by surveying the actual practice of firms with respect to hedging. Following their survey, Jesswein et al. (1995) documented the extent of knowledge and use of foreign exchange risk management products by 500 US firms. The products considered were forward contracts, currency swaps, currency futures, straight options, futures options, synthetic forwards, synthetic options and other more sophisticated instruments, such as compound options and lookback options. The results of the survey showed that 93% of the respondents used forward contracts followed by swaps and options. Only 5.1% and 3.8% used lookback options and compound options respectively.

Joseph (2000) obtained a measure of the degree of utilisation of hedging techniques on the basis of a survey of 109 companies belonging to the top 300 category of The Times 1000: 1994. The following results were obtained:

1. British firms utilise a narrow set of techniques to hedge exposure.
2. They place much more emphasis on currency derivatives than on internal hedging techniques. This result is not consistent with the approach that is suggested in the academic literature (McRae and Walker, 1980) and the implications of prior empirical work (for example, Hakkarainen et al., 1988).
3. Firms place more emphasis on transaction exposure and economic exposure and much less on translation exposure.
4. There is strong cross-sectional variation in the characteristics of firms that hedge.

Marshall (2000) surveyed the foreign exchange risk practices of 179 large British, American and Asia Pacific multinational firms. The following results were obtained:

1. There are some notable variations between British and American firms and in particular respondents from Asia Pacific firms. Differences pertain to the importance and objectives of foreign exchange risk, emphasis on translation and economic exposure, the use of internal/external hedging techniques, and the policies used to manage economic exposure.
2. The percentage of overseas business is not a significant factor (but size and the industry sector are significant) in determining the importance of foreign exchange risk, emphasis on economic and translation exposure, or the methods used for hedging.
3. The main objectives of managing foreign exchange risk are the minimisation of fluctuations in earnings and seeking certainty of cash flows.
4. Firms place more emphasis on transaction exposure and less emphasis on translation exposure, particularly in the USA.
5. For translation exposure, the main internal method used is balance sheet hedging, whereas matching and netting are the predominantly internal methods used for managing transaction exposure.
6. The most popular external method for managing translation and transac­tion exposure is the forward contract, although swaps are popular with
British firms.
7. The majority of firms do not favour exchange-traded instruments, such as
currency futures and options on currency futures.
8. The industry sector is an important determinant of the use of external
derivatives, particularly exchange-traded derivatives.
9. Reasons for not managing economic exposure include the difficulty of
quantifying the exposure and the lack of effective tools to deal with the
complexity of this exposure.
10. Pricing strategies and the currency of invoicing are the most widely used
methods to deal with economic exposure.

The question of why firms hedge has been dealt with by Francis and
Stephan (1993), who tested eight different hypotheses about the hedging
behaviour of firms. The hypotheses are:

1. Hedging firms are more likely to have binding debt restrictions than non-
hedging firms.
2. Hedging firms have a higher probability of bankruptcy than non-hedging firms.
3. Hedging firms are smaller than non-hedging firms.
4. Hedging firms have higher tax rates than non-hedging firms.
5. Managers of hedging firms are of higher ability than managers of non-
hedging firms.
6. Over time, hedging firms are more likely to experience reductions in the
restrictiveness of their debt covenants, expected bankruptcy costs and tax
rates than non-hedging firms.
7. Over time, hedging firms are more likely to experience increases in proxies
for managerial ability than non-hedging firms.
8. Over time, hedging firms are more likely to experience (a) an increase
in size (as measured by total assets, net sales and market value of equity) if
managers hedge to reduce bankruptcy costs; or (b) a reduction in size if
managers hedge to avoid political costs, than are non-hedging firms.

The results of univariate tests support the debt covenant, political cost and
signalling hypotheses, but they do not provide strong evidence consistent
with the tax motivations to hedge or the theory that firms hedge to avoid
bankruptcy costs. The multivariate tests do not support the covenant or bank-
ruptcy cost explanations, but provide strong evidence favouring the political
cost explanation. The time series results show some evidence that over time
hedging firms experience reductions in the restrictiveness of debt covenants,
the probability of bankruptcy and tax rates, as well as increases in size and
managerial ability.

Other studies have found a variety of results. Nance et al. (1993) and Dolde
(1995) used a questionnaire survey, which required respondents to indicate
whether or not they use one or more of four currency derivatives: forward, futures, swaps and options. In contrast, Berkman and Bradbury (1996) classified firms according to the hedging information contained in their audited financial reports. Joseph (2000) criticised both of these approaches because firms are only required to disclose exposure information if such information is material, in which case the second approach may not fully capture the hedging activities of firms. Also, the first approach is restrictive because firms use a wide range of techniques and instruments to hedge exposure to foreign exchange risk. Dolde (1993) found that firms may or may not hedge or may partially hedge, depending on their perception about the behaviour of exchange rates and/or their confidence in handling derivatives.

It has been found that firms use a wide variety of techniques to hedge exposure (for example, Hakkarainen et al., 1998). Although newer financial innovations can reduce the demand for traditional types of hedging techniques, empirical evidence shows that firms are not that receptive to complex types of derivatives (Tufano, 1995). This is because firms are concerned about the banks’ commitment to those products and their ability to provide real solutions to exposure problems (for example, Fairlamb, 1988; Galum and Belk, 1992).

It has also been found that managers tend to adjust their hedging decisions to reflect their expectations of changes in financial prices. Thus, if the forward rate is a biased predictor of the spot rate, managers can alter their hedging strategies to accommodate this effect. Berg and Moore (1991) and Schooley and White (1995) argue that a partial, no hedge or fully hedged strategy can be optimal in this case for both transaction and economic exposures. Houry and Chan (1988) and Joseph and Hewins (1991) argue that the use of hedging techniques may reflect the types of exposure they hedge (in general caring more about transaction exposure than about economic or translation exposure).

Giddy and Dufey (1995) argue that options are not ideal hedging instruments because the gains/losses arising from their use are not linearly related to changes in exchange rates. But Ware and Winter (1988) argue that forward contracts can only hedge economic exposure in an optimal manner if managerial decision regarding inputs and outputs are fixed, otherwise options are more appropriate. Based on an analysis of the foreign exchange exposure of the Australian equity market, De Iorio and Faff (2000) present some evidence for asymmetry, which they attribute to the use of currency options, as they limit the downside exposure while permitting the potential upside gains.

Despite the widespread use and importance of options in risk management, it is argued that the optimality of options being a hedging instrument remains largely unexplained (Broll et al., 2001). On the one hand, it is argued by Lapan et al. (1991) that currency options are useful for hedging only if the forward market and/or option premiums are biased. However, Moschini and Lapan (1995) show that production flexibility of the competitive firm under price certainty leads to an \textit{ex post} profit function that is convex in prices, thereby
inducing the firm to use options for hedging. Sakong et al. (1993) and Moschini and Lapan (1995) show that production uncertainty provides another rationale for using options, because it is related to the multiplicative interaction between price and yield uncertainty, which affects the curvature of the firm’s profit function. Lence et al. (1994) show that forward-looking firms would use options as a hedging instrument because they are concerned about the effects of future prices on profit from future production cycles. Finally, Broll et al. (2001) offer yet another rationale for the hedging role of options when the underlying uncertainty is nonlinear.

Another issue that needs to be brought up here is the extent to which firms use operational as opposed to financial hedging. There seems to be a mixture of views on this issue. In its 1995 annual report, Schering-Plough argues in support of the exclusive use of operational hedging by saying that “to date, management has not deemed it cost effective to engage in a formula-based program of hedging the profitability of these operations using derivative financial instruments. Some of the reasons for this conclusion are: the company operates in a large number of foreign countries; the currencies of these countries generally do not move in the same direction at the same time”. On the other hand, it is well known that many corporations with large worldwide networks, such as IBM and Coca-Cola, make extensive use of derivative financial instruments.

An example: Microsoft

Microsoft uses the currency of invoicing to hedge its foreign exchange exposure. In some regions it uses the US dollar (the company’s base currency) to bill its customers. This approach is used in Latin America, Eastern Europe and South-East Asia. The problem with this approach is that Microsoft becomes exposed to economic risk if the domestic currencies in these areas depreciate against the US dollar. In other parts of the world, Microsoft conducts its business in the local currency. Microsoft’s 2000 annual report states that “finished goods sales to international customers in Europe, Japan, Canada and Australia are primarily billed in local currencies”.

Microsoft also has substantial expenses in Europe associated with its manufacturing, sales and services. These expenses are denominated in local currencies. Every month Microsoft’s profits/losses are converted into US dollars at the average exchange rate for the month. Because of this feature, what matters is the average exchange rate for the month, not the end of the month rate. To hedge this risk, Microsoft makes extensive use of average rate options.

The problem with the policy of using the US dollar as the currency of invoicing in some parts of the world is that a significant depreciation of the local currency against the dollar implies a significant rise in the local currency price of the products, which may adversely affect demand and revenue, particularly if demand is elastic. To soften the impact of a strong US dollar,
Microsoft enters a long forward contract to buy the dollar against the local currency. If the local currency depreciates, profit will be made on the forward contract, and some of this profit is channelled to the distributors to relieve the pressure on their profit margins. In 1999, Microsoft used this approach to mitigate the impact of the depreciation of the Brazilian real.
6.1 THE CONCEPT OF THE HEDGE RATIO

Hedging is an attempt to reduce the risk of adverse price changes, such as the exchange rate implicit in a spot position on a currency. Financial hedging, as we have seen, entails taking an offsetting position on another asset or a hedging instrument (say, a forward position on the same or another currency, with the latter constituting cross hedging). The position must be offsetting in the sense that if the unhedged position is long (say receivables) then the position on the hedging instrument must be short, and vice versa. The idea is that if a loss is incurred on the unhedged position, it will be offset by profit on the position in the hedging instrument, and vice versa. Among others, two important questions are involved in the hedging operation: (i) to hedge or not to hedge; and (ii) if the decision to hedge is taken, should the full position be hedged?

We dealt with the first question in the previous chapter, reaching the conclusion that the hedging decision may or may not be taken, because only a completely risk-averse firm will always hedge exposure to foreign exchange risk irrespective of exchange rate expectations. This view is supported by theoretical reasoning and survey evidence. However, we have so far assumed that hedging always involves the full position. This means that the assumption so far is that a hedge ratio of one is always chosen. The problem here is that even if the agent is risk averse, a hedge ratio of one may not be optimal, in the sense that it will not eliminate the risk completely or it may result in a smaller risk reduction than under a different hedge ratio.

Determining the hedge ratio amounts to choosing the size of the position on the hedging instrument that is used to hedge the unhedged (spot) position. If the size of the position on the hedging instrument is equal to the value of the spot position, then we have a hedge ratio of one. Formally, consider the rate of return on a hedged position, with a hedge ratio of $h$:

$$R_H = R_U - hR_A$$  \hspace{1cm} (6.1)
where $R_H$ is the rate of return on the hedged position, $R_U$ is the rate of return on the unhedged position and $R_A$ is the rate of return on the asset used as a hedging instrument. These rates of return are measured over the period between the point in time when the hedge is taken and that when the spot position is liquidated or materialises (for example, when payables and receivables become due). For a perfect hedge, in the sense that the loss (gain) on the unhedged position is completely offset by the gain (loss) on the hedging instrument, $R_H = 0$, which means that

$$h = \frac{R_U}{R_A}$$  \hspace{1cm} (6.2)

Generally speaking, a perfect hedge is obtained with a hedge ratio of one, only if $R_U = R_A$, which means that the prices of the unhedged asset and the hedging instrument are perfectly correlated. If $R_U \neq R_A$, then for $R_H = 0$, the condition $h \neq 1$ must be satisfied. Specifically, if $R_U > R_A$, then for a perfect hedge the condition $h > 1$ must be satisfied, and if $R_U < R_A$, then a perfect hedge requires $h < 1$.

Now, assume that $R_U \neq R_A$, such that $R_U$ and $R_A$ are related by the equation

$$R_U = \alpha + \beta R_A$$  \hspace{1cm} (6.3)

By substituting equation (6.3) into equation (6.1) we obtain

$$R_H = \alpha + \beta R_A - h R_A$$  \hspace{1cm} (6.4)

which means that for an optimal hedge we have

$$h = \frac{\alpha}{R_A} + \beta$$  \hspace{1cm} (6.5)

or equivalently that

$$h = \frac{\beta R_U}{R_U - \alpha}$$  \hspace{1cm} (6.6)

Consider now the special case of using forward hedging. If the unhedged position is a long position on currency $y$, then the hedging instrument is a short forward position on $y$. The rate of return on the unhedged spot position and between $t-1$ and $t$ is given by

$$R_U = \frac{S_t}{S_{t-1}} - 1 = \hat{S}$$  \hspace{1cm} (6.7)

Let us for the time being define the rate of return on the forward position as

$$R_A = \frac{S_t}{F_{t-1}} - 1$$  \hspace{1cm} (6.8)
in the sense that return arises by buying a currency at $F_{t-1}$ and selling it at $S_t$. Since $F_{t-1} = (1 + f)S_{t-1}$, where $f$ is the forward spread, it follows that

$$R_A = \frac{1 + \hat{S}_t}{1 + f} - 1 = \frac{\hat{S}_t - f}{1 + f} \quad (6.9)$$

For a perfect hedge, we have

$$h = \frac{\hat{S}_t}{(\hat{S}_t - f)/(1 + f)} = \frac{\hat{S}(1 + f)}{\hat{S}_t - f} \quad (6.10)$$

It follows from equation (6.10) that for $h = 1$, the condition $f = 0$ must be satisfied. This means that if the full hedge is to be a perfect hedge, the forward spread must be equal to zero, and this would be so if the interest rates on the two currencies are equal, which is not normally the case. If a money market hedge is used instead of a forward hedge, the same arguments hold, except that the price of the hedging instrument will in this case be the interest parity forward rate, which is the forward rate consistent with covered interest parity.

Consider now a comparison between the hedge ratio under a direct forward hedge and a cross forward hedge. Let us for this propose introduce a third currency, $z$, and rewrite equations (6.7) and (6.8) as

$$R_U = \frac{S_t(x/y)}{S_{t-1}(x/y)} - 1 = \hat{S}(x/y) \quad (6.11)$$

$$R_A = \frac{S_t(x/z)}{F_{t-1}(x/z)} - 1 = \frac{\hat{S}(x/z) - f(x/z)}{1 + f(x/z)} \quad (6.12)$$

Therefore, the optimal hedge ratio is given by

$$h = \frac{\hat{S}(x/y)[1 + f(x/z)]}{\hat{S}(x/z) - f(x/z)} \quad (6.13)$$

which is obviously equal to one only if $f(x/z) = 0$ and $\hat{S}(x/y) = \hat{S}(x/z)$.

If cross hedging involves two spot positions, then $R_A = \hat{S}(x/z)$, in which case the optimal hedge ratio is given by

$$h = \frac{\hat{S}(x/y)}{\hat{S}(x/z)} \quad (6.14)$$

which is equal to one only if $\hat{S}(x/y) = \hat{S}(x/z)$.

**6.2 MEASURING THE OPTIMAL HEDGE RATIO**

It is important for firms to employ the most effective model to calculate the optimal hedge ratio. In reality it may never be possible to get a perfect hedge,
in which case the objective may be to minimise the variance of the rate of return on the hedged position. This variance is given by

\[
\sigma^2 (R_H) = \sigma^2 (R_U) + h^2 \sigma^2 (R_A) - 2h\sigma(R_U, R_A)
\]  

(6.15)

The minimum-risk hedge ratio is calculated from the first-order condition

\[
\frac{d\sigma^2 (R_H)}{dh} = 2\sigma^2 (R_A)h - 2\sigma(R_U, R_A) = 0
\]  

(6.16)

Hence

\[
h = \frac{\sigma(R_U, R_A)}{\sigma^2 (R_A)}
\]  

(6.17)

The minimum-risk hedge ratio can be calculated from historical data by estimating the regression equation

\[
R_{U,t} = \alpha + hR_{A,t} + \epsilon_t
\]  

(6.18)

The early literature focused on estimating the hedging ratio by using OLS to estimate equation (6.18) (for example, Ederington, 1979). However, this approach is subject to two criticisms. One problem is that risk minimisation without regard to the effect of expected return cannot be optimal. Hedging away the risk must also hedge away the expected return to bearing that risk. Thus, the optimal hedge ratio should be the one that maximises utility, which is a function of risk and return. Different hedge positions can be compared directly by examining their certainty equivalent returns (for example, Cecchetti et al., 1988).

The other problem is that the joint distribution of \((R_U, R_A)\), and therefore the hedge ratio, is estimated incorrectly since there is no adjustment for the fact that it changes over time. Thus, regression employing past data will not correctly estimate the current risk minimisation ratio.

Kahneman and Tversky (1979) and Tversky and Kahneman (1986, 1992) argue that individuals do not maximise expected utility but behave according to a set of rules known as “prospect theory”. One of the suggested rules is that individuals care for changes in wealth, instead of wealth itself. Lien (2001) suggests another rule, which is loss aversion. According to him, there are two versions of loss aversion. Strong loss aversion implies that individuals tend to be more sensitive to wealth losses than to wealth gains. This approach postulates constant sensitivity to losses. The other version is that of weak loss aversion, which accounts for diminishing sensitivity to losses. Albuquerque (1999) considers futures hedging under loss aversion, showing that it simply reduces to downside risk minimisation. Lien attempts to explore the effects of loss aversion on futures hedging with constant absolute-risk aversion. Specifically, he examined the effect of loss aversion on the futures trading behaviour of a short hedger.
Because the OLS estimation method does not take into account the time-varying nature of variances and covariances that make up the hedge ratio, economists started using the ARCH model of Engle (1982) and the GARCH model of Bollerslev (1986) to calculate a time-varying hedge ratio. Myers and Thompson (1989) argued that conventional procedures for estimating the hedge ratio are based on the unconditional rather than the more appropriate conditional variances and covariances. They further argue that basing hedging decisions on deviations from the conditional moments reflect more accurately the risk faced by hedgers. Myers (1991) and Baillie and Myers (1991) also suggested that optimal hedge ratio estimation may require the conditional variance and covariance of futures and spot prices (prices of the hedging instrument and the unhedged position respectively) to vary over time, resulting in time-varying hedge ratios. Recent literature deals mostly with estimating the hedge ratio by employing bivariate GARCH models, as demonstrated by Kroner and Sultan (1991, 1993) who used currency futures as the hedging instrument. When the variances and the covariances are allowed to vary over time, we use the conditional variances and covariances. Therefore

\[ s^2_t(R_{H,t}|\Omega_{t-1}) = s^2_t(R_{U,t}|\Omega_{t-1}) + h^2 s^2_t(R_{A,t}|\Omega_{t-1}) \]

\[ -2h \sigma_t(R_{U,t}, R_{A,t} | \Omega_{t-1}) \]  

(6.19)

where \( \Omega_{t-1} \) is the information set available at time \( t-1 \), \( s^2_t(\cdot) \) is the conditional variance and \( \sigma_t(\cdot, \cdot) \) is the covariance. Hence

\[ h_t|\Omega_{t-1} = \frac{\sigma_t(R_{U,t}, R_{A,t} | \Omega_{t-1})}{s^2_t(R_{A,t} | \Omega_{t-1})} \] 

(6.20)

The conditional variances and covariances can be calculated from various models as outlined by Brooks and Chong (2001), which include EWMA, univariate GARCH (GARCH, EGARCH, GJR, GARCH-t) and multivariate GARCH (VECH, BEKK). Otherwise, the ratio can be calculated by estimating the regression in a TVP framework via state space modelling and the Kalman filter.

### 6.3 EMPIRICAL MODELS OF THE HEDGE RATIO

Several empirical models are used to estimate the hedge ratio. These models are described in turn.

**The naïve model**

The naïve model simply implies that the hedge ratio is always 1. This is exactly the assumption we used to demonstrate the hedging decision in Chapter 5. Since the naïve model is naïve, we need to go no further in elaborating on it.
**The implied model**
The implied model allows the estimation of the conditional covariance by employing the implied volatilities derived from currency options. These volatilities may be readily available or they can be calculated from the Black–Scholes option pricing formula.

**The random walk model**
The random walk model assumes that the most appropriate forecast of future variance and covariance is the variance and covariance observed today. Again, the simplicity of this procedure requires us to go no further in our exposition.

**The conventional model**
Also called the simple model and the historical model, the conventional model amounts to estimating the hedge ratio from historical data by employing a linear OLS regression model. Let \( p_U \) and \( p_A \) be the logarithms of the prices of the unhedged position and the hedging instrument respectively, such that \( R_U = \Delta p_U \) and \( R_A = \Delta p_A \). The regression equation corresponding to equation (6.18) is

\[
\Delta p_{U,t} = \alpha + h \Delta p_{A,t} + \varepsilon_t
\]  

in which case \( h \) is the estimated hedge ratio and the \( R^2 \) of the regression measures the hedging effectiveness. Sometimes, the regression is written in levels rather than in first differences to give

\[
p_{U,t} = \alpha + h p_{A,t} + \varepsilon_t
\]  

in which case the optimal hedge ratio is defined as

\[
h = \frac{\sigma(p_U, p_A)}{\sigma^2(p_A)}
\]

(6.23)

The implication of equations (6.22) and (6.23) is that the objective of hedging is to minimise the variance of the price of (rather than the rate of return on) the hedged position.

**The error correction model**
One problem with the conventional OLS model is that equation (6.22) ignores short-run dynamics, whereas equation (6.21) ignores the long-run relationship as represented by (6.22). Specifically, if \( p_U \) and \( p_A \) are cointegrated such that \( \varepsilon_t \sim \text{I}(0) \), then equation (6.21) is misspecified, and the correctly specified model is an error correction model of the form

\[
\Delta p_{U,t} = \alpha + \sum_{i=1}^{n} \beta_i \Delta p_{U,t-i} + h \Delta p_{A,t} + \sum_{i=1}^{n} \gamma_i \Delta p_{A,t-i} + \theta \varepsilon_{t-1} + \xi_t
\]  

(6.24)
where \( \theta \) is the coefficient on the error correction term, which should be significantly negative for the model to be valid. This coefficient measures the speed of adjustment to the long-run value of \( P_U \), as implied by equation (6.22). In other words, it is a measure of the speed at which deviations from the long-run value are eliminated.

Lien (1996) argues that the estimation of the hedge ratio and hedging effectiveness may change sharply when the possibility of cointegration between prices is ignored. In Lien and Luo (1994) it is shown that although GARCH may characterise the price behaviour, the cointegration relationship is the only truly indispensable component when comparing the \textit{ex post} performance of various hedging strategies. Ghosh (1993) concluded that a smaller than optimal futures position is undertaken when the cointegration relationship is unduly ignored. He attributed the under-hedge results to model misspecification.

Lien (1996) provides a theoretical analysis of this conjecture by assuming a cointegrating relationship of the form \( \phi_t = P_{A,t} - P_{U,t} \). A simplified error correction model, which implies that prices adjust in response to disequilibrium, can be written as

\[
\Delta P_{U,t} = \alpha \phi_{t-1} + \xi_{1,t} \\
\Delta P_{A,t} = -\beta \phi_{t-1} + \xi_{2,t}
\]

A hedge ratio that minimises \( \sigma^2 (\Delta P_{U,t} - h \Delta P_{A,t}) \) is calculated as

\[
h = \frac{\sigma(\Delta P_{U,t}, \Delta P_{A,t})}{\sigma^2 (\Delta P_{A,t})} = \frac{\rho(\xi_{1,t}, \xi_{2,t})}{\sigma(\xi_{1,t}) \sigma(\xi_{2,t})}
\]

where \( \rho(\xi_{1,t}, \xi_{2,t}) \) is the correlation coefficient between \( \xi_{1,t} \) and \( \xi_{2,t} \). Alternatively it can be calculated from the regression equation

\[
\Delta P_{U,t} = \alpha + h \Delta P_{A,t} + \gamma \phi_{t-1} + \zeta_t
\]

If the cointegrating relationship is ignored, then the hedge ratio is calculated as

\[
h = \frac{\sigma(\Delta P_{U,t}, \Delta P_{A,t})}{\sigma^2 (\Delta P_{A,t})}
\]

From equations (6.25) and (6.26), we have

\[
\sigma(\Delta P_{U,t}, \Delta P_{A,t}) = \sigma(-\beta \phi_{t-1} + \xi_{2,t}, \alpha \phi_{t-1} + \xi_{1,t})
\]

\[
= -\alpha \beta \sigma^2 (\phi_{t-1}) + \rho(\xi_{1,t}, \xi_{2,t}) \sigma(\xi_{1,t}) \sigma(\xi_{2,t})
\]

and

\[
\sigma^2 (\Delta P_{A,t}) = \sigma^2 (-\beta \phi_{t-1} + \xi_{2,t}) = \beta^2 \sigma^2 (\phi_{t-1}) + \sigma^2 (\xi_{2,t})
\]
Hence the hedge ratio is measured as

\[ h = \frac{-\alpha \beta \sigma^2 (f_{t-1}) + \rho(\xi_{1,t}, \xi_{2,t}) \sigma(\xi_{1,t}) \sigma(\xi_{2,t})}{\beta^2 \sigma^2 (f_{t-1}) + \sigma^2 (\xi_{2,t})} \]  

(6.32)

Obviously, there is a difference between the expressions in equation (6.27) and equation (6.32). On the basis of these two expressions, Lien (1996) concludes that an errant hedger who mistakenly omits the cointegrating relationship always undertakes a smaller than optimal position on the hedging instrument.

By using a general specification of equations (6.25) and (6.26), we have

\[ \Delta p_{U,t} = \alpha \phi_{t-1} + \sum_{i=1}^{n} a_i \Delta p_{U,t-i} + \sum_{i=0}^{n} b_i \Delta p_{A,t-i} + \xi_{1,t} \]  

(6.33)

\[ \Delta p_{A,t} = -\beta \phi_{t-1} + \sum_{i=0}^{m} c_i \Delta p_{U,t-i} + \sum_{i=1}^{n} d_i \Delta p_{A,t-i} + \xi_{2,t} \]  

(6.34)

in which case the hedge ratio calculated on the basis of the correctly specified model is given by

\[ h = \frac{\sigma(\Delta p_{U,t}, \Delta p_{A,t})}{\sigma^2 (\Delta p_{A,t})} \frac{\rho(\xi_{1,t}, \Delta p_{U,t-i}, \Delta p_{A,t-i})}{\sigma^2 (\xi_{2,t})} = \rho(\xi_{1,t}, \xi_{2,t}) \left( \frac{\sigma(\xi_{1,t})}{\sigma(\xi_{2,t})} \right) \]  

(6.35)

whereas the errant hedger who does not take into account the cointegration relationship will choose a hedge ratio that is given by

\[ h = \frac{\sigma(\Delta p_{U,t}, \Delta p_{A,t})}{\sigma^2 (\Delta p_{A,t})} \frac{\rho(\xi_{1,t}, \Delta p_{U,t-i}, \Delta p_{A,t-i})}{\sigma^2 (\xi_{2,t}) + \beta^2 \sigma^2 (\phi_{t-1})} \]  

(6.36)

which means that the errant hedger will undertake a smaller than optimal position on the hedging instrument, incurring losses in hedging effectiveness.

**Bivariate ARCH/GARCH error correction models**

A bivariate ARCH/GARCH error correction model can be used to accommodate the two problems discussed so far, that of allowing for the possibility of cointegration and for the time-varying nature of the second moments and the hedge ratio. Kroner and Sultan (1993) use a bivariate GARCH error correction model of the form
\[ \Delta p_{U,t} = a + b(p_{U,t-1} - \Delta p_{A,t-1}) + \xi_{1,t} \quad (6.37) \]

\[ \Delta p_{A,t} = c + d(p_{U,t-1} - \Delta p_{A,t-1}) + \xi_{2,t} \quad (6.38) \]

\[ \begin{bmatrix} \xi_{1,t} \\ \xi_{2,t} \end{bmatrix} \sim N(0, H_t) \quad (6.39) \]

\[ H_t = \begin{bmatrix} \sigma_i^2 (\Delta p_{U,t} | \Omega_{t-1}) & \sigma_t (\Delta p_{U,t}, \Delta p_{A,t} | \Omega_{t-1}) \\ \sigma_t (\Delta p_{U,t}, \Delta p_{A,t} | \Omega_{t-1}) & \sigma_i^2 (\Delta p_{A,t} | \Omega_{t-1}) \end{bmatrix} \quad (6.40) \]

\[ \sigma_i^2 (\Delta p_{U,t} | \Omega_{t-1}) = \alpha_1 + \beta_1 \xi_{1,t-1}^2 + \delta_1 \sigma_i^2 (\Delta p_{U,t-1} | \Omega_{t-2}) \quad (6.41) \]

\[ \sigma_t^2 (\Delta p_{A,t} | \Omega_{t-1}) = \alpha_2 + \beta_2 \xi_{2,t-1}^2 + \delta_1 \sigma_i^2 (\Delta p_{A,t-1} | \Omega_{t-2}) \quad (6.42) \]

\[ h_t = \frac{\sigma(\Delta p_{U,t}, \Delta p_{A,t} | \Omega_{t-1})}{\sigma^2 (\Delta p_{A,t} | \Omega_{t-1})} \quad (6.43) \]

Again, the time subscripts on the hedge ratio and the second moments imply time-variation.

**The Kalman filter**

A time-varying hedge ratio can be estimated by applying the Kalman filter to equation (6.18), which can be written in a general form as

\[ R_U(t) = R_A(t)H(t) + u(t) \quad (6.44) \]

where \( H(t) \) is a vector of time-varying parameters (hedge ratios) and \( u(t) \) is normally distributed with \( E[u(t)] = 0 \) and \( \sigma^2 [u(t)] = V \). A common specification of the parameter variation is

\[ H(t) = AH(t-1) + w(t) \quad (6.45) \]

where \( w(t) \) is a vector random variable with \( E[w(t)] = 0 \) and \( \sigma^2 [w(t)] = W \). \( A \) is a diagonal matrix given by

\[ A = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix} \quad (6.46) \]

such that \( 0 < a_{ii} < 1, i = 1, 2, \ldots, n \). This restriction on the elements of \( A \) is necessary to guarantee the stability of the generating process. The random walk model can be obtained by setting \( A = I \), i.e., \( a_{ii} = 1 \) \( \forall i \). The estimation of the vector \( H(t) \) can be carried out recursively using the Kalman filter technique. This is because equations (6.44) and (6.45) form the state space representation of the system, in which equation (6.44) is the measurement equation and equation (6.45) is the transition equation, which allows for systematically varying
parameters. The state of the system $H(t)$ is not directly observable, but can be observed through $R_U(t)$. Let
\[ E[H(t) \mid R_U(t-1)] = H(t \mid t-1) \quad (6.47) \]
and
\[ E[H(t) - H(t \mid t-1)]^2 = \sigma^2 (t \mid t-1) \quad (6.48) \]
The Kalman filter equations are given by
\[ H(t \mid t-1) = AH(t-1 \mid t-1) \quad (6.49) \]
\[ \sigma^2 (t \mid t-1) = A\sigma^2 (t-1 \mid t-1)A' + W \quad (6.50) \]
\[ G(t) = \sigma^2 (t \mid t-1)R'_A(t)[R_A(t)\sigma^2 (t \mid t-1)R'_A(t) + V]^{-1} \quad (6.51) \]
\[ H(t \mid t) = H(t \mid t-1) + G(t)[R_U(t) - R_A(t)H(t \mid t-1)] \quad (6.52) \]
\[ \sigma^2 (t \mid t) = V(t \mid t-1) - G(t)R'_A(t)R_A(t)\sigma^2 (t \mid t-1) \quad (6.53) \]
The initial conditions are given by $H(0 \mid 0) = H(0)$ and $\sigma^2 (0 \mid 0) = \sigma^2 (0)$. This system of equations tells us that the optimal estimator of $H(t)$ at time $t$, $H(t \mid t)$, is represented by a linear combination of the previous estimator, $H(t \mid t-1)$ and the current observation, $R_U(t)$. Equation (6.52) shows the recursive nature of the computation. For more detail, see Cuthbertson et al. (1992).

**Nonlinear models**

Broll et al. (2001) suggest that the hedge ratio should be estimated from a nonlinear model of the form
\[ p_{U,t} = \alpha + h p_{A,t} + \gamma p^2_{A,t} + \epsilon_t \quad (6.54) \]
in which case the error correction term ($\epsilon_t = p_{U,t} - \alpha - h p_{A,t} - \gamma p^2_{A,t}$) becomes nonlinear. By applying this model to the futures market they found (i) prevalent nonlinearities in the relationship; (ii) that the relationship is likely to be convex (positive $\gamma$) rather than concave (negative $\gamma$); and (iii) that the order of magnitude of the nonlinear component is by and large relatively small. They concluded that the firm should export more (less) and adopt an over (under) hedge in an unbiased currency market if the spot-futures relationship is convex (concave).

The following is a more general treatment of nonlinearity, which we will undertake by referring to equation (6.22), that is $p_{U,t} = \alpha + h p_{A,t} + \epsilon_t$. There are two approaches to nonlinearity in a relationship like equation (6.22): by postulating either a nonlinear attractor, or a nonlinear adjustment to a linear attractor. The first approach is discussed in Granger (1991). Let us define two nonlinear functions, $f_1(p_U)$ and $f_2(p_A)$, both of which represent long-memory series. If we define $z_t$ as a short-memory series given by
\[ z_t = f_1(p_{U,t}) - \lambda f_2(p_{A,t}) \quad (6.55) \]
then the equation of the corresponding nonlinear attractor is

\[ f_1(p_{U,t}) = \lambda f_2(p_{A,t}) \] (6.56)

Hallman (1989) shows how the functions \( f_1(p_U) \) and \( f_2(p_A) \) are estimated.

Nonlinearity in the error correction model is discussed in Escribano (1987), and the procedure is applied to a model of the demand for money in Hendry and Ericsson (1991). Nonlinearity in this case is captured by a polynomial in the error correction term. Thus the error correction model corresponding to equation (6.22) is

\[ \Delta p_{U,t} = A(L)\Delta p_{U,t-1} + B(L)\Delta p_{A,t} + \sum_{i=1}^{k} \gamma_i \epsilon^t_{t-i} + \xi_t \] (6.57)

where \( A(L) \) and \( B(L) \) are lag polynomials. Hendry and Ericsson (1991) suggest that a polynomial of degree three in the error correction term is sufficient to capture the adjustment process.

### Multicurrency hedge ratios

Consider a model in which the percentage change in the base currency value of a firm, \( V_{x} \), is influenced by changes in the exchange rates of the base currency against other currencies. This model can be represented by the regression equation

\[ \hat{V}_x = \sum_{i=1}^{n} a_i \hat{S}(x/y_i) \] (6.58)

In this case, the hedge ratio corresponding to each exchange rate is equal to the regression coefficient on that exchange rate, which means that \( h_i = a_i \) for all \( i \). Suppose, for example, that \( V_x \) is the US dollar value of a firm and that this value is affected by changes in the exchange rates of the US dollar against three currencies: Canadian dollar, Japanese yen and pound. Equation (6.58) can be rewritten for this special case as

\[ \hat{V}_X = h_1 \hat{S}(USD/CAD) + h_2 \hat{S}(USD/JPY) + h_3 \hat{S}(USD/GBP) \] (6.59)

where \( h_1, h_2 \) and \( h_3 \) are the hedge ratios corresponding to the three currencies respectively. If, for example, \( h_1 > 0, h_2 < 0 \) and \( h_3 > 0 \), then hedging one US dollar of value requires taking a short position of \( h_1 \) Canadian dollars, a long position of \( h_2 \) yen and a long position of \( h_3 \) pounds.

Recall from Chapter 4 that if

\[ \hat{V}_{CAD} = h'_1 \hat{S}(CAD/USD) + h'_2 \hat{S}(CAD/JPY) + h'_3 \hat{S}(CAD/GBP) \] (6.60)

then \( h'_2 = h_2 \), \( h'_3 = h_3 \) and \( h'_1 = 1 - (h_2 + h_3) \). The hedge ratios applicable to the Canadian dollar value of the firm are \( h'_1, h'_2 \) and \( h'_3 \), which correspond to the US dollar, Japanese yen and pound respectively.
6.4 EVALUATING THE EFFECTIVENESS OF HEDGING

There are three approaches to the measurement of the effectiveness of hedging. These approaches will be discussed in turn.

**Correlation, variance ratio and variance reduction**

To start with, we have seen that perfect correlation between the prices of the asset to be hedged and the hedging instrument leads to a perfect hedge if the hedge ratio is one. If prices are less than perfectly correlated, then the optimal hedge ratio will be different from one and the hedge will be less than perfect. Hence, hedging effectiveness is indicated by the degree of correlation of the prices. One measure of the effectiveness of hedging is, therefore, the coefficient of determination ($R^2$) of a regression of the levels or the first differences of prices, as represented by equations (6.22) and (6.21) respectively. A perfect hedge would be indicated when the value of the coefficient of determination is one. This would be obtained, of course, when prices are perfectly positively or negatively correlated.

Hedging effectiveness can be measured by the variance of the rate of return on the hedged position compared with the variance of the rate of return on the unhedged position. To evaluate the performance of hedging resulting from using different estimated ratios (and even from using different hedging instruments) the criterion that is used is the variance of the rate of return on the hedged position, $\sigma^2 (R_H)$. The smaller the variance the more effective the hedge is. Of course, a mere comparison of the numerical values of the variances is inadequate and a formal test of the equality between the two variances must be conducted.

Consider first the effectiveness of a hedge against the alternative of leaving the underlying position unhedged. In this case we test the equality of the variance of the hedged position and that of the unhedged position. The null hypothesis is

$$H_0: \sigma^2 (R_H) = \sigma^2 (R_U) \quad (6.61)$$

against the alternative

$$H_1: \sigma^2 (R_H) \neq \sigma^2 (R_U) \quad (6.62)$$

If $H_1: \sigma^2 (R_U) > \sigma^2 (R_H)$, then the null is rejected if

$$VR = \frac{\sigma^2 (R_U)}{\sigma^2 (R_H)} > F(n - 1, n - 1) \quad (6.63)$$

where $VR$ is the variance ratio and $n$ is the sample size. This test can be complemented by calculating the variance reduction, which is calculated as

$$VD = 1 - \frac{1}{VR} = 1 - \frac{\sigma^2 (R_H)}{\sigma^2 (R_U)} = \frac{\sigma^2 (R_U) - hR_A}{\sigma^2 (R_U)} \quad (6.64)$$
The test can be conducted to compare the effectiveness of two hedging positions resulting from the use of different hedge ratios or different hedging instruments. In this case, the null hypothesis becomes \( H_0 : \sigma^2 (R_{H,1}) = \sigma^2 (R_{H,2}) \), where \( \sigma^2 (R_{H,1}) \) and \( \sigma^2 (R_{H,2}) \) are the rates of return on the hedged positions resulting from hedge number one and hedge number two respectively.

These tests can be done within-sample or out of sample. Suppose that the historical data that is used to estimate the hedge ratio is available for the period \( t = 1, 2, ..., n \). If the hedge ratio and the test statistics are calculated on the basis of the full sample, then this is an in-sample test. If, however, the hedge ratio is estimated over the sub-sample period \( t = 1, 2, ..., k \), where \( k < n \), then the test statistics are calculated from the observations \( t = k + 1, k + 2, ..., n \), which makes it an out-of-sample test.

Since correlation between the rates of return, \( R_U \) and \( R_A \), is crucial for the success of hedging, we will now attempt to find out how the hedge ratio, variance ratio and variance reduction are related to the correlation coefficient between the rates of return. The correlation coefficient is defined as

\[
\rho = \frac{\sigma(R_U, R_A)}{\sigma(R_U)\sigma(R_A)} \tag{6.65}
\]

which gives

\[
\sigma(R_U, R_A) = \rho \sigma(R_U)\sigma(R_A) \tag{6.66}
\]

By substituting equation (6.66) in equation (6.17), which defines the hedge ratio, we obtain

\[
h = \frac{\rho \sigma(R_U)\sigma(R_A)}{\sigma^2 (R_A)} = \rho \left( \frac{\sigma(R_U)}{\sigma(R_A)} \right) \tag{6.67}
\]

It is obvious from equation (6.67) that the hedge ratio is linearly related to the correlation coefficient. If \( \sigma(R_U) = \sigma(R_A) \), then \( h = \rho \).

To derive the relationship between the variance ratio and the correlation coefficient, we rewrite equation (6.63) as

\[
VR = \frac{\sigma^2 (R_U)}{\sigma^2 (R_U) + h^2 \sigma^2 (R_A) - 2h\sigma(R_U, R_A)} \tag{6.68}
\]

By substituting equations (6.66) and (6.67) into equation (6.68), we get

\[
VR = \frac{\sigma^2 (R_U)}{\sigma^2 (R_U) + \rho^2 \sigma^2 (R_U)\sigma^2 (R_A) - 2[\rho \sigma(R_U)\sigma(R_A)])/[\sigma^2 (R_A)\rho \sigma(R_U)\sigma(R_A)]} \tag{6.69}
\]

which can be simplified to obtain

\[
VR = \frac{1}{1-\rho^2} \tag{6.70}
\]
implying a positive relationship between $VR$ and $\rho$, because

$$\frac{d}{d\rho} (VR) = \frac{2\rho}{(1-\rho^2)^2} > 0 \quad (6.71)$$

As we can see from equations (6.70) and (6.71), the relationship between the variance ratio and the correlation coefficient is nonlinear. It also follows from equation (6.70) that

$$VD = \frac{1}{VR} = \rho^2 \quad (6.72)$$

which shows the variance reduction is equivalent to the coefficient of determination of the underlying regression.

To derive the relationship between the variance ratio and the hedge ratio, we substitute (6.67) into (6.70) to obtain

$$VR = \frac{1}{1-h^2 \left[ (\sigma^2 (R_A)) / (\sigma^2 (R_U)) \right]} \quad (6.73)$$

which gives

$$\frac{d}{dh} (VR) = \frac{2h[(\sigma^2 (R_A)) / (\sigma^2 (R_U))]}{(1-h^2 \left[ (\sigma^2 (R_A)) / (\sigma^2 (R_U)) \right])^2} \quad (6.74)$$

As we can see from equations (6.73) and (6.74) the variance ratio is a nonlinear positive function of the hedge ratio. Finally, we have

$$VD = 1 - \frac{1}{VR} = h^2 \left[ \frac{\sigma^2 (R_A)}{\sigma^2 (R_U)} \right] \quad (6.75)$$

which gives

$$\frac{d}{dh} (VD) = 2h \left[ \frac{\sigma^2 (R_A)}{\sigma^2 (R_U)} \right] > 0 \quad (6.76)$$

which again shows a positive and nonlinear relationship between variance reduction and the hedge ratio.

The base currency value of payables and receivables
Another criterion that is used for evaluating hedging effectiveness, particularly with contingent exposures involving foreign currency payables and receivables, is the base currency value of the payables or receivables. The hedger may wish to optimise the domestic currency values of payables and receivables (maximising receivables and minimising payables). If this is the case then one hedge will be preferred to another if the mean domestic currency value of payables (receivables) is lower (higher) than under the other hedge (see, for example, Moosa, 2002c). Let $\mu(U)$ and $\mu(H)$ be the population
means of the domestic currency values of the payables under no-hedge and the hedge decisions. The null hypothesis is

\[ H_0: \mu(U) = \mu(H) \]  

(6.77)

whereas the alternative hypothesis is written as

\[ H_1: \mu(U) \neq \mu(H) \]  

(6.78)

The null hypothesis is rejected if

\[ \frac{\bar{X}(U) - \bar{X}(H)}{\hat{\sigma}} \sqrt{\frac{n^2}{2n}} > t(2n - 2) \]  

(6.79)

where \( \bar{X}(U) \) and \( \bar{X}(H) \) are respectively the sample means of the domestic currency values of the payables under no-hedge and hedge decisions, \( n \) is the sample size, \( t(2n - 2) \) is the critical value of the \( t \) distribution with \( 2n - 1 \) degrees of freedom, and

\[ \hat{\sigma} = \sqrt{\frac{n[s^2(U) + s^2(H)]}{2n - 2}} \]  

(6.80)

where \( s^2 \) is the estimated sample variance. The sample mean domestic currency values of the payables or receivables are defined respectively as

\[ \bar{X}(U) = \frac{1}{n}\sum_{t=1}^{n} K_t S_{t+1} \]  

(6.81)

\[ \bar{X}(H) = \frac{1}{n}\sum_{t=1}^{n} K_t \bar{S}_t \]  

(6.82)

where \( \bar{S} \) is the conversion rate implicit in the hedge (which is equal to the forward rate in forward hedging and the interest parity rate in money market hedging). Naturally, the hedger may not only be interested in optimising the mean value of the payables or receivables but also in the variability of the cash flows. In this case, testing the equality of the variances must also be used. A final choice decision can be made, depending on the risk–return trade-off. Of course, the same procedure can be used to test the effectiveness of hedging under different hedge ratios.

**Out-of-sample forecasting**

Finally, the effectiveness of hedging may be tested by comparing the out-of-sample forecasting power of the underlying models that are used to calculate the hedge ratio. For example, Ghosh (1993) compares the first-difference model with the error correction model. Specifically, he calculates the root mean square error (RMSE) of the two models and concludes that the error correction model is associated with about 15% reduction in the RMSE of the first-difference model. The two models can be alternatively estimated on the basis of a likelihood ratio test between
the restricted and unrestricted models. He also concludes that (i) the error correction model provides better estimates of the optimal hedge ratio, which reduces risk as well as the cost of hedging; and (ii) that the hedge ratios from traditional models are underestimated.

The problem here is that economists tend to follow the faulty procedure of deriving inference on the superiority of the forecasting power of a model over another simply by comparing the numerical values of the mean square errors of the forecasts, without testing whether or not the difference between the mean square errors is statistically significant. Such tests are actually available, the simplest of which is the AGS test, designed by Ashley, Granger and Schmalensee (1980). The test requires the estimation of the linear regression

\[ D_t = \alpha_0 + \alpha_1 (M_t - \bar{M}) + u_t \]  

(6.83)

where \( D_t = w_{1t} - w_{2t}, M_t = w_{1t} + w_{2t}, \bar{M} \) is the mean of \( M, w_{1t} \) is the forecasting error at time \( t \) of the model with the higher MSE, \( w_{2t} \) is the forecasting error at time \( t \) of the model with the lower MSE. If the sample mean of the errors is negative, the observations of the series must be multiplied by \(-1\) before running the regression. The estimates of the intercept term \( (\alpha_0) \) and the slope \( (\alpha_1) \) are used to test the statistical difference between the MSEs of two different models. If the estimates of \( \alpha_0 \) and \( \alpha_1 \) are both positive, then a Wald test of the joint hypothesis \( H_0: \alpha_0 = \alpha_1 = 0 \) is appropriate. However, if one of the estimates is negative and statistically significant then the test is inconclusive. But if the estimate is negative and statistically insignificant the test remains conclusive, in which case significance is determined by the upper-tail of the \( t \)-test on the positive coefficient estimate.

The empirical evidence

The issue of estimating the hedge ratio and assessing the effectiveness of hedging has been the focus of considerable empirical research in the literature. A large number of studies have been carried out to evaluate the hedging effectiveness based on various methods for measuring the hedge ratio. Initially, hedge ratios were calculated as the slope coefficient from an OLS regression (Ederington, 1979). Kroner and Sultan (1993) estimated time-varying hedge ratios using a bivariate error correction model with a GARCH error structure. They showed that this model provides greater risk reduction than the conventional models. Brooks and Chong (2001) examined the ability of several models to generate optimal hedge ratios when cross currency hedging is used. By using four currency pairs they found that an exponentially weighted moving average model leads to lower portfolio variances than any of the GARCH-based, implied or time-invariant approaches.

Ghosh (1993) demonstrated that less than optimal hedging would result if the hedge ratio is estimated from a model that ignores the error correction mechanism, as shown by Lien (1996). However, Moosa (2002e) examined the sensitivity of the optimal hedge ratio estimates to four different model specifications
and found no evidence for any relationship between model specification and hedging effectiveness. What matters most for hedging effectiveness, he concludes, is the correlation between the prices of the unhedged position and the hedging instrument. In another paper, Moosa (2002d) examined the effectiveness of cross currency hedging compared with that of forward and money market hedging. He demonstrated that cross currency hedging is not only less effective than forward and money market hedging, but also that it is totally ineffective unless the exchange rate of the exposure currency and that of the third currency (the hedging instrument) are highly correlated. The results indicate that for effective cross currency hedging a correlation coefficient of 0.5 is required to reduce the variance of the rate of return on the unhedged position by 25%.

### 6.5 Static and Dynamic Hedging

A static hedge involves a constant hedge ratio, whereas a dynamic hedge involves a time-varying hedge ratio. Dynamic hedging may involve changing other aspects of the specification of the hedge. In general, we could say that dynamic hedging entails the adjustment of the hedge with the passage of time.

By referring to equations (6.1) and (6.2) we can see that a perfect hedge is obtained with a hedge ratio of one only if \( R_U = R_A \). If at a particular point in time, \( t \), we have \( R_{U,t} = R_{A,t} \), then \( h_t = 1 \). However, if things change such that \( R_{U,t+1} \neq R_{A,t+1} \), then \( h_{t+1} \neq 1 \).

There are several reasons why it may be necessary to adjust the hedge over time. The first of these reasons is that the hedge may require rolling over the position in the hedging instrument, and this particularly applies to long-term exposure. Suppose that the maturity of the exposure is five years whereas the maturity of the available hedging instruments is one year. In this case a position on the instrument is taken and rolled over to the second year, and so on, until the exposure matures at the end of the fifth year.

The second reason is that it is most likely that the exposure itself changes over time. A change in foreign sales means changing receivables, and the same can happen to payables. Also, the firm’s attitude towards risk or its ability to withstand adverse developments may change. However, this may boil down to using hedging instruments for speculative purpose if, for example, a firm sees profit/loss on a position involving a hedge, in which case the firm may decide to realise the profit or wind down the position to cut losses.

The third reason is that management may, with the passage of time, acquire new information about the effectiveness of a hedge or about the appropriate hedge ratio. For example, the delta of an option is the change in the price of an option relative to the change in the underlying exchange rate, which means that it is the hedge ratio. The delta of an option changes with the value of the
underlying exchange rate, which necessarily implies a changing hedge ratio. Hence the hedge ratio should be adjusted with changes in the delta of the option.

The fourth reason is that it is possible for the structure of the underlying asset to be changing. Suppose, for example, that a firm wants to hedge exposure arising from changes in the effective exchange rate of the base currency against a group of other currencies. The effective exchange rate is normally calculated as a weighted average of the bilateral exchange rate relatives against the currencies of the major trading partners. In this case the structure of the underlying asset changes because the currencies and/or their weights are revised every now and then. Kavussanos and Nomikos (2000) study the effectiveness of futures contracts whose underlying asset is an index when the structure of the index changes. They concluded that the effectiveness of the BFI contract (a freight futures contract), which is a hedging instrument that is traded on the BIFFEX (Baltic International Freight Futures Exchange) has increased as a result of making the index more homogenous.

Adjusting the hedge is not a free operation, as it involves transaction costs. However, it remains the case that monitoring requires an ongoing commitment. Kroner and Sultan (1993) suggest that investors will adjust the hedge only if the benefits of the operation offset its costs. Assuming that \( k \) is the percentage return that is lost due to transaction costs every time the hedge is adjusted, the return on the hedged position if there is an adjustment is

\[
R_{H,t+1} = R_{U,t+1} - h_t R_{A,t+1} - k
\]  

whereas the return on the hedged position without a readjustment is

\[
R_{H,t+1} = R_{U,t+1} - h'_t R_{A,t+1}
\]

where \( h'_t \) is the hedge ratio from the most recent adjustment. This means that the expected return is \(-k\) if there is adjustment and 0 otherwise. The conditional variances with and without adjustment are given respectively by

\[
\sigma^2_t (R_{H,t+1}) = \sigma^2_{t+1}(R_{U,t+1}) - 2h_t \sigma_{t+1}(R_{H,t+1}, R_{U,t+1}) + h^2 \sigma^2_{t+1}(R_{A,t+1})
\]

\[
\sigma^2_t (R_{H,t+1}) = \sigma^2_{t+1}(R_{U,t+1}) - 2h'_t \sigma_{t+1}(R_{H,t+1}, R_{U,t+1}) + h'^2 \sigma^2_{t+1}(R_{A,t+1})
\]

A mean–variance expected utility maximiser with an expected utility function of the form \( EU(R_{H,t+1}) = E(R_{H,t+1}) - \gamma \sigma^2 (R_{H,t+1}) \), where \( \gamma \) is the degree of risk aversion, will adjust the hedge if and only if

\[
-k - \gamma [\sigma^2_{t+1}(R_{U,t+1}) - 2h_t \sigma_{t+1}(R_{H,t+1}, R_{U,t+1}) + h^2 \sigma^2_{t+1}(R_{A,t+1})] > \sigma^2_{t+1}(R_{U,t+1}) - 2h'_t \sigma_{t+1}(R_{H,t+1}, R_{U,t+1}) + h'^2 \sigma^2_{t+1}(R_{A,t+1})
\]
In their study, Kroner and Sultan show that an investor in the pound would have adjusted the hedge 18 times during the 263 week period they examined.

6.6 AN ILLUSTRATION USING CROSS CURRENCY HEDGING

In cross currency hedging, both the position to be hedged and the hedging instruments are currencies. The position to be hedged is a spot position on a currency in which payables or receivables are denominated. The hedging instrument is a spot or a forward position on another currency. For the purpose of this illustration we will use a spot position as the hedging instrument.

If \( x \) is the base currency, \( y \) is the currency denominated assets or liabilities and \( z \) is the currency to be used as the hedging instrument, then \( R_U = \hat{S}(x/y) \), \( R_A = \hat{S}(x/z) \) and \( R_H = \hat{S}(x/y) - h\hat{S}(x/z) \). The hedge ratio is then calculated from the regression equation

\[
\hat{S}(x/y) = \alpha + h\hat{S}(x/z)
\]

which gives

\[
h = \frac{\sigma[\hat{S}(x/y), \hat{S}(x/z)]}{\sigma^2 [\hat{S}(x/z)]}
\]

Notice now what happens if the roles of currencies \( y \) and \( z \) are reversed by taking currency \( z \) to be the currency in which payables or receivables are denominated and \( y \) to be the hedging instrument. In this case the hedge ratio is calculated from the regression equation

\[
\hat{S}(x/z) = \alpha' + h'\hat{S}(x/y)
\]

which gives

\[
h' = \frac{\sigma[\hat{S}(x/y), \hat{S}(x/z)]}{\sigma^2 [\hat{S}(x/y)]}
\]

By combining equations (6.92) and (6.90), we obtain

\[
h' = \frac{h\sigma^2 [\hat{S}(x/z)]}{\sigma^2 [\hat{S}(x/y)]}
\]

In this exercise quarterly data on the exchange rates of nine currencies against the US dollar are used, covering the period 1991:1–2000:4. In the first instance the US dollar is taken to be the base currency, and all possible combinations for the exposure currency and the hedge are tried. The estimated hedge ratios and (in parentheses) the corresponding coefficient of
determination (variance reduction) are reported in Tables 6.1 and 6.2. From these two tables we can see that the best available cross hedge, when the base currency is the US dollar, involves the Danish and Swiss currencies and those involving the Norwegian and Danish currencies. Irrespective of which currency is the exposure currency and which is the hedging instrument, the same hedging effectiveness is produced (the same $R^2$). Notice, however, that the hedge ratio depends on which currency is the exposure and which one is the hedging instrument as demonstrated by equation (6.93). For example, if the DKK is the exposure currency and the CHF is the hedging currency, then the hedge ratio is 1.131. If, the roles of the two currencies are reversed, the hedge ratio becomes 0.718. In Tables 6.3 and 6.4 the corresponding results are reported when the base currency is the Swedish krona rather than the US dollar.

The results presented in Tables 6.1–6.4 are used to represent graphically the (stochastic version of the) relationships derived in Section 6.4. Figures 6.1–6.3 are scatter diagrams (with best-fit lines) for the relationship between the hedge ratio and the correlation coefficient as described by equation (6.67). It is obvious that the relationship is linear and positive as predicted by the equation. Figures 6.4 and 6.5 relate the variance ratio and variance reduction to the correlation coefficient. The relationship in both cases positive and nonlinear. Figures 6.6–6.7 are scatter diagrams of the variance ratio and variance reduction on the hedge ratio. These graphs are the empirical counterparts of the theoretical relationships represented by equations (6.73) and (6.75) respectively.

Figures 6.8 and 6.9 illustrate the comparative effectiveness of forward hedging and cross hedging respectively by plotting the rates of return on the unhedged and hedged positions. In Figure 6.8(a) the base currency is the US dollar, whereas the exposure currency is the pound. A hedged position formed by taking an opposite position on a forward contract produces a rather stable rate of return on the hedged position. Figure 6.8(b) shows the same when the exposure currency is the Canadian dollar and Figure 6.8(c) shows the effectiveness of forward hedging when the base currency is the pound and the exposure currency is the Canadian dollar. Now, compare this performance of forward hedging with the performance of cross currency hedging, as shown in Figure 6.9. By using three different currency combinations, we can see that cross hedging is not as effective as forward hedging. In fact, it is worse than remaining unhedged when the pound is used to hedge a Canadian dollar position when the base currency is the US dollar (Figure 6.9(b)).

Table 6.5 reports the variances of the rates of return on the hedged and unhedged positions for the six cases shown in Figures 6.8 and 6.9. It is obvious that forward hedging is more powerful and that, in one particular case, cross hedging leads to inferior results compared to what is obtained under the no-hedge decision.
TABLE 6.1 Cross currency hedge ratios: 1 (USD is the base currency).\textsuperscript{a}

<table>
<thead>
<tr>
<th></th>
<th>AUD</th>
<th>CAD</th>
<th>CHF</th>
<th>DKK</th>
<th>GBP</th>
<th>JPY</th>
<th>NOK</th>
<th>NZD</th>
<th>SEK</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD</td>
<td>–</td>
<td>0.883\textsuperscript{b}</td>
<td>-0.113</td>
<td>-0.023</td>
<td>0.055</td>
<td>0.018</td>
<td>0.041</td>
<td>0.672\textsuperscript{b}</td>
<td>0.074</td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>(0.176)</td>
<td>(0.025)</td>
<td>(0.001)</td>
<td>(0.003)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.482)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>CAD</td>
<td>–</td>
<td>-0.074</td>
<td>-0.061</td>
<td>0.046</td>
<td>-0.010</td>
<td>0.017</td>
<td>0.076</td>
<td>0.026</td>
<td></td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>(0.048)</td>
<td>(0.021)</td>
<td>(0.010)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.027)</td>
<td>(0.007)</td>
<td></td>
</tr>
<tr>
<td>CHF</td>
<td>–</td>
<td>1.131\textsuperscript{b}</td>
<td>0.785\textsuperscript{b}</td>
<td>0.409\textsuperscript{b}</td>
<td>1.102\textsuperscript{b}</td>
<td>0.314</td>
<td>0.627\textsuperscript{b}</td>
<td>–</td>
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<td></td>
<td>–</td>
<td>(0.813)</td>
<td>(0.344)</td>
<td>(0.206)</td>
<td>(0.664)</td>
<td>(0.053)</td>
<td>(0.462)</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>DKK</td>
<td>–</td>
<td>0.658\textsuperscript{b}</td>
<td>0.312\textsuperscript{b}</td>
<td>0.912\textsuperscript{b}</td>
<td>0.336\textsuperscript{b}</td>
<td>0.526\textsuperscript{b}</td>
<td>–</td>
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<td></td>
<td>–</td>
<td>(0.381)</td>
<td>(0.188)</td>
<td>(0.716)</td>
<td>(0.096)</td>
<td>(0.511)</td>
<td>–</td>
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</tr>
<tr>
<td>GBP</td>
<td>–</td>
<td>0.152</td>
<td>0.767\textsuperscript{b}</td>
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<tr>
<td></td>
<td>–</td>
<td>(0.051)</td>
<td>(0.576)</td>
<td>(0.080)</td>
<td>(0.558)</td>
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<tr>
<td>JPY</td>
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<td>0.427</td>
<td>0.520\textsuperscript{b}</td>
<td>0.168</td>
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<tr>
<td></td>
<td>–</td>
<td>(0.081)</td>
<td>(0.119)</td>
<td>(0.027)</td>
<td>–</td>
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<td></td>
</tr>
<tr>
<td>NOK</td>
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<td>0.338\textsuperscript{b}</td>
<td>0.547\textsuperscript{b}</td>
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<td></td>
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<tr>
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<td>–</td>
<td>(0.112)</td>
<td>(0.644)</td>
<td>–</td>
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<tr>
<td>NZD</td>
<td>–</td>
<td>0.151</td>
<td>–</td>
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<tr>
<td></td>
<td>–</td>
<td>(0.049)</td>
<td>–</td>
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<tr>
<td>SEK</td>
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<td>–</td>
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</tbody>
</table>

\textsuperscript{a}Rows represent the exposure currencies, whereas columns represent the hedging currencies. Hedge ratios are calculated from the regression $S_i = c + hS_j$, where $i$ and $j$ represent rows and columns respectively. The coefficient of determination, which measures hedging effectiveness, is placed in parentheses.

\textsuperscript{b}Statistically significant at the 5% level.

TABLE 6.2 Cross currency hedge ratios: 2 (USD is the base currency).\textsuperscript{a}

<table>
<thead>
<tr>
<th></th>
<th>AUD</th>
<th>CAD</th>
<th>CHF</th>
<th>DKK</th>
<th>GBP</th>
<th>JPY</th>
<th>NOK</th>
<th>NZD</th>
<th>SEK</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
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</tr>
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<td>0.061</td>
<td>0.224</td>
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<td>0.579\textsuperscript{b}</td>
<td>–</td>
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<td></td>
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<td>(0.010)</td>
<td>(0.344)</td>
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</tr>
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<td>0.603\textsuperscript{b}</td>
<td>0.335</td>
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<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.206)</td>
<td>(0.188)</td>
<td>(0.051)</td>
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<tr>
<td>NOK</td>
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<td>0.081</td>
<td>0.602\textsuperscript{b}</td>
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<td>(0.001)</td>
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<td>(0.716)</td>
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<td>(0.081)</td>
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</tr>
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<td>NZD</td>
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<td>0.278</td>
<td>0.229\textsuperscript{b}</td>
<td>0.334\textsuperscript{b}</td>
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</tr>
<tr>
<td></td>
<td>(0.482)</td>
<td>(0.027)</td>
<td>(0.053)</td>
<td>(0.096)</td>
<td>(0.080)</td>
<td>(0.119)</td>
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<tr>
<td>SEK</td>
<td>0.171</td>
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<td>0.736\textsuperscript{b}</td>
<td>0.973\textsuperscript{b}</td>
<td>1.084\textsuperscript{b}</td>
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<tr>
<td></td>
<td>(0.012)</td>
<td>(0.007)</td>
<td>(0.462)</td>
<td>(0.511)</td>
<td>(0.558)</td>
<td>(0.027)</td>
<td>(0.644)</td>
<td>(0.049)</td>
<td>–</td>
</tr>
</tbody>
</table>

\textsuperscript{a}Rows represent the hedging currencies, whereas columns represent the exposure currencies. Hedge ratios are calculated from the regression $S_j = a + hS_i$, where $i$ and $j$ represent rows and columns respectively. The coefficient of determination, which measures hedging effectiveness, is placed in parentheses.

\textsuperscript{b}Statistically significant at the 5% level.
### TABLE 6.3 Cross currency hedge ratios: 3 (SEK is the base currency).\(^a\)

<table>
<thead>
<tr>
<th></th>
<th>AUD</th>
<th>CAD</th>
<th>CHF</th>
<th>DKK</th>
<th>GBP</th>
<th>JPY</th>
<th>NOK</th>
<th>NZD</th>
<th>SEK</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD</td>
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<td>0.936b</td>
<td>0.316</td>
<td>0.754b</td>
<td>0.979b</td>
<td>0.484b</td>
<td>1.014b</td>
<td>0.927b</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>(0.740)</td>
<td>(0.043)</td>
<td>(0.200)</td>
<td>(0.305)</td>
<td>(0.285)</td>
<td>(0.272)</td>
<td>(0.802)</td>
<td>–</td>
</tr>
<tr>
<td>CAD</td>
<td>–</td>
<td>0.448b</td>
<td>0.788b</td>
<td>1.061b</td>
<td>0.499b</td>
<td>1.095b</td>
<td>0.761b</td>
<td>–</td>
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</tr>
<tr>
<td></td>
<td>–</td>
<td>(0.103)</td>
<td>(0.285)</td>
<td>(0.425)</td>
<td>(0.362)</td>
<td>(0.376)</td>
<td>(0.640)</td>
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<tr>
<td>CHF</td>
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<td>0.938b</td>
<td>0.533b</td>
<td>0.363b</td>
<td>0.922b</td>
<td>0.484b</td>
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<tr>
<td></td>
<td>–</td>
<td>(0.715)</td>
<td>(0.209)</td>
<td>(0.372)</td>
<td>(0.520)</td>
<td>(0.167)</td>
<td>(0.332)</td>
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<td>–</td>
</tr>
<tr>
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<td>–</td>
<td>(0.347)</td>
<td>(0.444)</td>
<td>(0.686)</td>
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<td>–</td>
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<tr>
<td>GBP</td>
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<td>0.274b</td>
<td>0.796b</td>
<td>0.362b</td>
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<tr>
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<td>JPY</td>
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<td>0.745b</td>
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<tr>
<td></td>
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<td>NOK</td>
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</tr>
</tbody>
</table>

\(^a\)Rows represent the exposure currencies, whereas columns represent the hedging currencies. Hedge ratios are calculated from the regression \(S_i = \alpha + hS_j\), where \(i\) and \(j\) represent rows and columns respectively. The coefficient of determination, which measures hedging effectiveness, is placed in parentheses.

\(^b\)Statistically significant at the 5% level.

### TABLE 6.4 Cross currency hedge ratios: 4 (SEK is the base currency).\(^a\)

<table>
<thead>
<tr>
<th></th>
<th>AUD</th>
<th>CAD</th>
<th>CHF</th>
<th>DKK</th>
<th>GBP</th>
<th>JPY</th>
<th>NOK</th>
<th>NZD</th>
<th>SEK</th>
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<td>AUD</td>
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<td>DKK</td>
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<td>0.762b</td>
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<td>0.569b</td>
<td>1.178b</td>
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<td>(0.394)</td>
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</tr>
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</tbody>
</table>

\(^a\)Rows represent the hedging currencies, whereas columns represent the exposure currencies. Hedge ratios are calculated from the regression \(S_j = \alpha + hS_i\), where \(i\) and \(j\) represent rows and columns respectively. The coefficient of determination, which measures hedging effectiveness, is placed in parentheses.

\(^b\)Statistically significant at the 5% level.
FIGURE 6.1 The hedge ratio as a function of the correlation coefficient (all data).

FIGURE 6.2 The hedge ratio as a function of the correlation coefficient (USD is the base currency).

FIGURE 6.3 The hedge ratio as a function of the correlation coefficient (SEK is the base currency).
FIGURE 6.4 The variance ratio as a function of the correlation coefficient (all data).

FIGURE 6.5 Variance reduction as a function of the correlation coefficient (all data).

FIGURE 6.6 The variance ratio as a function of the hedge ratio (all data).
FIGURE 6.7 Variance reduction as a function of the hedge ratio (all data).

<table>
<thead>
<tr>
<th>Hedging technique</th>
<th>Base currency</th>
<th>Exposure currency</th>
<th>Hedge currency</th>
<th>Variance (unhedged)</th>
<th>Variance (hedged)</th>
</tr>
</thead>
<tbody>
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<td>Forward</td>
<td>USD</td>
<td>GBP</td>
<td>GBP</td>
<td>23.5</td>
<td>0.15</td>
</tr>
<tr>
<td>Forward</td>
<td>USD</td>
<td>CAD</td>
<td>CAD</td>
<td>3.8</td>
<td>0.09</td>
</tr>
<tr>
<td>Forward</td>
<td>GBP</td>
<td>CAD</td>
<td>CAD</td>
<td>26.06</td>
<td>0.23</td>
</tr>
<tr>
<td>Cross</td>
<td>USD</td>
<td>GBP</td>
<td>CAD</td>
<td>23.5</td>
<td>23.06</td>
</tr>
<tr>
<td>Cross</td>
<td>USD</td>
<td>CAD</td>
<td>GBP</td>
<td>3.8</td>
<td>9.66</td>
</tr>
<tr>
<td>Cross</td>
<td>GBP</td>
<td>CAD</td>
<td>USD</td>
<td>26.06</td>
<td>5.82</td>
</tr>
</tbody>
</table>

It seems that it all depends on correlation. Forward hedging is more powerful than cross hedging because the percentage change in a spot rate is more highly correlated with the percentage change in the corresponding forward rate than with that of another spot rate.
FIGURE 6.8 The effectiveness of forward hedging.
(a) Base currency (USD), exposure currency (GBP), hedging currency (CAD)

(b) Base currency (USD), exposure currency (CAD), hedging currency (GBP)

(c) Base currency (GBP), exposure currency (CAD), hedging currency (USD)

FIGURE 6.9 The effectiveness of cross currency hedging.
CHAPTER 7

Speculation in the Spot and Currency Derivative Markets

7.1 DEFINITION OF SPECULATION

Speculation in the foreign exchange market is a subject that has been dealt with in the early writings on exchange rate economics. Early contributors to the post-war literature on exchange rate economics were to a large extent concerned with the role of speculation in the foreign exchange market. Nurkse (1945) warned against the dangers of “bandwagon effect” as a source of market instability. In his seminal work on the choice between fixed and flexible exchange rates, Friedman (1953) argued for flexible exchange rates by suggesting that speculation would be stabilising under flexible exchange rates because profit-making requires buying low and selling high, which is the action that makes speculation stabilising. A proof of this proposition will be presented later.

Tirole (1993) argues that the concept of speculation has always fascinated academics and practitioners alike, attributing this fascination to “inconsistent definitions, occasional misunderstanding and genuine economic importance”. Speculation may be defined as the assumption of risk for the sake of making profit. Speculators act on the basis of certain beliefs or expectations, making profit if their expectations are realised (or if their forecasts turn out to be accurate). Another definition that can be found in the *Oxford Universal Dictionary* is that it is “the action or practice of buying and selling goods, stocks and shares, etc. in order to profit by the rise or fall in the market value, as distinct from regular trading or investment”. Keynes (1930) and Hicks (1939) viewed speculation as a substitute for missing insurance markets, in that gains from trading are linked to differences in the trader’s willingness to take risk on an initial position. This is the so-called insurance motive for speculation, whereby speculation is viewed to be used to shift price risk from more to less risk-averse traders. Apart from the insurance motive, there are the following motives for speculation:
1. The liquidity motive, whereby traders may need to sell assets in periods of financial necessity or buy to diversify.

2. The divergence of priors, whereby assets are traded because agents disagree on their future value. Working (1953b) argued that the differences in beliefs are the key to speculative behaviour.

3. Speculators may be intermediaries acting on behalf of non-participating investors, in whose case the motivation for speculation would vary, depending on how they are compensated for their services.

### 7.2 SPOT SPECULATION

Speculation is the deliberate assumption of risk to obtain profit. In the case of spot speculation, the risk arises from changes in the spot exchange rate. We will initially ignore the interest rate factor by, for example, assuming that the interest rates on the two currencies that are bought and sold are equal.

#### The simple case

Spot speculation entails selling an amount, $K$, of currency $x$, which is expected to depreciate against currency $y$, which is expected to appreciate, and then reversing the operation in the future. Suppose that the present time is $t$ such that the exchange rate is $S_t$. If $y$ is expected to appreciate in the future such that $E(S_{t+1}) > S_t$, then a decision will be taken to sell an amount, $K$, of $x$ to obtain $KS_t$ of $y$. If the expectation turns out to be correct such that $S_{t+1} > S_t$, then the amount $K/S_t$ of $y$ is converted back into $x$ at $t + 1$ to obtain $KS_{t+1}/S_t$. Net profit (in terms of currency $x$) is

$$
\pi = K \left[ \frac{S_{t+1}}{S_t} - 1 \right]
$$

(7.1)

where the term in square brackets is the percentage change in the spot exchange rate. If $S_{t+1} > S_t$, it follows that $\pi > 0$, and vice versa.

The decision to speculate is based on the expectation that $y$ will appreciate, which is represented by the inequality $E(S_{t+1}) > S_t$. The expectation represented by the inequality pertains to the direction of the change in the exchange rate only, in the sense that it indicates that currency $y$ will appreciate without saying by how much. Suppose now that the expectation takes the form of a formal forecast that has a quantitative dimension as well, such that $E(S_{t+1}) = \theta S_t$ where $\theta > 1$. If the forecast turns out to be accurate such that $S_{t+1} = \theta S_t$, and it is acted upon, then the resulting profit will be given by

$$
\pi = K(\theta - 1)
$$

(7.2)
Again, \( \pi > 0 \) because \( \theta > 1 \). The difference in this case is that the forecast not only indicates that \( y \) will appreciate and that profit will be made, but also indicates the extent of the appreciation and hence the size of the profit.

Figure 7.1 shows this relationship diagrammatically when \( K = 1 \). First, the equation \( S_{t+1} = \theta S_t \) is represented in the \( S_t - S_{t+1} \) space by a straight line falling above the 45° line \( S_{t+1} = S_t \) (because \( \theta > 1 \)). In this part of the diagram profit is measured by the gap between the two lines \( S_{t+1} = \theta S_t \) and \( S_{t+1} = S_t \). The second part of the diagram is a 45° line that is used to transfer \( S_{t+1} \) from the vertical axis to the horizontal axis. The third part of the diagram is used to plot profit, \( \pi \), against the exchange rate at \( t + 1 \) represented by the equation \( \pi = -1 + (1 / S_t) S_{t+1} \), which means that it is a straight line with an intercept of –1 and a positive slope of \( 1/S_t \). Notice that the intercept indicates the maximum possible loss, which is realised when \( S_{t+1} = 0 \). If the exchange rate does not change (\( \theta = 1 \)), then profit would be zero. In Figure 7.2, it is shown that when \( \theta < 1 \) a loss would be made.

**FIGURE 7.1** A diagrammatic representation of speculative profit (\( \theta > 1 \)).
What if the forecast is not accurate, magnitude-wise, such that $S_{t+1} = \lambda S_t$, where $\theta > \lambda > 1$? This means that the exchange rate has risen as the forecast indicated but not to the same extent. In this case profit will be lower, since $0 < K(\lambda - 1) < K(\theta - 1)$. Figure 7.3 provides a diagrammatic representation of this case, in which profit associated with $\lambda$ is larger than the profit associated with $\theta$.

The forecast as a probability distribution

If the forecast is given in terms of a probability distribution, then the situation becomes as follows. Suppose that there are $q$ point forecasts given by $E(S_{t+1,i})$, each materialising with a probability $p_i$, where $i = 1, 2, \ldots, q$. Consider two forecasts, $E(S_{t+1,j})$ and $E(S_{t+1,k})$, such that $E(S_{t+1,j}) < S_t < E(S_{t+1,k})$. If the decision maker acts on the basis of the lower forecast, $E(S_{t+1,j})$, which materialises with a probability $p_j$, then a decision will be taken to buy currency $x$ and sell currency $y$, since the forecast indicates that the former will appreciate. Conversely, if the decision maker acts on the basis of the higher forecast,
7.2 SPOT SPECULATION

$E(S_{t+1,k})$, which materialises with a probability $p_k$, then a decision will be taken to buy currency $y$ and sell currency $x$, since the forecast indicates that the former will appreciate. If the decision is based on one forecast, then this naturally should be the forecast with the higher probability. Thus, if $p_j > p_k$ a decision will be taken to buy $x$ and sell $y$, and vice versa.

More appropriately, the decision is made on the basis of a weighted average of the forecasts where the weights are the probabilities. This is given by

$$E(S_{t+1}) = \sum_{i=1}^{q} p_i E(S_{t+1,i}) \quad (7.3)$$

The outcome will depend on the actual exchange rate at $t + 1$ in the same manner as before. Figure 7.4 shows four possibilities for the level of speculative profit with five different values for the exchange rate at $t + 1$. 

**FIGURE 7.3** A Diagrammatic representation of speculative profit ($\theta > \lambda > \eta$).
**Introducing the bid–offer spread**

We will now introduce the bid–offer spread. Let the bid and offer exchange rates at time \( t \) be \( S_{b,t} \) and \( S_{a,t} \), such that \( S_{a,t} = (1 + m)S_{b,t} \), where \( m \) is the bid–offer spread expressed as a percentage of the bid rate. In the presence of the bid–offer spread, the speculator buys currency \( y \) at the higher offer rate and sells it at the lower bid rate. If the forecast bid and offer rates are \( E(S_{b,t+1}) \) and \( E(S_{a,t+1}) \), and assuming that there is no change in the bid–offer spread between time \( t \) and time \( t + 1 \), then

\[
E(S_{a,t+1}) = (1 + m)E(S_{b,t+1})
\]  

(7.4)

To act on the basis of the forecast, a decision to buy \( y \) and sell \( x \) at time \( t \) will be taken if \( y \) is forecast to appreciate such that \( E(S_{b,t+1}) > S_{a,t} \). If the forecast is accurate such that \( S_{b,t+1} > S_{a,t} \), then the speculator can sell currency \( y \) at time \( t + 1 \) at \( S_{b,t+1} \) to realise profit (per unit of \( y \)) that is given by
\[ \pi = S_{b,t+1} - S_{a,t} \]  

or

\[ \pi = S_{b,t+1} - (1+m)S_{b,t} \]  

The profit realised is positive if

\[ S_{b,t+1} - (1+m)S_{b,t} > 0 \]  

or if

\[ \frac{S_{b,t+1}}{S_{b,t}} > 1+m \]  

which gives

\[ S_b > m \]  

The condition represented by equation (7.9) says that the operation will be profitable only if the percentage rise in the bid exchange rate is greater than the bid–offer spread. The forecast in this case must indicate not only the direction but also the size of the change. The decision to buy \( y \) and sell \( x \) at time \( t \) will be taken only if

\[ \frac{E(S_{b,t+1})}{S_{b,t}} > 1+m \]  

Conversely, the decision to buy \( x \) and sell \( y \) at time \( t \) will be taken if

\[ E(S_{a,t+1}) < S_{b,t} \]  

If the expectation is realised such that \( S_{a,t+1} < S_{b,t} \) then the speculator can sell currency \( x \) (buy currency \( y \)) at \( S_{b,t+1} \) to realise profit that is given by

\[ \pi = S_{b,t} - S_{a,t+1} \]  

or

\[ \pi = S_{b,t} - (1+m)S_{b,t+1} \]  

For this profit to be positive, the following condition must be satisfied

\[ (1+m)S_{b,t+1} - S_{b,t} < 0 \]  

or

\[ \frac{S_{b,t+1}}{S_{b,t}} < \frac{1}{1+m} \]  

which means that the operation will be profitable only if the bid exchange rate falls by a factor that is related to the bid–offer spread. Again, the forecast must
indicate the direction and the size of change. The decision to buy $x$ and sell $y$ will be taken only if

$$\frac{E(S_{b,t+1})}{S_{b,t}} < \frac{1}{1+m} \quad (7.16)$$

The matter becomes more complicated if the bid–offer spread also changes between $t$ and $t+1$. So, let

$$S_{a,t} = (1+m)S_{b,t} \quad (7.17)$$

and

$$E(S_{a,t+1}) = [1 + E(m_{t+1})]E(S_{b,t+1}) \quad (7.18)$$

A decision to buy $x$ and sell $y$ will be taken if

$$[1 + E(m_{t+1})]E(S_{b,t+1}) < S_{b,t} \quad (7.19)$$

For this operation to be profitable, the following condition must be satisfied

$$\frac{E(S_{b,t+1})}{S_{b,t}} < \frac{1}{1 + E(m_{t+1})} \quad (7.20)$$

which means that the change in the bid–offer spread must also be forecast.

**Introducing interest rates**

If interest rates are introduced, the condition for making profit by selling $x$ and buying $y$ will change to the following

$$1 + i_x < \frac{E(S_{t+1})}{S_t} (1 + i_y) \quad (7.21)$$

or

$$1 + i_x < [1 + E(\hat{S})] (1 + i_y) \quad (7.22)$$

which says that the rate of return on a position in $x$ should be lower than the rate of return on a position in $y$. Otherwise, it will be profitable to buy $y$ and sell $x$.

Without the bid–offer spreads it does not make any difference for the formula whether or not the position in $x$ is available or that the funds have to be borrowed, because the interest rates in (7.21) and (7.22) can be taken to be the lending or the borrowing rates. If we introduce the bid–offer spreads in interest and exchange rates, the condition given by (7.21) becomes

$$1 + i_{x,b} \frac{E(S_{b,t+1})}{S_{a,t}} < (1 + i_{y,b}) \quad (7.23)$$
if the position in $x$ is available. If the funds have to be borrowed, the condition changes to

$$1 + i_{x,a} < \frac{E(S_{b,t+1})}{S_{a,t}} (1 + i_{y,b})$$  \hspace{1cm} (7.24)$$

If we look carefully at the conditions given by (7.21)–(7.24), we find that they are equivalent to the uncovered interest arbitrage condition, which brings us back to the question of whether uncovered arbitrage is actually arbitrage or speculation. This question was discussed in Chapter 2, where we showed that under certain conditions this operation can be regarded as arbitrage. It also brings us to an issue that we will discuss later in this chapter, which is whether or not arbitrage is a risky or riskless operation.

### 7.3 SPOT SPECULATION BASED ON SPECIAL EVENTS

Sometimes profit can be made by speculating in the spot market based on special events. For example, George Soros made billions of dollars during the European Monetary System (EMS) crisis in September 1992, based on his forecast that it would be very difficult for the Bank of England to maintain the pound’s fixed exchange rate within the EMS. Another example is the May 1986 devaluation of the Norwegian krone, following the collapse of oil prices in the mid-1980s. These two episodes will be described briefly here.

#### The EMS crisis of 1992

The UK joined the exchange rate mechanism (ERM) of the EMS in October 1991 at the central rate of 2.95 (DEM/GBP) against the German mark, a rate that many perceived to be artificially high (the pound was overvalued). Other events of the early 1990s started to unfold. First, the removal of capital controls was completed by the members of the European Economic Community in the spirit of the programme to unify financial markets within the Community. Then, there was the recession of 1992 and German reunification. The chain of events went as follows. German reunification put upward pressure on German interest rates, as the demand for funds rose with the need to finance the operation. As a result, the German mark started to appreciate against other currencies, at a time when other countries could not defend their currencies by raising interest rates, since their economies were in recession. For the UK, the situation was even more difficult, as the task of defending the overvalued pound was extremely demanding. Although the Bank of England spent almost all of its reserves to defend the pound, it was not possible to maintain the rate required by the ERM, and the pound was forced to exit in September...
1992 at a lower rate. By selling the pound short in anticipation of these events, Soros made billions in profit at the expense of British taxpayers.

The Norwegian krone: May 1986
In the mid-1980s the oil market was characterised by a chronic glut resulting from a drop in demand, as industrial countries found ways to reduce oil consumption per unit of GDP. At that time, both the Norwegian krone and the pound were described as petrocurrencies because the UK and Norway had become oil exporters. The currencies were sensitive to changes in oil prices because of the dependency of the respective economies on oil revenues. Since the Norwegian economy was more dependent on oil than the British economy, it was felt that a collapse of oil prices would adversely affect both currencies, but that the Norwegian currency would be affected even more. This would also be the case because the Norwegian currency was pegged to a basket, and the defence of the rate implied by the basket required intervention in the foreign exchange market. Intervention, as we know, costs money, and if the money required for intervention is obtained from oil revenue, then a collapse of oil prices would make the Norwegian authorities less able or less willing to defend the exchange rate. At that time, the interest rates on both currencies were close at around 11%. With the Norwegian krone expected to depreciate against the pound, taking a short position on the krone and a long position on the pound would result in net profit equal to the percentage change in the exchange rate between the two currencies.

This operation can be formalised as follows. First, the cost of going short on the Norwegian krone is the interest rate on the currency, \( i_{NOK} \). The return from the long position on the pound is the interest rate on the pound plus the percentage change in the exchange rate between the two currencies, \( i_{GBP} + \hat{S}(NOK/GBP) \). The net profit from this operation would be

\[
\pi = (i_{GBP} - i_{NOK}) + \hat{S}(NOK/GBP) \tag{7.25}
\]

If \( i_{GBP} \approx i_{NOK} \), it follows that

\[
\pi \approx \hat{S}(NOK/GBP) \tag{7.26}
\]

Now, we have the cross exchange rate relationship

\[
\hat{S}(NOK/GBP) = \frac{\hat{S}(NOK/USD)}{\hat{S}(GBP/USD)} \tag{7.27}
\]

If the Norwegian krone falls more proportionately against the US dollar than the British pound does, such that \( \hat{S}(NOK/USD) > \hat{S}(GBP/USD) \), it follows that

\[
\hat{S}(NOK/GBP) = \hat{S}(NOK/USD) - \hat{S}(GBP/USD) > 0 \tag{7.28}
\]

which means that the operation would be profitable.
Spot–forward speculation involves operations in both the spot and forward markets. Let $F_{t+1}^t$ be the forward rate applicable to a forward contract initiated at time $t$ for delivery at time $t + 1$. If a speculator believes that the spot exchange rate prevailing at $t + 1$ will be higher than the forward rate then he will react by buying currency $y$ forward at time $t$ and selling it spot at time $t + 1$ when the forward contract matures. The speculator will do this if

$$E(S_{t+1}) > F_t$$

(7.29)

By dividing both sides of the inequality by $S_t$, we obtain

$$E(S_{t+1}) > f_t$$

(7.30)

where $f$ is the forward spread measured as the difference between the forward and spot rates as a percentage of the spot rate. Thus, the decision rule involves forecasting the spot exchange at time $t + 1$ or the percentage change in the spot exchange rate between $t$ and $t + 1$. If the forecast is accurate such that $E(S_{t+1}) = S_{t+1}$, then profit (in terms of currency $x$) per unit of currency $y$ is given by

$$\pi = S_{t+1} - F_t$$

(7.31)

If, on the other hand, the speculator believes that the spot rate at $t + 1$ will be lower than the forward rate, then he will respond by short selling currency $y$ spot (that is, by borrowing and selling currency $y$) and buying it forward. Hence, the condition to indulge in spot–forward speculation is

$$E(S_{t+1}) < F_t$$

(7.32)

or

$$E(S_{t+1}) < f_t$$

(7.33)

If the forecast is accurate such that $E(S_{t+1}) = S_{t+1}$, then profit (per unit of $y$) is given by

$$\pi = F_t - S_{t+1}$$

(7.34)

**Spot–forward speculation in the presence of bid–offer spreads**

If the bid–offer spreads in the spot and forward rates are taken into account, then the problem becomes the following. If currency $y$ is expected to appreciate, a speculator will buy it forward at the offer forward rate and sell it spot at the expected bid spot rate. Hence, the condition necessary to indulge in spot–forward speculation is

$$E(S_{b,t+1}) > F_{a,t}$$

(7.35)

By definition
\[ F_{a,t} = (1 + m)F_{b,t} \]  
(7.36)

where \( m \) in this case is the bid–offer spread on the forward rate. Therefore the condition becomes

\[ E(S_{b,t+1}) > (1 + m)F_{b,t} \]  
(7.37)

The expected value of the profit is

\[ E(\pi) = E(S_{b,t+1}) - (1 + m)F_{b,t} \]  
(7.38)

which is positive when

\[ E(S_b) > f + m \]  
(7.39)

because \( F_{b,t} = S_{b,t}(1 + f) \). Thus, the operation will be profitable when the bid exchange rate is expected to rise by more than the sum of the forward spread and the bid–offer spread. Similar conditions can be derived for speculation by short selling \( y \) at \( t \). Allowance can also be made for changes in the bid–offer spreads between \( t \) and \( t + 1 \).

### 7.5 FORWARD SPECULATION

It is possible to speculate by combining two offsetting forward transactions contracted at two different points in time but for the same delivery date; that is, by using two forward contracts with two different maturities. Consider two forward (or futures) contracts maturing at time \( t + 2 \): one is a two-period contract initiated at time \( t \) to mature at time \( t + 2 \) and the other is a one-period contract initiated at time \( t + 1 \) to mature at time \( t + 2 \). At time \( t \), the speculator will indulge in forward speculation, by taking a long position on \( y \), if

\[ E(F_{t+2}^{t+2}) > F_{t+2}^{t+2} \]  
(7.40)

where \( F_{t+2}^{t+2} \) is the two-period forward rate applicable to a forward contract initiated at \( t \) to mature at \( t + 2 \) and \( F_{t+1}^{t+2} \) is the one-period forward rate applicable to a forward contract initiated at \( t + 1 \) for delivery at \( t + 2 \). Thus, \( E(F_{t+2}^{t+2}) \) is the one-period forward rate expected to prevail at \( t + 1 \). If this condition is satisfied, then the speculator can make profit by buying currency \( y \) on the two-period contract at \( t \) and selling the one-period contract at \( t + 1 \). At \( t + 2 \), the speculator will take delivery of currency \( y \) in accordance with the two-period contract and deliver to the counterparty in accordance with the one-period contract.

If we consider the three-period contract, then the speculator has even greater flexibility. She can speculate on the basis of the expected value of the one-period forward rate prevailing at \( t + 2 \), \( E(F_{t+2}^{t+3}) \), or the expected value of the two-period forward rate prevailing at \( t + 1 \), \( E(F_{t+3}^{t+3}) \). Suppose that both of these rates are expected to be higher than \( E(F_{t+3}^{t+3}) \). The speculator can buy currency \( y \) at
time $t$ for delivery at time $t + 3$ and sell it at $t + 1$ and $t + 2$ for delivery at $t + 3$, acting on the expected values of the forward rates, $E(F^t_{t+3})$ and $E(F^t_{t+2})$. Suppose also that she sells a fraction, $w$, of the amount at $E(F^t_{t+3})$ and the rest of the amount, $1 - w$, at $E(F^t_{t+2})$. The expected value of the profit is

$$E(\pi) = w[E(F^t_{t+3}) - F^t_{t+3}] + (1 - w)[E(F^t_{t+2}) - F^t_{t+3}]$$ (7.41)

If there are $n$ periods, then forward speculators can speculate on a set of forward rates ranging from the one-period rate expected to prevail at $t + n - 1$ (one period before the maturity of the $n$-period contract) to the $(n - 1)$-period rate expected to prevail at $t + 1$ ($n - 1$ periods before the maturity of the $n$-period contract). Again, if the speculator buys an amount at $F^t_{t+n}$ and sells a fraction, $w_i$, at $E(F^t_{t+i})$ at time $t + i$ for $i = 1$ to $i = t + n - 1$ then the expected profit is

$$E(\pi) = \sum_{i=1}^{t+n-1} w_i [E(F^t_{t+i}) - F^t_{t+i}]$$ (7.42)

It is obvious that the decision rule to indulge in forward speculation requires the forecasting of the forward exchange rates prevailing between $t$ and $t + n$.

### 7.6 Speculation with Currency Options

Let us consider call and put options on currency $y$. The call gives the holder the right to buy currency $y$ at the exercise exchange rate, $S_t$, assuming that the option is at the money, whereas the put option gives the holder the right to sell the currency at the exercise exchange rate. Let us also assume, for the time being, that we are dealing with European options, which can be exercised on the maturity of the contract only (that is on the expiry date of the option). A speculator will buy a call or sell a put if she expects currency $y$ to appreciate

Consider first the case of buying a call at time $t$, which expires at time $t + 1$. A speculator will do this if she expects the exchange rate at time $t + 1$ to be higher than the exercise exchange rate plus the premium, $m$; that is, if

$$E(S^t_{t+1}) > S_t + m$$ (7.43)

If the forecast is accurate then the speculator can make profit by exercising the option at $t + 1$, buying the currency at $S_t$ and selling it spot at $S_{t+1}$, in which case the net profit earned per unit of $y$ is $S_{t+1} - S_t - m$. Based on the same forecast, the speculator may decide to sell a put. If the forecast is correct the holder of the put will not exercise and the speculator makes profit by keeping the premium paid to her (by the holder) up front.
If, on the other hand, the forecast indicates that the expected spot rate will be below the exercise exchange rate, then profit can be made by buying a put option. Thus, the condition required for buying a put option is

\[ E(S_{t+1}) < S_t - m \] (7.44)

If the forecast is accurate, then on the expiry date, the speculator buys currency \( y \) spot at \( S_{t+1} \) and sells it at the exercise exchange rate \( S_t \), earning net profit of \( S_t - S_{t+1} - m \). Alternatively, the speculator can sell a call. If the forecast is accurate, the call will not be exercised and profit will be gained since the premium received up front can be kept. Notice that the forecast is not necessarily that currency \( y \) will appreciate or depreciate from the present level, but that it ends up higher or lower than the exercise exchange rate on the expiry date.

**Speculation with combined option positions**

We will consider two combined option positions: a short straddle and a long straddle. A short straddle involves selling a call and a put with the same exercise exchange rate. This position is taken when the underlying currency is not expected to move much. Suppose that at time \( t \), a forecast indicates that \( E(S_{t+1}) = S_t \). In this case, a short straddle position should be taken, because if the forecast is accurate then neither the call nor the put will be exercised and profit will be realised that is equal to the premiums received for the two options.

If, on the other hand, a forecast indicates that \( E(S_{t+1}) > (S_t + m_c + m_p) \) or that \( E(S_{t+1}) < (S_t - m_c - m_p) \), where \( m_c \) and \( m_p \) are the premiums on the call and the put respectively, then a long straddle position should be taken. If the first forecast is accurate, the call option is exercised while the put option is left to expire, in which case the profit will be \( S_t + m_c + m_p \). If, on the other hand, the second forecast is accurate, then the put option is exercised while the call option is left to expire, in which case net profit will be \( S_t - (S_{t+1} + m_c + m_p) \).

**Speculation with exotic options**

We now consider an example of exotic options, the average rate option. This option gives the right to sell an amount of the underlying currency at the exercise exchange rate if the average spot rate over the period between \( t \) and \( t + n \) is less than the exercise exchange rate. Suppose that the average exchange rate, \( \overline{S} \), is calculated on the basis of \( k \) observations taken at times \( t + 1, \ldots, t + 2, \ldots, t + k \), where \( k < n \). Hence

\[ \overline{S} = \frac{1}{k} \sum_{i=1}^{k} S_{t+i} \] (7.45)
At time $t$, the exchange rate at $t + 1, ..., t + 2, ..., t + k$ is not known, so $\overline{S}$ is not known. The expected value of $\overline{S}$ is

$$E(\overline{S}) = \frac{1}{k} \sum_{i=1}^{k} E(S_{t+i})$$  \hspace{1cm} (7.46)$$

The option will be exercised if $\overline{S} < S_t$, in which case the holder can sell the underlying currency at $S_t$. If the speculator buys the currency spot at the average exchange rate, $\overline{S}$, then the profit realised is $S_t - (\overline{S} + m)$. Still better is if the speculator buys the currency spot at $S_{t+1}$, where $S_{t+1} < \overline{S}$. The speculator will buy an average rate option at time $t$ if

$$\frac{1}{k} \sum_{i=1}^{k} E(S_{t+i}) < S_t$$  \hspace{1cm} (7.47)$$

which obviously requires forecasting the spot exchange rate at several points in time between $t$ and $t + k$.

7.7 COMBINING SPECULATION WITH ARBITRAGE AND HEDGING

The so-called “modern theory of forward exchange” has been presented as another explanation for the observed deviations from CIP by, inter alia, Spraos (1953), Tsiang (1959) and Grubel (1968). This theory is labelled “modern”, although it was developed in the 1950s, because it is modern in relation to its predecessor, the CIP theory. These economists assert that operations other than arbitrage, such as hedging and speculation, also exert a determining impact on the forward exchange rate. Even earlier proponents of the CIP theory, such as Keynes (1923), Einzig (1937) and Kindleberger (1939), did not rule out the possibility of speculative pressure and offered considerations of how various speculative forces alter the relationship. According to this theory, the equilibrium forward rate is determined not only by the interest parity forward rate but also by the future spot rate expected to prevail at the maturity of the forward contract. This theory has been tested under fixed and flexible exchange rates by several economists, including Stoll (1968), Kesselman (1971), Haas (1974), McCallum (1977), Stokes and Neuberger (1979), Callier (1980, 1981), Taylor (1987a), Moosa and Bhatti (1994, 1997) and Moosa (1996a). The results of these tests have been mixed, with McCallum (1977), Callier (1981), Moosa and Bhatti (1994, 1997) and Moosa (1996a) rejecting the modern theory in favour of the traditional CIP theory.

Stoll (1968, p. 68) argues that under a flexible exchange rate system there is considerable uncertainty about the future spot rate that makes speculative demand more inelastic and, therefore, speculation less effective. This view is
shared by Spraos (1953), who argues that speculative demand is highly inelastic under flexible exchange rates because speculators have no single value of the exchange rate as a reference point. While Kesselman (1971, p. 297) argues that these findings cannot be generalised for all flexible exchange markets, Moosa and Bhatti (1994) provide evidence indicating that speculation has no role to play in forward rate determination under the current system of flexible exchange rates. Only Taylor (1987a) found highly contrasting evidence under the flexible exchange rate period of 1973–1980.

The model
The model presented here is derived by specifying excess demand functions for forward (foreign currency) contracts. According to this model, the forward rate is determined by arbitrage, speculation and hedging. Arbitragers’ excess demand for forward exchange, \( X^a_t \), can be defined as

\[
\begin{equation}
X^a_t = a(F^{t+1}_t - F^t_t), \quad a > 0
\end{equation}
\]

where \( F^{t+1}_t \) is the actual one-period forward rate, which is determined at time \( t \), but is applicable to delivery at time \( t + 1 \), and \( F^t_t \) is the corresponding interest parity forward rate that is derived by adjusting the spot rate by a factor that reflects the interest rate differential. Equation (7.48) tells us that if \( F^{t+1}_t > F^t_t \) then \( X^a_t > 0 \), implying that arbitragers will be net buyers of forward contracts. Similarly, an excess demand function can be specified for speculators which may take the form

\[
\begin{equation}
X^s_t = b[E(S_{t+1}) - F^t_t], \quad b > 0
\end{equation}
\]

Thus, \( E(S_{t+1}) \) is the spot rate expected to prevail at the maturity of the forward contract (at time \( t + 1 \)) conditional on the information set available at time \( t \). If \( E(S_{t+1}) > F^t_t \) then \( X^s_t > 0 \), implying that spot speculators will be net buyers of forward contracts.

Equilibrium cannot exist if all market participants are net buyers or net sellers of forward contracts, since it requires zero excess demand. It is achieved when there is zero excess demand, a condition that is represented by

\[
\begin{equation}
X^a_t + X^s_t = 0
\end{equation}
\]

By substituting equations (7.48) and (7.49) into equation (7.50) and solving for the forward rate we obtain

\[
\begin{equation}
F^{t+1}_t = \left( \frac{a}{a+b} \right) F^t_t + \left( \frac{b}{a+b} \right) E(S_{t+1})
\end{equation}
\]

which tells us that the forward rate is a weighted average of the interest parity forward rate and the expected spot rate. If the coefficient on \( F^t_t \) is greater than the coefficient on \( E(S_{t+1}) \), this means that arbitrage plays a more effective role than speculation in determining the forward rate. Moreover, if the
coefficient on $F_{t+1}^t$ is not significantly different from unity, while the coefficient on $E(S_{t+1})$ is not significantly different from zero, this means that CIP holds and that speculation plays no role in determining the forward rate.

It is also possible to incorporate the role of hedgers by specifying the following excess demand function

$$X_t^h = \gamma - hF_t, \quad \gamma, h > 0$$

(7.52)

In this case equilibrium is represented by the equation

$$F_{t+1}^t = \left( \frac{\gamma}{a+b+h} \right) + \left( \frac{a}{a+b+h} \right) F_{t+1}^t + \left( \frac{h}{a+b+h} \right) E(S_{t+1})$$

(7.53)

There are, however, two reasons why the role of hedgers may be ignored, or at least not taken into account explicitly for the purpose of specifying the model. First, Stoll (1968) argues that traders can be considered either as arbitragers or as speculators. Indeed, it is often argued that the decision whether to hedge or not is a speculative decision that depends largely on expectation. Second, McCallum (1977) concludes that if $\gamma$ and $h$ are small, equation (7.53) will not be distinguishable from equation (7.51).

Figure 7.5 is a diagrammatic representation of the determination of the forward rate by arbitrage and speculation. AA is the excess demand function of arbitragers (equation 7.48) whereas SS is the excess demand function of speculators (equation 7.49). The equilibrium forward rate, $F_{t+1}^t$, is the one consistent with the condition of zero excess demand, which is the rate.

\[ \text{FIGURE 7.5 Determination of the forward rate by arbitrage and speculation.} \]
satisfying the condition $X^a = -X^s$. It is important to notice that $F^{t+1}_t$ is closer to $F^{t+1}_t$ than to $E(S_{t+1})$, implying that arbitrage plays a bigger role than speculation in determining the forward rate (that is, a situation of arbitrage dominance). This is because the arbitragers’ excess demand function is more elastic than the speculators’ function, a situation that seems plausible under a system of flexible exchange rates.

**Introducing forward speculation**

Following the discussion in Section 7.5, we may specify an excess demand function for forward speculators. Considering a two-period and a one-period forward contract, this excess demand function can be written as

$$X^f_t = c[E(F^{t+2}_{t+1}) - F^{t+2}_t]$$  \hspace{1cm} (7.54)

If $E(F^{t+2}_{t+1}) > F^{t+2}_t$, then $X^f_t > 0$, implying that forward speculators will be net buyers of the two-period forward contract. In this case, the speculator can make a profit by buying the two-period contract at $t$ and selling the one-period contract at $t + 1$. At $t + 2$ the speculator will take delivery of the foreign currency in accordance with the two-period contract and deliver to the counterparty in accordance with the one-period contract. This situation is illustrated by Figure 7.6. AA, FF and SS represent the excess demand functions.

**FIGURE 7.6** Determination of the forward rate by arbitrage, spot speculation and forward speculation.
of arbitragers, forward speculators and spot speculators respectively. The configuration that \( E(S_{t+2}) < F_{t+2}^t < F_{t+2}^t < E(F_{t+1}) \) is only an assumption that is used for illustrative purposes. Any other possibility can be used instead, making no difference to the description of the equilibrium position. The equilibrium two-period forward rate is determined by the point that satisfies the condition \( X^a + X^f = -X^s \). Notice that the equilibrium forward rate, \( F_{t+2}^t \), is closer to \( F_{t+2}^t \) than to either \( E(F_{t+2}^t) \) or \( E(S_{t+2}) \), implying the dominance of arbitrage.

In general mathematical terms, equilibrium is achieved when there is zero excess demand, a condition that can be written as

\[
X_t^a + X_t^s + X_t^f = 0 \quad (7.55)
\]

in which case we obtain

\[
F_{t+2}^t = \left( \frac{a}{a+b+c} \right) F_{t+2}^t + \left( \frac{b}{a+b+c} \right) E(S_{t+2}) + \left( \frac{c}{a+b+c} \right) E(F_{t+1}^t) \quad (7.56)
\]

where the coefficients on \( F_{t+2}^t \), \( E(S_{t+2}) \) and \( E(F_{t+1}^t) \) signify the effectiveness of arbitrage, spot speculation and forward speculation respectively.

If we consider the three-period contract, forward speculators have even greater flexibility. They can speculate on the basis of the expected value of the one-period forward rate prevailing at \( t+2 \), \( E(F_{t+3}^{t+2}) \), or the expected value of the two-period forward rate prevailing at \( t+1 \), \( E(F_{t+3}^{t+1}) \). In this case the equilibrium condition becomes

\[
F_{t+3}^t = \left( \frac{a_3}{a_3+b_3+c_{31}+c_{32}} \right) F_{t+2}^t + \left( \frac{b_3}{a_3+b_3+c_{31}+c_{32}} \right) E(S_{t+3}) + \left( \frac{c_{31}}{a_3+b_3+c_{31}+c_{32}} \right) E(F_{t+2}^{t+2}) + \left( \frac{c_{32}}{a_3+b_3+c_{31}+c_{32}} \right) E(F_{t+1}^{t+3}) \quad (7.57)
\]

The coefficient \( c_{31} \) measures the effectiveness of forward speculation in determining the three-period forward rate when the one-period forward rate is used as the variable on which the (forward) speculative decision is based. Likewise, the coefficient \( c_{32} \) measures the effectiveness of forward speculation when speculative decisions are based on the two-period forward rate. The coefficients \( a_3 \) and \( b_3 \) measure the effectiveness of arbitrage and spot speculation in determining the three-period forward rate.

In general, if there are \( n \) periods then forward speculators can speculate on a set of forward rates ranging from the one-period rate expected to prevail at \( t+n-1 \) (one month before the maturity of the one-period contract) to the \( (n-1) \)-period price expected to prevail at \( t+1 \) (\( n-1 \) periods before the maturity of the \( n \)-period contract). The generalisation of equation (7.57) may, therefore, be written as
\[ F_{t+n} = \left( \frac{a_n}{a_n + b_n + \sum_{i=1}^{n-1} c_{ni}} \right) F_{t+n} + \left( \frac{b_n}{a_n + b_n + \sum_{i=1}^{n-1} c_{ni}} \right) E(S_{t+n}) \]
\[ + \sum_{i=1}^{n-1} \left( \frac{c_{ni}}{a_n + b_n + \sum_{i=1}^{n-1} c_{ni}} \right) E(F_{t+n-i}) \] (7.58)

where the coefficient \( c_{ni} \) measures the effectiveness of forward speculation in determining the \( n \)-period forward rate, such that the speculative decision is based on the \( i \)-period forward rate expected to prevail at \( t + n - i \).

**Speculation as a source of deviation from CIP**

The role of speculation as a source of deviations from CIP can be demonstrated by rewriting equation (7.51) as

\[ F_{t+1} = \beta_0 + \beta_1 F_{t+1} + \beta_2 E(S_{t+1}) \] (7.59)

which can be rewritten as

\[ F_{t+1} = \left( \frac{1}{\beta_1} \right) F_{t+1} - \frac{\beta_0}{\beta_1} - \left( \frac{\beta_2}{\beta_1} \right) E(S_{t+1}) \] (7.60)

Since CIP holds precisely for \( F_{t+1} \), the modified CIP condition prevailing at time \( t \) becomes

\[ 1+i_x = \left[ F_{t+1} - \frac{\beta_0}{\beta_1} - \beta_2 E(S_{t+1}) \right] \left( \frac{1+i_y}{\beta_1} \right) \] (7.61)

or

\[ 1+i_x = \frac{F_{t+1}}{S_t} \left( \frac{1+i_y}{\beta_1} \right) \left( \frac{1+i_y}{\beta_1} \right) \left( \frac{1+i_y}{\beta_1} \right) \left( \beta_0 + \beta_2 E(S_{t+1}) \right) \] (7.62)

By manipulating equation (7.62), we obtain

\[ 1+i_x = \frac{F_{t+1}}{S_t} \left( 1+i_y \right) \left( \frac{1+i_y}{\beta_1} \right) \left( \beta_0 + \beta_2 E(S_{t+1}) \right) \]
\[ - \left( \frac{(1+i_y)(\beta_1-1)}{\beta_1} \right) \left( \frac{F_{t+1}}{S_t} \right) \] (7.63)

Equation (7.63) implies the presence of a neutral band of \( \pm D \), where

\[ D = \left( \frac{1+i_y}{\beta_1} \right) \left[ \beta_0 + \beta_2 E(S_{t+1}) - (\beta_1-1) \left( \frac{F_{t+1}}{S_t} \right) \right] \] (7.64)
Therefore, the effect of speculation is to distort the CIP relationship, such that the covered margin must exceed $D$ in order to trigger arbitrage. Otherwise, equilibrium will be maintained with the presence of what may appear to be deviation from CIP.

### 7.8 HEDGING AS A SPECULATIVE ACTIVITY

Hedging and speculation are normally defined to imply that they are diametrically opposite to each other, and the same applies to hedgers and speculators. Hedging is the covering of risk, whereas speculation is the deliberate assumption of risk in anticipation of profit. Hedgers hate risk and therefore they cover it, whereas speculators love risk and strive to bear it. Hedgers and speculators are, therefore, two different species with diametrically opposite tastes for risk. However, it can be argued that this is not the case and that hedging can be viewed as a speculative activity. It will also be demonstrated that, at least in some cases, hedgers and speculators may behave in similar manners and react to changes in the same parameters.

The definition of hedging and the description of hedgers stated earlier are based on the assumption that the agent’s objective is to minimise risk rather than to maximise expected utility, the latter being dependent on risk as well as expected return. It is arguable that risk minimisation without any regard to the effect on expected return cannot be optimal (see, for example, Cecchetti et al., 1988). This is because risky assets are priced to earn expected premium over the riskless rate. Hence, hedging away the risk must also hedge away the expected return to bearing that risk, which may not be desirable. It is indeed the case that only a totally risk averse agent can make an optimal hedging decision without taking the impact on both risk and return into account. In practice, hedgers are aware of the trade-off between risk and return, and this is why they may hedge partially or selectively. This is also why they may remain exposed to market risk on part of their position or part of the time. Therefore, hedgers do not hedge automatically but take a decision whether or not to hedge on the basis of expectation with respect to the decision variable.

There is some body of theoretical and empirical literature indicating that the decision to hedge is not as automatic as the decision to pray for a religious person. For example, Dolde (1993) argues that firms with exposure to foreign exchange risk may not hedge, partially hedge or stay unhedged depending on their perception about the behaviour of exchange rates. This is essentially speculative behaviour, and it is obvious that there is significant overlapping between hedging and speculation in this case. Joseph (2000) and Marshall (2000) provide some survey evidence on this issue. Furthermore, Culp and Miller (1995) argue that “most value-maximising firms do not hedge”. Finally, Stulz (1995) argues that a firm with little debt or with highly-rated debt has no
need to hedge, as the risk of it getting into financial trouble is tiny. Therefore, hedging is invariably a speculative activity.

It is also possible to show that speculators and hedgers use the same decision variables. For this purpose we refer the model of pricing futures contracts proposed by Moosa and Al-Loughani (1995) and its extension in Moosa (2000b), a version of which was presented in the previous section. In this model speculators have an excess demand function for crude oil futures contracts that can be written as $X^s_t = \beta \left( E(S_{t+1}) - F_{t+1}^t \right)$, where $\beta > 0$. Speculators have an incentive to enter the market whenever there is a difference between $E(S_{t+1})$ and $F_{t+1}^t$. If $E(S_{t+1}) > F_{t+1}^t$ there will be excess demand by speculators who will buy the commodity futures at $t$ and sell it spot at $t + 1$, making profit of $S_{t+1} - F_{t+1}^t$ if their expectations are realised. Otherwise, they will buy futures and sell spot.

It can be demonstrated that the behaviour of hedgers is identical to the behaviour of speculators. Consider first long hedgers who buy futures contracts. For these hedgers, the expected cost of hedging is the difference between the futures price and the expected spot price, $F_{t+1}^t - E(S_{t+1})$, which is the expected cost they are willing to accept to avoid uncertainty. Naturally, the smaller the expected cost, the greater will be the demand for futures contracts, implying that excess demand by long hedgers is a positive function of $E(S_{t+1})$. Conversely, the expected cost of hedging for short hedgers is $E(S_{t+1}) - F_{t+1}^t$. Since short hedgers are suppliers of futures contracts, supply will decrease (excess demand will increase) as the expected cost of hedging increases. Again, excess demand is a positive function of $E(S_{t+1}) - F_{t+1}^t$. It is obvious that the expected cost of long hedgers, $F_{t+1}^t - E(S_{t+1})$, is equivalent to the profit made by speculators who buy spot and sell futures, whereas the expected cost of short hedgers, $E(S_{t+1}) - F_{t+1}^t$, is equivalent to the profit made by speculators who buy futures and short sell spot. Hedgers and speculators act upon the same variables, and this is why this model does not distinguish between them.

There may be only one difference between hedgers and speculators in this particular case. Hedgers are more likely to be market participants who actually require the physical commodity (for example, industrial companies). Speculators, on the other hand, are the participants who are not interested in the physical commodity per se but rather in generating speculative profit from holding ownership titles in that commodity (for example, financial institutions). This is an institutional difference. As far as behaviour is concerned, hedgers and speculators belong to the same species.

7.9 STABILISING AND DESTABILISING SPECULATION

Speculators participate in the foreign exchange market, buying and selling currencies on the basis of certain expectations about the future movements of exchange rates. By their actions, speculators affect the supply of and demand for currencies and, therefore, exchange rates.
Speculation can be stabilising and destabilising. Figure 7.7 shows the effect of destabilising and stabilising speculation on exchange rate volatility. Destabilising speculation, which leads to an increase in exchange rate volatility, occurs when speculators buy a currency when it is high and sell it when it is low. This kind of behaviour arises, for example, when speculators believe that there are “bubbles” in the market. Thus, when a currency appreciates, they think that it will keep on appreciating, so they will buy it until the bubble bursts for one reason or another. Conversely, when a currency depreciates they believe that it will keep on depreciating, so they will keep on selling it, inducing a run on the currency. Krugman (1989) identifies two episodes of destabilising speculation on the US dollar: the 1984–1985 appreciation and the surge of April–June 1989.

Stabilising speculation works the other way round. When the demand for a currency increases, it appreciates, and as it appreciates further speculators start thinking that it is overvalued so they sell it, causing it to depreciate. Conversely, when a currency depreciates, because of an increase in its supply, speculators will start thinking that it is undervalued and hence it is a bargain. Therefore they will buy it, reversing the trend in the foreign exchange market. Stabilising speculation, therefore, reduces exchange rate volatility.

**Is profitable speculation stabilising?**

Following Farrell (1966), it can be shown that profitable speculation is stabilising. Let $S_t$ and $S_{t+1}$ be the exchange rates in two successive periods in the
absence of speculation. Speculation takes the form of buying \( Q \) units of the foreign currency at \( t \) and selling it at \( t + 1 \). Speculative purchases and sales produce the exchange rates \( \tilde{S}_t \) and \( \tilde{S}_{t+1} \), which are given by

\[
\tilde{S}_t = S_t + \alpha Q \tag{7.65}
\]

\[
\tilde{S}_{t+1} = S_{t+1} - \alpha Q \tag{7.66}
\]

where \( \alpha > 0 \). The mean value of the exchange rate in the absence of speculation is given by

\[
\overline{S} = \frac{(S_t + S_{t+1})}{2} = \frac{\tilde{S}_t + \tilde{S}_{t+1}}{2} \tag{7.67}
\]

which means that speculation (in this case) does not change the mean value of the exchange rate. Speculative profit is given by

\[
\pi = Q(\tilde{S}_{t+1} - \tilde{S}_t) \tag{7.68}
\]

By substituting equations (7.65) and (7.66) into (7.68) we obtain

\[
\pi = Q(S_{t+1} - S_t) - 2\alpha Q^2 \tag{7.69}
\]

Exchange rate volatility is measured by the variance of the exchange rate. Without speculation, the variance is given by

\[
\sigma_n^2 = \frac{1}{2} [(S_t - \overline{S})^2 + (S_{t+1} - \overline{S})^2] = \frac{1}{4} (S_t - S_{t+1})^2 \tag{7.70}
\]

On the other hand, the variance of the exchange rate with speculation is given by

\[
\sigma_s^2 = \frac{1}{2} [(\tilde{S}_t - \overline{S})^2 + (\tilde{S}_{t+1} - \overline{S})^2] \tag{7.71}
\]

By substitution and manipulation we obtain

\[
\sigma_s^2 = \frac{1}{4} (S_t - S_{t+1})^2 + (S_t - S_{t+1})\alpha Q + \alpha^2 Q^2 \tag{7.72}
\]

or

\[
\sigma_s^2 = \sigma_n^2 - \alpha \pi - \alpha^2 Q^2 \tag{7.73}
\]

Equation (7.73) shows that if \( \pi > 0 \), then \( \sigma_s^2 < \sigma_n^2 \). Hence, profitable speculation is stabilising.

### 7.10 SPECULATIVE BUBBLES

A bubble is a self-reinforcing movement in prices (including exchange rates) away from their equilibrium level that is determined by fundamentals. This happens, for example, when everyone thinks that the price is too high (low),
but no-one expects it to fall (rise). Sometimes, bubbles are explained in terms of trader irrationality, market psychology, mass hysteria and so on. However, bubbles can be reconciled with rational behaviour, hence the term “rational bubbles”.

Once a bubble starts, it becomes a reality that traders have to live with. The equilibrium value or fair value, as envisaged by an economist, becomes irrelevant. Traders take the attitude whereby they will buy as long as the market believes that prices will keep on rising. The perceived continuation of the price rise provides compensation for risk bearing. This is why traders kept on buying the US dollar in the 1980s although it was overvalued by all measures. In hindsight, we know that the 1980–85 appreciation of the dollar was a spectacular bubble (Figure 7.8).

If the foreign exchange market is not experiencing a bubble, the exchange rate is determined by fundamentals, the expected values of the fundamentals to be precise. Thus

\[ S_t = f[X_t, E(X_{t+1}), E(X_{t+2}), ..., E(X_{t+k})] \]  

(7.74)

In the presence of a bubble, equation (7.74) becomes

\[ S_t = f[X_t, E(X_{t+1}), E(X_{t+2}), ..., E(X_{t+k})] + B_t \]  

(7.75)

where the bubble term represents the extent of the deviation from the fundamental equilibrium rate. The bubble, \( B_t \), has the property

\[ E(B_{t+1}) = \beta^{-1}B_t \]  

(7.76)

where \( \beta < 1 \). Existing theory has nothing to say about how and why a bubble develops. However, two propositions stand out. The first is that it is sufficient for traders to perceive the bubble factor to be important for it to become

---

**FIGURE 7.8** The US dollar’s effective exchange rate.
important. Second, a bubble can be viewed as a variable that is unobservable to economists but observable to traders.

At any point in time, there must be some perceived probability that the bubble will burst in the next period. If this happens, the term $B_t$ will disappear from equation (7.75), which means that the exchange rate will revert back to its fundamental equilibrium level. By introducing this probability, the behaviour of the bubble can be represented as follows:

\[
\begin{align*}
E(B_{t+1}) &= \beta^{-1}B_t \\
E(B_{t+1}) &= 0
\end{align*}
\]

with probabilities $p$ and $1 - p$ respectively. In this case, as long as the bubble persists the exchange rate will rise sufficiently in order to compensate traders for the possibility of loss when the bubble bursts. The greater the exchange rate’s deviation from fundamentals, the further it has to fall and hence the greater the prospective capital gains needed to sustain the process.

Bubbles may be triggered by fundamental changes, actual or perceived, but they may be spontaneous. Empirical research reveals, for example, that the behaviour of the dollar until the mid-1980 was a rational bubble. The presence of bubbles is often suggested as a reason for the failure of fundamental models of exchange rates, such as the monetary model (for example, Lane 1991).
CHAPTER 8

Speculation: Generating Buy and Sell Signals

8.1 SPECULATION ON THE BASIS OF EXPECTATION FORMATION

Foreign exchange market participants may act on the basis of expectations on the future change in the exchange rate, such that the expectation is formed in a mechanical manner following one or more expectation formation mechanisms. These mechanisms will be specified in terms of the percentage change in the exchange rate, $\dot{S}$, which may be approximated by the first log difference of the exchange rate. Again, it will be assumed that $t$ is the present time, when the expectation is formed, whereas $t + 1$ is future point in time at which the actual value of the expected variable materialises and which coincides with the outcome of the decision taken at time $t$. For the purpose of the following analysis, the words “forecasting” and “prediction” may be used interchangeably with the word “expectation”. Strictly speaking, however, forecasting may be defined as a formal way of forming expectations. Another point that is worthy of mentioning here is that the expectation formation mechanisms may be written in terms of the level rather than the percentage change in the exchange rate.

Extrapolative expectations
There are four versions of extrapolative expectations, which are represented by the equations

$$E(\dot{S}_{t+1}) = \delta \dot{S}_t \quad \delta > 0$$  \hspace{1cm} (8.1)

$$E(\dot{S}_{t+1}) = (1-\theta)\dot{S}_t + \theta \dot{S}_{t-1} \quad 0 < \theta < 1$$  \hspace{1cm} (8.2)

$$E(\dot{S}_{t+1}) = \frac{1}{n} \sum_{i=0}^{n} \dot{S}_{t-i}$$  \hspace{1cm} (8.3)
where \( E(\hat{S}_{t+1}) \) is the expected change in the exchange rate during the period extending between \( t \) and \( t + 1 \), such that the expectation is made at time \( t \) (the present). In general, we define extrapolative expectations to imply positive dependence of period-to-period changes in the exchange rate, such that a rise in the exchange rate is expected to be followed by another rise and vice versa. Equation (8.1) says that the expected change in the exchange rate is a positive fraction of the current change. Equation (8.2) tells us that the expected change in the exchange rate is a weighted average of the current change and the previous period’s change, such that the higher the value of the parameter \( \theta \) the greater the weight assigned to the previous period’s change. Equation (8.3) simply postulates that the expected change in the exchange rate is an \( n \)-period moving average of previous actual changes. Finally, equation (8.4) says that the expected exchange rate is a geometrically declining distributed lag of previous actual changes. The difference between (8.3) and (8.4) is that (8.3) assigns the same weight to all of the actual changes, whereas (8.4) assigns greater weights to the most recent changes.

Pilbeam (1995a) suggests a simple representation of the extrapolative expectations hypothesis, based on the notion that a rise in the exchange rate is followed by another rise and vice versa. Formally, extrapolative expectations are represented by

\[
E(\hat{S}_{t+1}) = \begin{cases} \geq 0 & \text{if } \hat{S}_t > 0 \\ < 0 & \text{if } \hat{S}_t < 0 \end{cases}
\]  

(8.5)

When traders base their decisions on extrapolative expectations, a buy signal is given by \( \hat{S}_t > 0 \), whereas a sell signal is given by \( \hat{S}_t < 0 \). This representation only indicates the direction, not the magnitude, of the expected change.

**Regressive expectations**

There are two versions of the regressive expectation formation mechanism, which are given by

\[
E(\hat{S}_{t+1}) = \alpha \hat{S}_t, \quad \alpha < 0
\]  

(8.6)

\[
E(\hat{S}_{t+1}) = -\lambda \left( \frac{S_t}{\bar{S}_t} - 1 \right), \quad 0 < \lambda < 1
\]  

(8.7)

Equation (8.6) tells us that the expected change in the exchange rate is a negative fraction of the current change, implying that a rise in the exchange rate is expected to be followed by a fall and vice versa. Equation (8.7) says that the exchange rate tends to converge on a long-run equilibrium value, \( \bar{S}_t \). If the current level of the exchange rate is above (below) the long-run equilibrium...
8.1 SPECULATION ON THE BASIS OF EXPECTATION FORMATION

value, then the exchange rate is expected to fall (rise) by a fraction of the gap between the two rates. This is the specification popularised by Dornbusch (1976) since he first introduced it as an element of the sticky-price monetary model of exchange rate determination. Notice that the right-hand side of equation (8.7) is the percentage deviation of the actual exchange rate from its long-run equilibrium value. Several methods can be used to estimate the long-run equilibrium value of the exchange rate, one of which is PPP.

Some economists (for example, Takagi, 1991) take equations (8.1) and (8.6) to represent extrapolative expectations such that equation (8.1) represents destabilising (or bandwagon) expectations, whereas equation (8.6) represents stabilising expectations. The two concepts correspond to the concepts of stabilising and destabilising speculation, which we came across in Chapter 7. Furthermore, if \( \delta \) or \( \alpha \) is equal to zero, then we have static expectations, whereas if \( \delta > 1 \), then we have explosive expectations (extremely strong destabilising expectations). We prefer to distinguish between extrapolative and regressive expectations on the basis of whether a rise in the exchange rate is followed by a rise or a fall. This is why equation (8.6) in our view represents one version of regressive expectations. It is interesting to note that Takagi (1991, p. 171) uses the term “extrapolative” for “the obvious reason that the expected currency movement for the next period is given by the past currency movement for the most recent period”, implying that only equations (8.1) and (8.6) represent extrapolative expectations. However, he then argues that extrapolative expectations imply that the expected change in the exchange rate is a weighted average of the changes in the current and last period as represented by equation (8.2). If this is the case, then there is no reason why extrapolative expectations cannot be represented by an \( n \)-period simple moving average or by an exponentially weighted moving average as in (8.3) and (8.4) respectively.

According to Pilbeam (1995a), regressive expectations are the opposite of extrapolative expectations, in the sense that the exchange rate is expected to rise if it falls in the current period, and vice versa. Formally, this representation can be written as

\[
E(\hat{S}_{t+1}) > 0 \quad \text{if} \quad \hat{S}_t < 0 \\
E(\hat{S}_{t+1}) < 0 \quad \text{if} \quad \hat{S}_t > 0
\]

which again shows the expected direction of the change only. If this is the underlying expectation formation mechanism, then a buy signal is given by \( \hat{S}_t < 0 \), whereas a sell signal is given by \( \hat{S}_t > 0 \).

Although regressive expectations are the opposite of extrapolative expectations, both mechanisms (as in (8.6) and (8.1) respectively) can be represented on the same diagram, as shown in Figure 8.1. This is the normal four-quadrant diagram, with two lines passing through the origin defining the conditions representing the two mechanisms. The line with the positive slope (\( \delta > 0 \)) represents extrapolative expectations. Any point on the extrapolative
expectations line implies that the expectation has been fulfilled (that is, accurate prediction). Otherwise, points falling off the line in the first and third quadrants indicate either overestimation or underestimation of the percentage change in the exchange rate. The same descriptions are valid for regressive expectations (points falling in the second and fourth quadrants).

Adaptive expectations
Adaptive expectations can be shown to be rather similar to one version of extrapolative expectations, which can be demonstrated very easily as follows. The adaptive expectations mechanism can be written as

$$E_t(\dot{S}_{t+1}) = \kappa [\dot{S}_t - E_{t-1}(S_t)] + (1-\kappa)E_{t-1}(\dot{S}_t)$$  \hspace{1cm} 0 < \kappa < 1 \hspace{1cm} (8.9)$$

where $E_{t-1}(\dot{S}_t)$ is the percentage change of the exchange rate at $t$ as expected at time $t-1$. Since the specification of adaptive expectations requires expectation formation at more than one point in time, the expectations operator appears in equation (8.9), and some equations that follow, with a time subscript. Equation (8.9) can be rewritten as

$$E_t(\dot{S}_{t+1}) = \kappa \dot{S}_t + (1-\kappa)E_{t-1}(\dot{S}_t)$$

$$\hspace{1cm} (8.10)$$
By applying the lag operator to equation (8.10) period by period, while at the same time multiplying by \((1 - \kappa)^j\), where \(j\) is the number of periods involved in the lag process, we obtain

\[
(1 - \kappa)E_{t-1}(\hat{S}_t) = \kappa(1 - \kappa)\hat{S}_t + (1 - \kappa)^2 E_{t-2}(\hat{S}_{t-1})
\]

(8.11)

\[
(1 - \kappa)^2 E_{t-2}(\hat{S}_{t-1}) = \kappa(1 - \kappa)^2 \hat{S}_{t-1} + (1 - \kappa)^3 E_{t-3}\hat{S}_{t-2}
\]

(8.12)

and so on. By substituting the resulting equations into (8.10) and combining the terms, we obtain

\[
E_t(\hat{S}_{t+1}) = \kappa[\hat{S}_{t+1} + (1 - \kappa)\hat{S}_t + (1 - \kappa)^2 \hat{S}_{t-1} + \ldots] = \kappa \sum_{j=0}^{\infty} (1 - \kappa)^j \hat{S}_{t-j+1}
\]

(8.13)

which resembles the GDL model represented by equation (8.4).

According to Pilbeam (1995a), adaptive expectations mean that if the exchange rate rises in at least two of the latest three periods then it should be expected to rise in the coming period. Formally, adaptive expectations have the following representation

\[
\begin{align*}
E(S_{t+1} > 0) & \quad \text{if} \quad \hat{S}_{t-i} > 0 \\
E(S_{t+1} < 0) & \quad \text{if} \quad \hat{S}_{t-i} < 0
\end{align*}
\]

(8.14)

for at least two values of \(i = 0, 1, 2\).

**Rational expectations**

The rational expectations hypothesis postulates that the expected value of the change in the exchange rate is equal to the actual change plus a random error term that may be positive or negative. The underlying idea is that the trader collects and processes all available information and that she does not make systematic errors, which is possible under the previous expectation formation mechanisms. This point is illustrated with the help of Figure 8.2, which is based on hypothetical data generated from equations (8.1), (8.2) and (8.10) under four behavioural patterns of the percentage change in the exchange rate (rising, falling, constant and fluctuating). As we can see, the expectation error is systematic in all four cases, in the sense that it is always positive. The expected value is consistently below what the actual value turns out to be, although this is not necessarily the case.

As a result, the trader converges on the “correct” underlying model in the long run, forming accurate expectations on average. Hence, the process is specified as

\[
E(\hat{S}_{t+1}) = \hat{S}_{t+1} + \xi_{t+1}
\]

(8.15)

where \(\xi_{t+1}\) is white noise, which is totally random.

It is arguable that the rational expectations hypothesis does not presuppose any particular expectation formation mechanism. The hypothesis stipulates
that regardless of how forecasts are generated, rationality of agents combined with the discipline of the market should eliminate persistent errors and force all participants to make efficient use of available information. Although Takagi (1991) considers rational expectations, he does not regard it as an expectation formation mechanism like extrapolative, adaptive and regressive expectations. Likewise, MacDonald (2000, p. 73) considers the rationality of survey expectations to be of interest in terms of explaining the poor predictive power of the forward rate. However, he refers to the kind of expectation process by asking whether financial market expectations are adaptive, extrapolative or regressive.

There are problems with the rational expectations hypothesis. For example, Ito (1990) argues that to the extent that individuals are not likely to possess private information, the presence of individual effects may reflect the failure of the hypothesis. Davidson (1982) argues against the rational expectations hypothesis by saying that it is a poor guide to real-world economic behaviour,
because it assumes that market participants passively forecast events rather than cause them.

There is also the problem of picking the “correct” exchange rate determination model, given that most fundamental models have been shown to be inadequate (Moosa, 2000a). Pilbeam (1995a), for example, uses the forward rate to proxy rational expectations, an option that we do not find that appealing. Takagi (1991) argues against using the forward rate to represent expectations because this procedure presupposes that there is no risk premium, which is a testable hypothesis. Another problem with the rational expectations hypothesis is that it rules out the existence of differences between the predictions of various participants, since the “true stochastic process” should be unique. However, MacDonald (2000) presents survey evidence on unbiasedness, orthogonality and expectation formation that reveals differences between forecasters. The absence of heterogeneity in expectation formation is
inconsistent with the market microstructure literature that stresses the presence of heterogeneity and its implications for volatility (see, for example, Lyons, 2001).

One way to use rational expectations to generate buy and sell signals is to follow Pilbeam (1995a), who assumes that all of the market information is reflected in the forward spread, such that if a currency is selling at a premium, then it should be expected to appreciate and vice versa. This in fact is a prediction of the version of the flexible price monetary model that incorporates covered interest parity (see, for example, Moosa (2000b, Chapter 4)). Hence, rational expectations may have the representation

\[
\begin{align*}
E(S_{t+1}^*) > 0 & \quad \text{if} \quad F_t - S_t > 0 \\
E(S_{t+1}^*) < 0 & \quad \text{if} \quad F_t - S_t < 0
\end{align*}
\]

(8.16)

Thus a buy signal is indicated when there is a forward premium and a sell signal is indicated when there is a forward discount.

The learning mechanism
Marcet and Nicolini (1998) suggested an expectation formation mechanism of the form

\[
E_t(\hat{S}_{t+1}) = E_{t-1}(\hat{S}_t) + \frac{1}{\alpha_t} [\hat{S}_{t-1} - E_{t-1}(\hat{S}_t)]
\]

(8.17)

which says that the expected change in the exchange rate is updated by a term that depends on the last error weighted by \(1/\alpha_t\). The behaviour of \(\alpha_t\) is represented by

\[
\alpha_t = \alpha_{t-1} + 1
\]

(8.18)

where \(\alpha_0\) is set exogenously to be equal to zero. In this case \(\alpha_t = t\), and it follows that

\[
E(S_{t+1}) = \frac{1}{t} \sum_{i=1}^{t} \hat{S}_i
\]

(8.19)

which means that the expected change in the exchange rate is equal to the sample mean of past changes. Equivalently, it is the result of regressing \(\hat{S}_t\) on a constant.

Mixed and contrarian expectations
Mixed and contrarian expectations are based on the postulation that market participants pay attention to the “convention that is formed by the others” (Hodgson, 1985, p. 13). The mixed (or heterogenous) expectation formation mechanism is based on the assumption that some market participants use different expectation formation mechanisms at different points in time. The simplest way to model this expectation formation mechanism is to take the
expected change in the exchange rate to be the average of the expected changes obtained from other mechanisms, or by following the majority signal. Contrarian expectations are formed by market participants who hold a diametrically opposite view to the market as a whole. They can be proxied by the negative of what is obtained under mixed expectations.

**Evidence on expectation formation in the foreign exchange market**

Studies on expectation formation in the foreign exchange market follow two approaches. The first approach is based on survey data, whereas the second approach is based on estimating demand for money functions that incorporate various expectation formation mechanisms.

Broadly speaking, two sets of results are reported in studies based on survey data: one set deals with testing for rational expectations, whereas the other deals with the expectation formation mechanisms. Testing for rational expectations boils down to testing for (expectation) unbiasedness (that the expected exchange rate is an unbiased predictor of the spot rate that will prevail in the future) and orthogonality (that expectations incorporate all available information). Takagi (1991) surveyed the studies conducted by Dominguez (1986), Frankel and Froot (1987a,b), Bank of Japan (1989), Wakita (1989), Froot and Frankel (1990) and Ito (1990). He concludes, on the basis of his survey, that the evidence overwhelmingly rejects rational expectations. For example, Dominguez (1986) almost unanimously rejected rational expectations, concluding that forecasters missed the direction of exchange rate movements, and that forecasters overpredicted the size of future dollar depreciation. Takagi (1991) obtained what he calls the crucial result that whereas short-run expectations tend to move away from some long-run “normal value”, long-run expectations tend to move back towards it. He calls this reversal in the direction of expectation a “twist”. The evidence in general supports extrapolative and regressive expectations, but not static expectations.

MacDonald (2000) surveyed some of the same studies covered by Takagi (1991) as well as more recent studies. These studies include Dominguez (1986), MacDonald and Torrance (1988), Frankel and Froot (1987a, 1989), Cavaglia et al. (1993), Chinn and Frankel (1994), Prat and Uctum (1996) and Kim (1997). His results strongly rejected both unbiasedness and orthogonality. He also found evidence for the “twist” in expectation formation, concluding that forecasting at horizons longer than three months exhibit clear evidence for stabilising expectation.

Most of the studies following the second approach (of estimating demand for money functions incorporating expectation formation mechanisms) impose rather than test for the mechanisms. One exception is the study of Moosa (1999) in which he tests for expectation formation under the German hyperinflation of the 1920s using static, extrapolative, adaptive, regressive and rational expectations. The results of this analysis reject rational expectations, lending most support to extrapolative and adaptive expectations. It is
also revealed that expectation is destabilising under conditions of hyperinflation.

Moosa and Shamsuddin (2002) used historical data to conduct a study in which they related the dominance, or otherwise, of an expectation formation mechanism to the profitability of a trading rule based on it. By calculating the accumulated profit generated over a long period of time by trading on the basis of buy and sell signals generated by various mechanisms, they reached the following conclusions. First, the dominant mechanism is extrapolative expectations, indicating that most market participants believe that a rise in the exchange rate will be followed by a rise and vice versa. Second, the superiority of mixed expectations over contrarian expectations, indicating that market participants follow some sort of a herd behaviour and that they pay attention to the expectations of other participants.

8.2 SPECULATION ON THE BASIS OF TECHNICAL ANALYSIS

Definitions and principles
Technical analysis can be defined as the study of exchange rates (and other financial prices) based on supply and demand. Technical analysts (also called technicians or chartists) record, normally in chart form, historical exchange rates and try to deduce from the pictured history the probable future trend, generating buy and sell signals in the process. They may also use quantitative technical indicators. The basic idea is that exchange rates as observed in the foreign exchange market are determined by supply and demand, and this is all that is needed to be known.

Traditionally, economists have rejected the propositions put forward by technical analysts, since they believe that economic fundamentals are the only determinants of exchange rates. Given the notion of market efficiency, old information (particularly publicly available information) must be useless because it is already incorporated in the current exchange rate. Malkiel (1996) puts forward the cynical view that “technical strategies are usually amusing, often comforting, but of no real value”. A question that seeks an answer is that if this is the case, why is it that technical analysts are still in demand by financial institutions? Malkiel has a ready answer: they are hired by brokers largely in order to generate trades and hence commissions. Allen and Taylor (1993) put forward the counter argument that even if this is the case, “the results of this policy manifest themselves as self-fulfilling chartist strategies”. The fact remains, however, that the failure of fundamental models has forced economists to take technical analysis more seriously.

Lo et al. (2000) evaluated the status quo of technical analysis in terms of the separation between technical analysts and their academic critics, which they describe as “one of the greatest gulfs between academic finance and industry
practice”. To emphasise this point, they cite anti-technical analysis slogans, such as the following:

1. The difference between technical analysis and fundamental analysis is not unlike the difference between astrology and astronomy.
2. Technical analysis is “voodoo finance”.
3. Under scientific scrutiny, chart-reading must share a pedestal with alchemy (Malkiel, 1996).

Indeed, some economists have gone as far as saying that one reason for the popularity of technical analysis is that it uses rather “erotic terms”, such as “penetration points”. But, Lo et al. (2000) defend technical analysis, arguing that it may be an effective means for extracting useful information from market prices. They base this argument on the findings of a large number of studies revealing that financial markets are not fully efficient and rejecting the random walk hypothesis. These findings provide some support for using technical analysis. Furthermore, they argue that technical analysis and fundamental analysis share the premise that past prices contain information for predicting future prices, except that they put that differently. Campbell et al. (1997, pp. 43–44) provide two statements to demonstrate the difference between technical analysis and academic finance. While technical analysts talk about “support and resistance levels”, “retrenchment parameter”, and so on, finance academics talk about “the first twelve autocorrelations” and the “statistical significance of the Box–Pierce Q-statistic”.

Technical analysis is based on the postulation that exchange rates do not move randomly but rather in repeated and identifiable patterns. The exchange rate series that is recorded over time reflects all available information on which supply and demand decisions are based. This information pertains to economic fundamentals as well as to other non-quantifiable factors such as expectations, sentiment and psychology. It is these factors that lead to the emergence of the patterns observed by technical analysts. This postulation does not go without a challenge, however. For example, Cootner (1964) and Malkiel (1996) argue that the apparent cycles in stock prices (and financial prices in general) are no more regular than the run of chance, and that they can be replicated by the toss of a coin.

Technical analysis is based on the following principles:

1. All of the factors affecting exchange rates are discounted: they are reflected in the actual behaviour of exchange rates as shown on a chart.
2. The movements of exchange rates simply reflect changes in the forces of supply and demand.
3. Exchange rates move in trends that persist. The supply and demand balance sets a trend in motion, and this trend remains intact until it ends.
4. Market behaviour is repetitive. This is so because human nature means that people react to similar situations in a consistent manner. Since the foreign
exchange market is a reflection of the actions of people (the traders), technical analysts study these actions to determine how people react under certain conditions and thus how exchange rates move.

The empirical evidence on technical analysis
Studies on the extent of the use of technical analysis have produced results showing extensive use of this technique, particularly for short-term trading. Allen and Taylor (1989, 1990) and Taylor and Allen (1992) present evidence on the use of technical analysis based on a survey of some 240 foreign exchange dealers in London. The survey revealed a broad consensus view on the weights given to technical analysis at different time horizons. At short horizons (ranging from intra-day to weekly) 90% of the participants reported using technical analysis. Some 60% of the participants said that they regarded technical analysis at least as important as fundamental analysis. The results also revealed that the weight given to technical analysis is greater at short horizons. There was a very small minority (2%) who claimed never to have used fundamental analysis. The overall conclusion that can be derived from the survey is that technical analysis and fundamental analysis are complimentary.

Menkhoff (1997) surveyed the practices of foreign exchange traders in Germany, reaching the conclusion that technical analysis is used extensively. In contrast with Taylor and Allen, Menkhoff found that technical analysis was not primarily associated with short-term trading. Whether forecasting horizons were limited to an intraday perspective or to 2–6 month view, respondents gave technical analysis a similar weight between 33.9% and 40.4%. Menkhoff argued that the findings of Taylor and Allen that reliance on technical analysis decreases at longer time horizons is due to the exclusion of information on flows (what traders are doing and what orders exist). However, Menkhoff also agrees with the finding that the use of fundamentals is associated with long-term trading.

Lui and Mole (1998) conducted a similar survey involving 153 foreign exchange dealers in the Hong Kong market. This survey revealed that a very high proportion of the respondents placed some weight on both technical and fundamental analysis at all time horizons. At shorter horizons, however, there exists a skew towards reliance on technical analysis. Moreover, a view was revealed that technical analysis is only slightly more useful than fundamental analysis in predicting trends, but significantly more useful in predicting turning points.

Oberlechner (2001) presented findings of a survey of the perceived importance of technical versus fundamental analysis among foreign exchange traders and financial journalists in European financial centres. The results confirm the proposition that most traders use both forecasting approaches and that the shorter the forecasting horizon the more important technical analysis becomes. Results also indicate that the importance of technical
analysis has increased since the 1990s. Financial journalists seem to put more emphasis on fundamental analysis than do foreign exchange traders.

Cheung and Chinn (1999) report the findings of a survey of US foreign exchange traders. The results show that technical trading best characterises about 30% of traders, with the proportion rising steadily, and that the importance of fundamental analysis increases at longer horizons.

As to the effectiveness and usefulness of technical analysis, Lo et al. (2000) found that several technical indicators provide incremental information, which means that they should have value. They reached this conclusion by comparing the unconditional empirical distribution of daily stock returns with the conditional distributions conditioned on specific technical indicators, such as head and shoulders and double bottoms. This is probably why economists have started to assign some role to the activity of technical analysts in the process of exchange rate determination. This is legitimate, particularly over short periods and in the absence of major changes in the fundamentals. Under these conditions, acting on technical analysis becomes a self-fulfilling prophecy. When a trader is convinced, by a certain chart pattern, that the exchange rate is going to rise after a long decline (that is a trend reversal upwards is expected) he will buy the currency, with the resulting increase in demand causing the exchange rate to rise as indicated by the chart. The opposite is also true.

The behaviour of traders who act on the basis of fundamental analysis and those who act on the basis of technical analysis may differ drastically. Fundamental analysts watch deviations from an equilibrium level of the exchange rate as implied by a fundamental model (for example, the PPP model and the monetary model). If the exchange rate is above the equilibrium level the currency is sold, which should lead to currency depreciation. This is not necessarily the case, however. If technical analysts believe that there is no indication of a trend reversal they will keep on buying it, lending support to the currency. What happens to the exchange rate depends on the net effect of the forces of supply and demand by fundamental analysts and technical analysts. The same argument is valid if the exchange rate falls below the equilibrium value.

Several economists have formalised the idea put forward in the preceding paragraph. Goodhart (1988) suggests that exchange rate misalignment is determined by the balance of the predictions of technical analysts and fundamental analysts. Likewise, Frankel and Froot (1990b) specify a model based on the same idea to explain the sharp rise in the demand for the dollar in the first half of the 1980s. The explanation provided on the basis of this model is that the increase in the demand for the dollar is the overwhelming role of technical analysts during that period. Kirman (1991) presents an extension of the Frankel–Froot model.

Moosa and Korczak (2000) carried out similar work. A model relating changes in the exchange rate to the activities of fundamentalists and technicians is
specified and estimated using some exchange rates involving major currencies. The results show that both types of traders play a role in exchange rate determination and that fundamentalists play a bigger role. The results of model selection tests reveal that models based on the activities of fundamentalists only or technicians only are misspecified. Moosa and Al-Loughani (2002) applied a modified version of the model (that takes into account the effect of the exchange rate arrangement) to the exchange rate of a currency that is pegged to a basket (the Kuwaiti dinar) against the Japanese yen. The results show that market forces, as represented by the activities of traders, play a role in the determination of the exchange rate although this role is secondary to the effect of the exchange rate arrangement as represented by changes in the exchange rate of the Kuwaiti dinar against the US dollar. Non-nested model selection tests reveal that models that are based on market forces only or the exchange rate arrangement only are misspecified. They also found some evidence indicating that the activity of technicians is more important for this process than the activity of fundamentalists.

**Generating buy and sell signals from chart formations**

While chartists observe a wide range of chart formations or patterns, we will concentrate on the so-called reversal patterns, which can be used to generate buy and sell signals because they (allegedly) identify directional changes in exchange rates. Currency traders using charts can use reversal patterns to sell a currency before considerable depreciation and cover short positions prior to considerable appreciation. Trends change direction because of the interaction of the forces of supply of and demand for the underlying currency. The forming of a top and the subsequent reversal occur as a result of supply overcoming demand, or excess supply. The opposite occurs when demand overcomes supply.

Figures 8.3 and 8.4 show respectively how buy and sell signals are generated by chart formations (reversal patterns) known as head and shoulders, triangles, rectangles, double tops and bottoms, triple tops and bottoms and wedges (for a detailed treatment, see Moosa (2000a, Chapter 7)).

**Generating buy and sell signals from quantitative technical indicators**

Quantitative technical indicators have numerical values that can be calculated from certain equations. Two quantitative technical indicators are described in turn: oscillators and the relative strength index.

Oscillators are typically constructed with lower and upper boundaries, such as –1 to +1 or 0 to 100. A momentum oscillator is designed to measure the speed or the rate of change in the exchange rate. Hence, a momentum oscillator measures the acceleration or deceleration in the exchange rate. The value of a $k$-period momentum oscillator at a particular point in time is calculated as

$$O_t^k = S_t - S_{t-k}$$  (8.20)
where \( k \) may also be called the order of the momentum oscillator. When the momentum oscillator goes up in the current time period it means that the exchange rate rose by more or declined by less than what happened \( k \) periods ago. Similar interpretations are assigned to flat and downward movements in momentum oscillators. One of the benefits of momentum oscillators is that they lead exchange rate movements at market turning points. A momentum oscillator flattens out while the current exchange rate trend is still in effect.

On the other hand, a rate of change oscillator of order \( k \) is calculated as
The relative strength index (RSI) is calculated as follows

\[ (RSI)_t = 100 - \frac{100}{1 + (RS)_t} \]  

(8.22)
(8.23) \[
(RS)_t = \frac{\frac{1}{n} \sum_{t=1}^{n} \Delta S_t^+}{\frac{1}{n} \sum_{t=1}^{n} |\Delta S_t^-|}
\]

where \( n \) is the number of periods over which the RSI is calculated, \( \Delta S_t^+ \) is a positive change and \( |\Delta S_t^-| \) is the absolute value of a negative change. Notice that, for the purpose of this calculation, when \( \Delta S_t \geq 0 \), then \( |\Delta S_t^-| = 0 \), and when \( \Delta S_t \leq 0 \), then \( \Delta S_t^+ = 0 \).

Oscillators and the RSI can be used to generate buy and sell signals as follows (Figure 8.5). A buy signal is generated when the oscillator cuts the midway line from below and when the RSI goes below 30. Sell signals are

FIGURE 8.5 Generating buy/sell signals from oscillators and the RSI.
generated when the oscillator cuts the midway line from above and when the 
RSI line goes above 70 (for more details, see Moosa (2000a, Chapter 7)).

8.3 SPECULATION ON THE BASIS OF TRADING RULES

In technical trading rules, a buy signal is indicated if it is established that an 
upward trend will continue and a sell signal is indicated if a downward trend 
is perceived to continue for some time in the future. We will consider filter 
rules and moving average rules.

Filter rules
Filter rules were initially developed by Alexander (1961) who applied them to 
the stock market. A similar attempt was made by Fama and Blume (1966). 
Subsequently, filter rules were applied to other financial markets, including 
the foreign exchange market. Later attempts have been made by several econ­
omists including Dooley and Shafer (1976, 1983) and Sweeney (1986, 1988).

The working of the filter rule depends on the recognition of peaks and 
troughs (ex post) in the exchange rate series. Assume that the exchange rate is 
observed at points in time \( t - k, t - k + 1, ..., t - 1, t, t + 1 \). The value of the 
exchange rate at time \( t, S_t \) defines a trough if

\[ S_{t+1} - S_t > 0 \]  
and

\[ S_{t-i} - S_{t-i-1} < 0 \quad \text{for } i = 0, 1, ..., k - 1 \]  

Equations (8.24) and (8.25) imply that for \( S_t \) to define a trough, the necessary 
condition is that \( \Delta S_{t+1} > 0 \), whereas the sufficient condition is that \( \Delta S_{t-i} < 0 \) for 
\( i = 0, 1, ..., k - 1 \). On the other hand, \( S_t \) defines a peak if

\[ S_{t+1} - S_t < 0 \]  
and

\[ S_{t-i} - S_{t-i-1} > 0 \quad \text{for } i = 0, 1, ..., k - 1 \]  

Equations (8.26) and (8.27) imply that for \( S_t \) to define a peak, the necessary 
condition is that \( \Delta S_{t+1} < 0 \), whereas the sufficient condition is that \( \Delta S_{t-i} > 0 \) for 
\( i = 0, 1, ..., k - 1 \).

A \( g \) per cent filter rule can be used to generate buy and sell signals as follows. 
If there is a trough at \( S_t \), then a buy signal emerges at \( S_{t+i} \) if

\[ S_{t+i} \geq (1 + g)S_t \]  
On the other hand, if there is a peak at \( S_t \), then a sell signal emerges at \( S_{t+i} \) if

\[ S_{t+i} \leq (1 - g)S_t \]
The representation of the $g-h$ filter rule (such that $g > h$ or $g < h$) is similar. A buy signal emerges when the exchange rate rises by $g$ per cent from the trough as represented by equation (8.28). A sell signal, on the other hand, emerges when the exchange rate falls by $h$ per cent from its most recent peak. This is represented by equation (8.29) with $h$ replacing $g$. The buy and sell signals generated from a $g$ per cent rule and a $g-h$ per cent rule are shown in Figure 8.6. If the peak occurs at $S_t(1+\theta)$, then the profit realised from the $g$ and $g-h$ filter rules is

$$\pi(g) = S_t[(1+\theta)(1-g)-(1+g)]$$

$$\pi(g-h) = S_t[(1+\theta)(1-h)-(1+g)]$$

which means that if $h > g$, as in Figure 8.6, then $\pi(g) > \pi(g-h)$.

If we allow for the bid–offer spread in exchange rates, then currency $y$ must be bought at the offer rate and sold at the bid rate. Hence profit will be

$$\pi(g) = S_{b,t}(1+\theta)(1-g) - S_{a,t}(1+g)$$

$$\pi(g-h) = S_{b,t}(1+\theta)(1-h) - S_{a,t}(1+g)$$

We can modify these rules by allowing for interest rates, and hence the possibility of borrowing $x$ to finance the purchase of currency $y$. Assume that a buy signal arises at $t+j$ and that there are $j$ time periods between buying and selling in the $g$ filter rule and $k$ time periods in the $g-h$ filter rule. For each unit of $x$ borrowed, profit realised from the trading rule is

$$\pi(g) = \frac{(1+\theta)(1-g)(1+i_y)^j}{(1+g)} - (1+i_x)^j$$

![Figure 8.6 Generating buy/sell signals from filter rules.](image-url)
\[ \pi(g-h) = \frac{(1+\theta)(1-h)(1+i_y)^k}{(1+g)} -(1+i_x)^k \]  

(8.35)

And, of course, the matter becomes more complicated if we allow for bid–offer spreads in both interest and exchange rates. In this case

\[ \pi(g) = \frac{S_{b,t}((1+\theta)(1-g)(1+i_{y,b})^j}{S_{a,t}(1+g)} -(1+i_{x,a})^j \]  

(8.36)

\[ \pi(g-h) = \frac{S_{b,t}((1+\theta)(1-h)(1+i_{y,b})^k}{S_{a,t}(1+g)} -(1+i_{x,a})^k \]  

(8.37)

**Moving average rules**

The moving average rule is a mechanical trading rule that is based on buy–sell signals derived from the behaviour of the exchange rate relative to one or more moving averages. Suppose that we observe the exchange rate, \( S_t \), over discrete points in time \( t = 0, 1, 2, 3, ..., n \). A moving average of order (length) \( q \) at time \( t \) can be measured as

\[ M_t(q) = \frac{1}{q} \sum_{i=0}^{q-1} S_{t-i} \]  

(8.38)

According to the single moving average rule, buy and sell signals are indicated by the intersection of the time paths of the exchange rate and a moving average of some order, as shown in Figure 8.7. A buy signal is generated when the moving average cuts the exchange rate from above, that is when

\[ S_t = M_t \quad \text{or} \quad S_t - M_t = 0 \]  

(8.39)

and

\[ S_{t-1} \leq M_{t-1} \quad \text{or} \quad S_{t-1} - M_{t-1} \leq 0 \]  

(8.40)

whereas a sell signal is generated when the moving average cuts the exchange rate from below, that is when

\[ S_t = M_t \quad \text{or} \quad S_t - M_t = 0 \]  

(8.41)

and

\[ S_{t-1} \geq M_{t-1} \quad \text{or} \quad S_{t-1} - M_{t-1} \geq 0 \]  

(8.42)
The double moving average rule works in exactly the same way except that the buy and sell signals are indicated by the intersection of two moving averages: a short moving average and a long moving average. The buy and sell signals are generated in the same way as in the single moving average rule when the exchange rate is replaced with the short moving average and the short moving average is replaced with the long moving average.

**Empirical evidence on the profitability of trading rules**

A large number of studies have been conducted to find out whether or not mechanical trading rules are profitable. Studies applying filter rules to the foreign exchange market using daily data produced some evidence for the profitability of these rules. These studies include Cornell and Dietrich (1978), Dooley and Shafer (1983) and Logue et al. (1978). Sweeney (1986) used a risk-adjusted test to present evidence for the profitability of filter rules. Some evidence has also been found for the profitability of moving average rules in the foreign exchange market. Surajaras and Sweeney (1992) found that single and double moving average rules produced significant out-of-sample profit that was on average larger than what is produced by following a filter rule. Moreover, Bilson (1981) and Sweeney and Lee (1990) found profit from trading rules in the forward foreign exchange market. Further favourable but indirect evidence is provided by Engle and Hamilton (1990) who demonstrated that the dollar from the early 1970s to the late 1980s was susceptible to “long swings” or largely uninterrupted trends, which are susceptible to mechanical trading rules. Martin (2001) puts forward the proposition that central bank intervention makes trading rules more profitable in the foreign
exchange markets of developing countries. By using a moving average rule, she showed that statistically significant profits were generated in most of the markets that were examined.

An important study was conducted by Levich and Thomas (1993b), who argue that a drawback to most studies is that they do not measure the statistical significance of their results (the generated profit), and if they do the measure is based on the assumption that exchange rate volatility is constant and that exchange rates are drawn from stationary distributions. Therefore they suggest a new method for testing the profitability of trading rules and the randomness of the exchange rate series. This test does not depend on the assumptions of the distribution of exchange rate changes as in other studies, but rather apply bootstrapping methods to examine the profits generated by using a filter rule. By using filters of 0.5, 1, 2, 3, 4 and 5%, they found substantial risk-adjusted profit. On the other hand, Cheung and Wong (1997) provided evidence that filter rules may not generate significant profits. This piece of work is based on an explicit modelling of the risk premium, which is expressed as a function of investors’ preference and the level of risk they are willing to accept.

Other studies concentrated on the use of moving average rules. Schulmeister (1988) examined the profitability of selected technical trading rules based on moving averages, including (i) a momentum model, (ii) a double moving average model, and (iii) a combination of the two. He reported that most of the technical trading rules generated profits even after adjusting for interest expenses and transaction costs. Kho (1996) suggested that moving average rules may not generate significant levels of profit when time-varying expected risk and return are taken into account. Neely et al. (1997) used a complex, computer-intensive genetic algorithm procedure to select optimal rules, some of which are based on moving averages. They found strong evidence for out-of-sample excess returns adjusted for transaction costs.

Although evidence has been found supporting the proposition that technical trading rules can yield significant risk-adjusted profits across a wide range of currencies, most studies also concluded that there is no stable linear relationship between successive changes in exchange rates. In other words, the estimated serial correlation coefficients are small and often insignificant. For example, Dooley and Shafer (1983) found evidence for positive serial correlation, but its pattern was generally unstable over time. Levich and Thomas (1993b) found small and insignificant serial correlation coefficients after adjusting for heteroscedasticity. Takagi (1988) found that some serial dependence in successive exchange rate movements is almost always present (particularly for monthly data), but the estimated serial correlation coefficients were small. Rosenberg (1996, pp. 349–350) suggests that the reason for this finding is that serial correlation tests tend to yield biased results when (changes in) exchange rates are not normally distributed, which is invariably the case. Similarly, Hsieh (1989) notes that traditional serial correlation tests
are flawed because they seek to determine only if a stable linear relationship exists. He argues that although successive exchange rate changes may be found to be linearly independent, they may still exhibit nonlinear dependence. Surajaras and Sweeney (1992, p. 27) reinforce this point by arguing that “unlike serial correlation tests, or any other direct statistical tests in the literature, technical models [rules] include the possibilities of both linear and nonlinear relationships”.

### 8.4 SPECULATION ON THE BASIS OF FUNDAMENTALS

Fundamentals are defined here to be the variables that determine exchange rates according to fundamental models. The following are examples of fundamental models of exchange rates: the flow model, the flexible price monetary model, the sticky price monetary model, the real interest differential model and the Hooper–Morton model. The models are represented respectively by the following functional relationships:

\[
\begin{align*}
    s_t &= f_1(y_t - y_t^*, \Delta p_t - \Delta p_t^*, i_t - i_t^*) \\
    s_t &= f_2(m_t - m_t^*, y_t - y_t^*, i_t - i_t^*) \\
    s_t &= f_3(m_t - m_t^*, y_t - y_t^*, i_t - i_t^*) \\
    s_t &= f_4(m_t - m_t^*, y_t - y_t^*, E(\Delta p_t) - E(\Delta p_t^*), r_t - r_t^*) \\
    s_t &= f_5(m_t - m_t^*, y_t - y_t^*, E(\Delta p_t) - E(\Delta p_t^*), i_t - i_t^*, c_t - c_t^*)
\end{align*}
\]

where \(y\) is the logarithm of real output, \(\Delta p_t\) is the inflation rate, \(i\) is the interest rate, \(m\) is the logarithm of the money supply, \(r\) is the real interest rate, \(c\) is the current account, and \(E(\Delta p_t)\) is the expected inflation rate.

Two issues are discussed in this section: (i) the debate about whether or not fundamentals do matter, and (ii) how buy and sell signals are generated by referring to fundamentals.

**Do fundamentals matter?**

Economists have for some time been debating the relevance of macroeconomic fundamentals to exchange rate determination. Research in international finance has revealed that fundamental models (like those represented by equations (8.43)–(8.47)) have failed in two respects: out-of-sample forecasting (for example, Meese and Rogoff, 1983) and the ability to explain exchange rate volatility (for example, Flood and Rose, 1999).

The failure to overturn the Meese–Rogoff (1983) results, showing that fundamental models cannot outperform the random walk model in out-of-sample forecasting, has led Rogoff (1999, p. F655) to conclude that there is no “systematic relationship between exchange rates and macroeconomic fundamentals”. Similarly, Frankel and Rose (1995) conclude that “the Meese and
Rogoff analysis at short horizons has never been convincingly overturned or explained”. As a result of this conclusion they advocate a move away from fundamental-based models. Flood and Rose (1999, p. F662) reach the same conclusion by stating that “macroeconomics is an inessential piece of the exchange rate volatility puzzle” (p. F662). In general, these conclusions are based on the empirical observation that there is a disparity between the actual behaviour of exchange rates and what is implied by fundamental models. Hence, the argument goes, these models do not have any explanatory or predictive power, and this view is taken as far as saying that fundamentals do not matter.

Not only mainstream economists make remarks like these, as post-Keynesian economists also cast doubt on the importance of fundamentals. For example, Harvey (1991, p. 63) talks about the “apparent separation of the short-term movements (and possibly the medium-term movements) [of exchange rates] from the influence of fundamentals”. Schulmeister (1988, p. 346) concludes that “foreign exchange dealing has largely emancipated itself from the direct forces implied by market fundamentals”. And Davidson (1994, p. 237) even questions the very existence of a long-run equilibrium exchange rate. Sometimes, post-Keynesian economists go as far as saying that it is better to avoid using the term “fundamentals” completely because they are not well defined (for example, Harvey, 2001).

Some economists (for example, MacDonald, 1999) disagree with the view that fundamentals do not matter. MacDonald’s argument is based on the ability to overturn the Meese–Rogoff results by demonstrating that fundamental models can outperform the random walk model in out-of-sample forecasting, as in MacDonald and Marsh (1997) and Wolff (1987). However, these results cannot do anything to change the (justified) perception of the appalling empirical performance of these models as documented by Lane (1991) and Kwiecein (2000). While there is no doubt that the fundamental models of exchange rate determination are inadequate, this inadequacy cannot be interpreted to imply the irrelevance of fundamentals. Moosa (2002a) argues that this interpretation is far-fetched, because it implies that the foreign exchange market is governed by the iron law embedded in the underlying fundamental model, and that this iron law is observed and obeyed by all market participants.

These arguments overlook the fact that the foreign exchange market is not a mechanical system that moves according to a predetermined, hitherto-undiscovered formula, given the empirical failure of existing exchange rate models. This flawed line of reasoning results from the wrong perception, which Harvey (1993, p. 679) describes by saying that “markets are perceived as quasi-physical phenomena composed of a system of deterministic laws leading to predictable outcomes”. The problem with this line of reasoning is that when these predictable outcomes do not materialise, then the conclusion that jumps to the forefront is that fundamentals do not drive the foreign exchange
market. Rather, it is some other force that is yet to be discovered: this is a very convenient excuse if one rejects a proposition and cannot provide an alternative one.

A question that is frequently asked is the following (for example, Dixon, 1999): “Are exchange rates ultimately tied down by economic fundamentals, or are they free to drift at random on a sea of speculation?”. Asking this question is a reflection of the belief that speculation in the foreign exchange market cannot be based on fundamentals. This proposition is at odds with the available survey and econometric evidence, which indicates that foreign exchange traders base their speculative decisions (related to exchange rate forecasting) on fundamentals, technical analysis or a combination of both (see, for example, Allen and Taylor, 1989, 1990; Taylor and Allen, 1992; Lui and Mole, 1998; Frankel and Froot, 1990b; Moosa and Korczak, 2000). Moosa (2000a, Chapter 2) presents a comprehensive description of decision-making situations involving speculation in the foreign exchange market. The relevant decision rules involve the expected exchange rate as a decision variable. Forecasting the unknown decision variable may be based on fundamental models, technical models or even market-based models. Hence speculation does not necessarily preclude the use, and hence the relevance, of fundamentals. Moreover, the most successful currency speculator, George Soros, has made his billions by speculating on the basis of fundamental factors. This observation alone is a testimony in favour of the proposition that fundamentals are important. Moreover, “drifting at random” does not necessarily mean that fundamentals do not matter. Fundamentals are important as long as market participants act upon them. When they do, they change the forces of supply and demand and hence affect the exchange rate.

Generating buy and sell signals on the basis of fundamentals
Traders who base their buy–sell decisions on fundamentals can use rules or discretion. Fundamental trading rules are based on one or more fundamental models that describe the relationship between the exchange rate and its determining factors. The profitability or otherwise of these rules therefore depends on the validity of the underlying economic models. These models presumably define an equilibrium exchange rate, and the trading rule is based on deviations of the actual rate from the equilibrium rate. The trading rule generates a buy signal when the currency is undervalued (that is, when the exchange rate is below the level predicted by the model) and a sell signal when the currency is overvalued. The underlying idea is that if the currency is undervalued it is likely to appreciate, and if it is overvalued then it is likely to depreciate.

Formally, the equilibrium level of the exchange rate is determined by a vector of variables, X. Thus

$$\bar{S}_t = f(X_t)$$  \hfill (8.48)
where the vector of variables varies according to the underlying fundamental model. The current level of the exchange rate may deviate from the equilibrium level because of the effect of random shocks that tend to have a temporary effect. Hence the current level of the exchange rate may be represented by

$$S_t = \bar{S}_t + \varepsilon_t$$

(8.49)

where $\varepsilon_t$ is a random term. Thus, the currency is undervalued when $S_t < \bar{S}_t$ and overvalued when $S_t > \bar{S}_t$. If a quantitative dimension is added to the rule then a buy signal is generated when the exchange rate is lower than the equilibrium rate by a certain percentage, say $g$. Hence the buy signal is generated when

$$S_t = (1-g)\bar{S}_t$$

(8.50)

whereas the sell signal is generated when

$$S_t = (1+g)\bar{S}_t$$

(8.51)

Alternatively, the conditions for buy and sell signals can be written respectively as

$$S_t - (1-g)\bar{S}_t = 0$$

(8.52)

and

$$S_t - (1+g)\bar{S}_t = 0$$

(8.53)

Figure 8.8 shows how buy and sell signals are generated from a fundamental rule as represented by equations (8.50) and (8.51). Figure 8.9, on the

![Figure 8.8 Generating buy/sell signals from a fundamental rule.](image-url)
other hand, shows how the signals are generated from a fundamental rule as represented by equations (8.52) and (8.53).

Now, we turn to using fundamentals in a discretionary manner. Those using discretion react to changes in individual fundamental variables as announced. Thus, if the change in a fundamental variable is perceived to have a positive effect on the exchange rate, the underlying currency will be bought, and vice versa. Whether the effect of the change in a fundamental variable on the exchange rate is positive (hence a buy signal) or negative (hence a sell signal) depends on the fundamental model to which the trader subscribes. This is because economic theory can be used to show that a change in a certain fundamental variable can be good or bad for a particular currency. A rise in income may be interpreted to be a bullish signal, since growth means profitability and a thriving stock market, but it can be interpreted to be a bearish signal, since it leads to growth of imports and hence deterioration in the current account. Likewise, a rise in the interest rate may be taken to be a bullish signal, since it implies that domestic assets have become more attractive or as a bearish signal since a higher interest rate depresses the economy. Table 8.1 shows how two traders, A and B, may react differently to the same announcements.

Figure 8.10 shows how buy and sell signals are generated by discretion according to announcements pertaining to economic fundamentals. The announcements A1, A3, A5 and A7 are favourable, generating buy signals.
### TABLE 8.1 Two alternative scenarios for reacting to announcements.

<table>
<thead>
<tr>
<th>Announcement</th>
<th>Trader A’s reaction</th>
<th>Trader B’s reaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>The domestic money supply increased by 10% last month.</td>
<td>Sell domestic currency. Monetary expansion leads to inflation, which is bad for the domestic currency.</td>
<td>Buy domestic currency. Monetary expansion leads to inflation. Central bank reacts by raising the interest rate, which is good for the domestic currency.</td>
</tr>
<tr>
<td>The domestic economy is expected to grow by 5% in the coming year.</td>
<td>Sell domestic currency. Strong growth leads to higher level of imports and deterioration in the current account.</td>
<td>Buy domestic currency. Stronger growth leads to higher interest rates and booming financial markets, thus attracting capital flows.</td>
</tr>
<tr>
<td>The budget deficit as a percentage of GDP is expected to decline.</td>
<td>Buy domestic currency. A smaller deficit in the absence of a change in saving-investment balance leads to improvement in the current account.</td>
<td>Sell domestic currency. Lower borrowing requirements by the government eases pressure on interest rates.</td>
</tr>
<tr>
<td>Energy economists expect the price of oil to rise.</td>
<td>Buy domestic currency because it is a petrocurrency.</td>
<td>Sell domestic currency. Higher oil prices lead to an increase in the demand for US dollar.</td>
</tr>
</tbody>
</table>

Conversely, the announcements A2, A4, A6 and A8 are unfavourable, generating sell signals.

**Empirical evidence on the relative profitability of fundamental and technical rules**

Pilbeam (1995b) assesses the profitability of trading rules based on the monetary model of exchange rates in relation to other trading strategies. Specifically, he examines the profitability of trading rules used by (i) chartists who use past movements of exchange rates, (ii) fundamentalists, who base their decisions on the predictions of the monetary model and (iii) the simpletons, who use *ad hoc* trading rules. In general, the results suggest the failure of fundamental models to add any value to an investment strategy. On average he found that chartists generated greater levels of profit than fundamentalists. In his other paper
(Pilbeam, 1995a) he combined some fundamental models with expectation formation mechanisms.

Moosa and Shamsuddin (2002) examined the profitability of 19 trading rules based on expectation formation mechanisms, filter and moving average rules, and fundamental rules. The results suggested that trading strategies based on fundamental rules can be profitable. Out of the 19 different trading strategies, a rule based on the real interest differential model was the most profitable. This is followed by two single moving average rules, and then by trading rules based on the Hooper–Morton model and the Hodrick–Prescott filter. Three trading strategies produced losses, including those based on the flow model, adaptive expectations and regressive expectations. Glavas (2001) examined the profitability of eight different strategies including a filter rule, two moving average rules, two ARIMA models, the Hodrick–Prescott filter and two versions of the monetary model. The most profitable trading strategies turned out to be those based on the monetary model, with the best performance produced by using the filter rule.

8.5 HETEROGENEITY OF SPECULATORS AS A SOURCE OF EXCHANGE RATE VOLATILITY

In this section we discuss the hypothesis that exchange rate volatility may result from the heterogeneity of traders (speculators) with respect to the trading strategies they use to generate buy and sell signals. We will first deal
with heterogeneity in financial markets, and then we present a simple descriptive model of how trader heterogeneity cases exchange rate volatility.

**Heterogeneity in financial markets**
Moosa (2002a) presents a simple theoretical model that is based on the micro foundations of exchange rate determination to illustrate the relationship between heterogeneity of traders and volatility. The model is founded on the idea that observed exchange rate volatility can only result from erratic shifts in the market’s excess demand function that is made up of the excess demand functions of heterogenous traders. The heterogeneity of traders means that they have different sentiments and different expectations at any point in time. Hence they are likely to react differently to new developments: some want to buy (thus raising excess demand) and some want to sell (thus reducing excess demand). The net effect of their actions is to shift the aggregate excess demand function by a certain amount in a certain direction. In describing the model, Moosa assumes that there are four kinds of traders: technicians using filter rules, technicians using moving average rules, fundamentalists using rules, and fundamentalists using discretion.

Studies of the microstructure of the foreign exchange market seem to support this hypothesis. MacDonald (2000, p. 87) argues that it seems impossible to explain the huge daily trading volume in the foreign exchange market in terms of standard open economy models using rational expectations, since this volume must rely on a dispersion of beliefs about the future path of exchange rates. The rational expectations hypothesis rules out the existence of heterogeneity, since it assumes that the true stochastic process generating exchange rates is unique. The market microstructure literature takes as its starting point the proposition that agents are heterogenous and seeks to build models that capture the interrelationships between information flows, heterogeneity, trading volume and price volatility (for example, Lyons, 1991, 1993, 2001). In Melvin and Yin (2000), exchange rate volatility is implicitly attributed to trader heterogeneity resulting from trading on the basis of public information, private information, noise or a combination thereof.

The microstructural hypotheses have been tested by using survey data on exchange rate expectation. MacDonald and Marsh (1997) found very strong evidence for heterogeneity. They further examined the effect of heterogeneity, as measured by the standard deviation of the consensus expectation, on foreign exchange turnover and found the dispersion of expectations to be significantly positive, thus confirming the microstructural hypotheses. Chionis and MacDonald (1997) pushed the market microstructure test of MacDonald and Marsh further by testing for causality between volume, volatility and dispersion. They reported “strong evidence of heterogeneity causing both volume and volatility”.

Studies that are not directly concerned with the market microstructure also imply heterogeneity. Pilbeam (1995a,b) based his study of the profitability of
foreign exchange trading on the notion of trader heterogeneity. In Pilbeam (1995b) traders are supposed to follow three different exchange rate determination models (the flexible price monetary model, the sticky price monetary model and the sticky price portfolio model) in conjunction with six expectation formation mechanisms (static, extrapolative, adaptive, regressive, rational and heterogeneous). In Pilbeam (1995a) traders are classified into chartists, fundamentalists and simpletons. The same idea forms the basis of the post-Keynesian theory of exchange rate determination (see for example, Harvey, 1993).

Heterogeneity is supported by the evidence based on survey data on exchange rate expectation indicating that expectations have a distribution (Takagi, 1991). In addition to the distributional factor, heterogeneity in expectation may reflect systematic individual or group effects. Wakita (1989) and Ito (1990) found significant industry-specific bias in expectation. For example, they found that while exporters had expectations of greater yen depreciation (or smaller appreciation), importers expressed exactly the opposite expectation. The heterogeneity of market participants with respect to expectation formation mechanisms means that buy and sell signals arise in a more or less random manner, causing erratic changes in exchange rates. By using eight different expectation formation mechanisms, Moosa and Shamsuddin (2002) reach the conclusion that “the heterogeneity of market participants with respect to expectation formation goes a long way towards explaining exchange rate behaviour and volatility”.

Trader heterogeneity has also found support in studies of the extent of the use of technical analysis, which show that traders use technical analysis, fundamental analysis or both. Allen and Taylor (1989, 1990) and Taylor and Allen (1992) present evidence on the use of technical analysis based on a survey of some 240 foreign exchange dealers in London. The survey revealed that traders give different weights to technical analysis at different time horizons. Lui and Mole (1998) conducted a similar survey involving 153 foreign exchange dealers in the Hong Kong market. This survey revealed that a very high proportion of the respondents placed some weight on both technical and fundamental analysis at all time horizons. By using an econometric model and proxies for the activities of technicians and fundamentalists, Moosa and Korczak (2000) found some evidence indicating that both technicians and fundamentalists play a role in exchange rate determination.

There is a vast literature disputing the validity of the representative agent hypothesis, rejecting it in favour of heterogeneity on the grounds that the former is inconsistent with observed trading behaviour and the existence of speculative markets. Indeed, it is arguable that there is no incentive to trade if all market participants are identical with respect to information, endowments and trading strategies (Frechette and Weaver, 2001). Brock and Hommes (1997), Cartapanis (1996) and Dufey and Kazemi (1991) have demonstrated that persistence of heterogeneity can result in boom and bust behaviour under
incomplete information. Furthermore, Harrison and Kreps (1978), Varian (1985), De Long et al. (1990), Harris and Raviv (1993) and Wang (1998) have shown that heterogeneity can lead to market behaviour that is similar to what is observed empirically.

In response to concerns about the representative agent hypothesis, financial economists started to model the behaviour of traders in speculative markets in terms of heterogeneity. Chavas (1999) views market participants to fall in three categories in terms of how they form expectations: naïve, quasi-rational and rational. Weaver and Zhang (1999) allowed for a continuum of heterogeneity in expectations and explained the implications of the extent of heterogeneity for price level and volatility in speculative markets. Frechette and Weaver (2001) classify market participants by the direction of bias in their expectations (their bullish or bearish sentiment) rather than by how they form expectations. The message that comes out of this research is loud and clear: homogeneity is conducive to the emergence of one-sided markets, whereas heterogeneity is more consistent with behaviour in speculative markets characterised by active trading and volatility.

**A descriptive model of heterogeneity and volatility**

By going back to basics, the basics of supply and demand in the foreign exchange market, we can see very clearly that exchange rate volatility arises from shifts in the supply and demand functions or shifts in the (aggregate) excess demand function. Subsequently, it can be demonstrated that the heterogeneity of speculators with respect to what they use to generate buy and sell signals leads to random and staggered shifts in the excess demand function, causing the observed random-like behaviour and volatility of exchange rates.

Let us start by examining Figure 8.11, which shows the pattern of shifts in the excess demand function ($E$) and the corresponding movements of the exchange rate over time. As the excess demand function shifts from $E_0$ to $E_1$, the exchange rate rises. And as the function shifts downwards from $E_1$ to $E_2$, the exchange rate falls. The erratic behaviour of the exchange rate reflects the erratic shifts in the excess demand function. The question that arises here pertains to the reasons for the erratic shifts in the excess demand function.

Consider Figure 8.12, which shows the time paths of the actual exchange rate, $S_t$, the equilibrium exchange rate, $\bar{S}_t$ and a moving average of the actual exchange rate, $M_t$. Assume that there are four kinds of speculators, classified according to the method used to generate buy and sell signals: (i) fundamentalists using rules, (ii) fundamentalists using discretion, (iii) technicians using a filter rule and (iv) technicians using a moving average rule. Given the behaviour of the exchange rate, its equilibrium value and the moving average, buy and sell signals arise at different points in time, twenty of them altogether. Table 8.2 shows what happens at each point in time: whether a buy or sell signal is generated, by what and why. For example, at point in time 1, the actual exchange rate is below its equilibrium level, implying that the currency
FIGURE 8.11 Shifts in the excess demand function and the time path of the exchange rate.

is undervalued in which case a buy signal arises for the fundamentalists using rules. At point in time 2, a favourable announcement is made, triggering a buy signal for the fundamentalists using discretion (the opposite happens at point in time 3). At time 4, a buy signal arises for those using filter rules because the exchange rate is g per cent above the trough, and at time 5 another buy signal arises for those using a moving average rule because the moving average cuts the exchange rate from above. As a result of these buy and sell signals and the actions taken accordingly, the excess demand functions of these speculators shift upwards and downwards in an erratic manner, as shown in Figure 8.13 (numbers correspond to those in Figure 8.12, with 0 implying the initial position). In reality, of course, the situation is much more complex as there is
greater diversity. The aggregate excess demand function, therefore, shifts in an erratic manner, causing the observed exchange rate volatility.

Moosa and Shamsuddin (2002) concluded that exchange rate volatility can indeed be explained in terms of the heterogeneity of traders. This conclusion is based on empirical evidence showing that an artificial exchange rate series that is simulated on the assumption of trader heterogeneity exhibits a similar volatility pattern to that of the actual series. They represented heterogeneity by differences in the trading strategies used by various traders. In all, 19 different strategies were used, falling in one of three categories: those based on expectation formation mechanisms, those based on mechanical trading rules, and those based on fundamentals. On the basis of the profitability of each trading strategy a weight was assigned to different traders, and these weights were subsequently used to simulate an artificial exchange rate series. Statistical testing showed that the actual and simulated exchange rate series belonged to the same statistical distribution. The conclusion reached in this paper supports the microstructural approach to exchange rate determination and volatility. Furthermore, the results shed some light on the question of whether or not fundamentals do matter for exchange rate determination. Based on their results, Moosa and Shamsuddin argue that fundamentals do matter in the sense that some traders act upon them, leading to changes in the forces of supply and demand and therefore in exchange rates.
TABLE 8.2 Shifts in the excess demand functions according to buy and sell signals.

<table>
<thead>
<tr>
<th>Point</th>
<th>Signal</th>
<th>Tool</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Buy</td>
<td>Fundamentals (R)</td>
<td>Currency is undervalued</td>
</tr>
<tr>
<td>2</td>
<td>Buy</td>
<td>Fundamentals (D)</td>
<td>Favourable announcement</td>
</tr>
<tr>
<td>3</td>
<td>Sell</td>
<td>Fundamentals (D)</td>
<td>Unfavourable announcement</td>
</tr>
<tr>
<td>4</td>
<td>Buy</td>
<td>Filter rule</td>
<td>$S_t$ is $g$ per cent above trough</td>
</tr>
<tr>
<td>5</td>
<td>Buy</td>
<td>MA rule</td>
<td>$M_t$ cutting $S_t$ from above</td>
</tr>
<tr>
<td>6</td>
<td>Buy</td>
<td>Fundamentals (D)</td>
<td>Favourable announcement</td>
</tr>
<tr>
<td>7</td>
<td>Sell</td>
<td>Fundamentals (R)</td>
<td>Currency is overvalued</td>
</tr>
<tr>
<td>8</td>
<td>Sell</td>
<td>Filter rule</td>
<td>$S_t$ is $g$ per cent below peak</td>
</tr>
<tr>
<td>9</td>
<td>Sell</td>
<td>MA rule</td>
<td>$M_t$ cutting $S_t$ from below</td>
</tr>
<tr>
<td>10</td>
<td>Buy</td>
<td>Fundamentals (R)</td>
<td>Currency is undervalued</td>
</tr>
<tr>
<td>11</td>
<td>Buy</td>
<td>Filter rule</td>
<td>$S_t$ is $g$ per cent above trough</td>
</tr>
<tr>
<td>12</td>
<td>Buy</td>
<td>MA rule</td>
<td>$M_t$ cutting $S_t$ from above</td>
</tr>
<tr>
<td>13</td>
<td>Sell</td>
<td>Fundamentals (R)</td>
<td>Currency is overvalued</td>
</tr>
<tr>
<td>14</td>
<td>Sell</td>
<td>Filter rule</td>
<td>$S_t$ is $g$ per cent below peak</td>
</tr>
<tr>
<td>15</td>
<td>Buy</td>
<td>Fundamentals (D)</td>
<td>Favourable announcement</td>
</tr>
<tr>
<td>16</td>
<td>Sell</td>
<td>MA rule</td>
<td>$M_t$ cutting $S_t$ from below</td>
</tr>
<tr>
<td>17</td>
<td>Buy</td>
<td>Fundamentals (R)</td>
<td>Currency is undervalued</td>
</tr>
<tr>
<td>18</td>
<td>Buy</td>
<td>Filter rule</td>
<td>$S_t$ is $g$ per cent above trough</td>
</tr>
<tr>
<td>19</td>
<td>Sell</td>
<td>Fundamentals (R)</td>
<td>Currency is overvalued</td>
</tr>
<tr>
<td>20</td>
<td>Sell</td>
<td>Filter rule</td>
<td>$S_t$ is $g$ per cent below peak</td>
</tr>
</tbody>
</table>

Figure 8.14 illustrates the situation when there is lack of volatility caused by the lack of heterogeneity. This figure explains the situation when all traders are fundamentalists, basing their decisions on deviations from a long-run equilibrium value. If the actual exchange rate has the tendency of persisting below or above the equilibrium value (as in the case with deviations from PPP) then not many buy and sell signals will be generated. Figure 8.14 shows only one buy signal and no following sell signal because the actual exchange rate remains below its equilibrium level for prolonged periods of time. The lack of heterogeneity, therefore, leads to a lack of volatility.

8.6 AN ILLUSTRATION

We will demonstrate how the heterogeneity of speculators with respect to the strategies they use to generate buy/sell signals can cause erratic shifts in the excess demand function, and hence exchange rate volatility. For this purpose, we use 33 quarterly observations on the exchange rate between the pound and
US dollar, measured as dollar/pound, covering the period 1992:4–2000:4. Four different trading strategies are used, based on: (i) a single moving average rule; (ii) a 2% filter rule; (iii) a fundamental rule of 1.5% deviation from the equilibrium exchange rate as predicted by the flexible price monetary model; and (iv) fundamental discretion, in which the buy/sell signals are generated by movements in the individual variables appearing in the flexible price monetary model (for example, a buy signal is indicated by a rise in the relative money supply, and so on).

Figures 8.15–8.18 illustrate these rules. Figure 8.15 shows the exchange rate, the moving average and the difference between the exchange rate and the moving average. A buy signal is indicated when the moving average cuts the exchange rate from above (the difference turns positive), and vice versa. Hence, we get four buy and four sell signals. In Figure 8.16, only the major peaks and troughs are used for the purpose of generating buy/sell signals. A buy signal is given when the exchange rate rises by 2% above the peak, whereas a sell signal is generated when the exchange rate falls by 2% below
the peak. Hence we get three buy and three sell signals. The dots represent peaks, troughs buy signals and sell signals (buy signals follow the troughs and sell signals follow the peak). In Figure 8.17 we show that actual and equilibrium exchange rates as well as the percentage deviation of the actual from the equilibrium rate. In this case a buy signal is generated when the actual exchange rate is 1.5% below the equilibrium level and vice versa. Finally, Figure 8.18 shows the behaviour of the relative money supply, relative income (growth differential) and the interest rate differential, the three explanatory variables in the monetary model of exchange rates. In this case a buy signal is generated when at least two of these variables give a buy signal. Nothing is done until there is a sell signal. Table 8.3 shows the buy and sell signals generated by various trading strategies.

If these buy and sell signals are translated into shifts in the excess demand function, these shifts will be erratic, leading to exchange rate volatility. Of course, such erratic behaviour will be more pronounced the more heterogeneity we introduce (by using more trading strategies).

One way of translating these buy and sell signals into shifts in the excess demand function, and consequently to what happens to the exchange rate quantitatively, is as follows. We can allocate a weight to each group of traders using a particular trading strategy, such that the weight is determined by the profitability of the strategy. Relating the market weight to profitability is...
FIGURE 8.15 The moving average rule.

FIGURE 8.16 Identifying peaks and troughs for a filter rule.
8.6 AN ILLUSTRATION

Justified in Moosa and Shamsuddin (2002). Then we estimate the maximum possible change in the exchange rate at any one point in time as the absolute mean plus or minus three standard deviations. The magnitude of the change can therefore be calculated by applying the weights of the groups of traders acting on buy/sell signals at that point in time to the maximum possible change. By doing this, it is possible to generate a simulated exchange rate series, which will invariably exhibit similar volatility pattern to that of the actual exchange rate series. A finding like this may be taken to imply that heterogeneity leads to volatility.
FIGURE 8.18 Variables determining the buy/sell signals under fundamental discretion.

TABLE 8.3 Buy/sell signals generated from four different trading strategies.

<table>
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<tr>
<th>Observation number</th>
<th>Buy signal</th>
<th>Sell signal</th>
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9.1 WHY FOREIGN CURRENCY FINANCING AND INVESTMENT?

International short-term financing and investment (also called international asset and liability management, working capital management and treasury management) involve the selection of the short-term assets and the short-term funds (liabilities) required to finance the assets, with the objective of maximising the value of the firm. The difference between domestic and international short-term investment and financing operations is that the latter include the impact of exchange rate fluctuations, potential exchange controls and multiple tax jurisdictions, in addition to the fact that they also involve a wider range of financing sources and investment outlets. We will consider these decisions with reference to a multinational firm that has subsidiaries located in various countries and operating with different base currencies.

Multinational firms typically resort to financing their short-term operations and investing their surplus cash balances by using short-term instruments denominated in a variety of currencies. This kind of behaviour is motivated by the desire to (i) minimise (or reduce) the cost of borrowing; (ii) maximise (or increase) the rate of return on short-term investment; (iii) minimise risk; and (iv) find a better risk–return trade-off. Another consideration is that markets for short-term funds offer different degrees of liquidity and wider ranges of the underlying assets. There are also some tax considerations, as the cost of borrowing and the rate of return on short-term assets may differ significantly for tax-related reasons. Last, but not least, short-term financial operations utilising currency portfolios (rather than single currencies) may involve lower risk resulting from diversification.

We have to bear in mind that foreign currency financing and investment introduce foreign exchange risk if the firm does not already have such an exposure. If, on the other hand, the firm already has exposure to a foreign
currency, this exposure may actually be reduced by foreign currency financing and investment operations. Suppose that a firm whose base currency is $x$ has payables and receivables in currencies $y$ and $z$ respectively due some time in the future. This firm is obviously subject to foreign exchange risk because the $x$-currency values of these payables and receivables are not known until they are realised. This risk can be reduced if the firm takes long and short positions on $y$ and $z$ respectively with the same maturity date on which the payables and receivables are due. In fact, even long and short positions on other currencies that are highly correlated with $y$ and $z$ against $x$ will reduce risk.

Of course, interest rates on various currencies may differ significantly, although they tend to be highly correlated. This naturally implies that the cost of borrowing and the rates of return on positions in various currencies may differ significantly. Figure 9.1 shows time plots of the three-month interest rates on four major currencies. While each of these interest rates represents the cost of borrowing for a borrower whose base currency is the currency in question, this is not true for a borrower with a different base currency. This is because foreign currency operations introduce another element to the cost of borrowing and the rate of return, which is the percentage change in the exchange rate. Thus, the cost of borrowing and the rate of return on foreign currencies are measured respectively by the effective financing rate and the effective rate of return. These concepts will be introduced later.

9.2 SOURCES OF FINANCING AND INVESTMENT OUTLETS

In this section we describe the sources of short-term financing and investment outlets (instruments) used to invest surplus funds. We start with the sources of short-term financing.

Sources of short-term financing
Before resorting to external financing, a business firm normally determines whether or not internal funds are available. A multinational firm with international subsidiaries can, for example, utilise internal financing by requesting a transfer of surplus funds from one of its subsidiaries. Another method, which produces the same effect, is to increase markups on supplies sent by the multinational firm to subsidiaries with surplus funds. In this case, transfer pricing is used as the means for transferring funds from the subsidiary to the parent firm. The same procedure can be used to transfer funds from the parent firm to a subsidiary or from a subsidiary to another. This is generally known as intercompany or inter-subsidiary financing. If internal funds are not available, then the firm would resort to external financing, which may take the form of bank loans or raising funds by issuing short-term securities. These sources of
FIGURE 9.1 Three month interest rates on major currencies.

External funds may be found in the domestic market or the Eurocurrency market.

Bank loans, which are typically unsecured, are the dominant form of short-term financing. The borrower signs a note documenting its obligation to
repay the loan when it is due along with the accumulated interest. The loan must be repaid over the specified period (say three months) or renewed, which is called rolling over the loan. The process of rolling over gives the bank some control over the use of funds. To ensure that the funds are not used for long-term objectives, the bank may require a cleanup clause, which requires the borrower to be completely out of debt for a minimum period every year.

Bank loans may take one of several forms. Term loans are unsecured straight loans extended for a fixed period of time (for example, three months). They are used by borrowers who have infrequent need for bank credit. Another form is the line of credit, which is an informal agreement that permits a firm to borrow up to a maximum amount. It involves an agreement whereby the firm can draw down the line of credit and pay when it can. This agreement (which is valid over a year and renewable) is used by frequent borrowers. An overdraft, on the other hand, is a line of credit against which the borrower can issue cheques. Revolving credit arrangements are similar to overdrafts except that the lending bank is obliged to provide the funds (which is not the case for overdrafts). The borrower pays interest on the outstanding amount plus a commitment fee. When the arrangement is renewed continuously the risk arises that it may be used for long-term purposes, in which case a cleanup clause may be required.

Discounting is a mode of bank financing associated with bankers’ acceptances. When a bill resulting from a transaction (promising payment some time in the future) is presented to a bank, and the bank endorses the bill, it becomes a bankers’ acceptance. The bill can then be sold at a discount to the bank or to a money market dealer. Another form of discounting is factoring, which arises when a firm’s receivables are bought at a discount, thereby accelerating their conversion into cash. A special kind of factoring is forfeiting, which is the discounting (at a fixed rate without recourse) of export receivables in fully convertible currencies.

Raising funds can be accomplished by issuing short-term securities in the domestic market or the Eurocurrency market. These securities may take the form of note issuance facilities (NIFs) or commercial papers (CPs). Note issuance facilities are short-term notes underwritten by banks or guaranteed by bank standby credit arrangements. They are attractive to investors because they offer high liquidity through an active secondary market. In the case of revolving underwriting facilities (RUFs), borrowers use a single bank to place their paper at a set price.

Commercial papers are short-term unsecured promissory notes that are generally sold by large firms on a discount basis to institutional investors and other firms. Because they are unsecured, only large well-known firms can issue them. By going directly to the market rather than through financial intermediaries, issuing firms can make substantial savings
Short-term investment outlets
International short-term investment is the activity of placing excess funds or
cash balances in the short-term instruments that are available in international
money markets and denominated in various currencies. The word “placing”
here has the same meaning as “lending”, “depositing” and “investing”.

Two instruments are predominantly used for short-term international
investment, which are basic types of deposit instruments: time deposits and
certificates of deposit (CDs), both of which may again be domestic or
Eurocurrency instruments. Other money market instruments include Treas­
ury bills, demand deposits, deposits with NBFIs, bankers’ acceptances and
commercial papers.

The placement of a Eurocurrency time deposit implies that the depositor
commits the underlying funds for a specified period of time at a specified
interest rate. On maturity, the depositor receives the amount invested (prin­
cipal) and the interest paid on the principal. The maturities of short-term time
deposits range between overnight and twelve months.

Certificates of deposit (CDs) comprise a smaller percentage of the total value
of these instruments than time deposits. The attractiveness of these instru­
ments as compared with time deposits stems from their property of being
negotiable (that is, they can be bought and sold on a secondary market). A CD
specifies the amount of the deposit (the principal), the date of maturity and
the interest rate applicable to the principal. There are normally three types of
CD: (i) tap CDs, which are large-denomination fixed-term deposits; (ii)
tranche CDs, which are divided into several portions, making them appealing
to smaller investors; and (iii) rollover CDs, which are renewed after maturity
at an interest rate reflecting market conditions.

9.3 INTERNATIONAL CASH MANAGEMENT

In this section we consider the issue of centralised versus decentralised cash
management, which often faces companies with overseas branches or multi­
national firms with subsidiaries in various countries. Centralised cash
management implies that receipts and payments in various currencies are
managed by a central body, normally in the company’s head office. Decentral­
ised cash management implies that these receipts and payments are managed
locally by the branches or subsidiaries. The related issue of netting is also
discussed.

Centralised cash management
Centralised cash management involves the transfer of a subsidiary’s cash in
excess of minimal operating requirements to a centrally managed account or a
cash pool. In this case each subsidiary needs to hold locally only the minimum
cash balance required for transaction purposes, whereas all precautionary balances are held by the parent firm or in the pool.

Most of the advantages of centralised cash management are associated with economies of scale. The first of these perceived advantages is netting. This operation involves the calculation of the overall corporate position in each currency by adding up the short and long positions of various branches and subsidiaries. Netting provides a natural hedge when there is a short position in one currency and an equivalent long position in the same currency.

Another advantage is currency diversification. Even if the combined position is not zero, centralised cash management may result in a combined position that is so diversified that the foreign exchange risk is sufficiently reduced, again removing the need to hedge individual positions. Specifically, if the combined exposure is well diversified, and if the exchange rates of these currencies against the base currency are not highly correlated, this will effectively provide a natural hedge. Alternatively, if exchange rates are positively correlated (as it is normally the case) then a natural hedge would be in place when long and short positions are taken on various currencies.

Pooling is another advantage of centralised cash management. By pooling cash balances in a centralised location, cash requirements by any branch or subsidiary anywhere can be met without having to keep balances denominated in various currencies in every locality. The requirements of a subsidiary that is short of cash in a certain currency can be met from the central “pool of resources”.

A centralised system has more advantages, including the following:

1. The firm is able to operate with a smaller amount of cash. Pools of excess liquidity are absorbed and eliminated.
2. By reducing total assets, profitability is enhanced and financing costs are reduced.
3. The headquarters staff, with knowledge of all corporate activity, can recognise problems and opportunities that an individual unit may not perceive.
4. All decisions are made using the overall corporate benefit as a criterion.
5. By increasing the volume of foreign exchange and other transactions done through the headquarters, firms encourage banks to provide better foreign exchange quotes and better service.
6. Greater expertise in cash and portfolio management exists if one group is responsible for these activities.
7. Less can be lost in the event of an expropriation or currency controls restricting the transfer of funds because the firm’s total assets at risk in a foreign country can be reduced.

A drawback of centralised cash management is that it can create motivational problems for local managers unless some adjustments are made to the way in which these managers are evaluated. One possible approach is to relieve local managers of profit responsibilities for their excess funds. An
alternative approach would be to present local managers with interest rates (for borrowing from or lending funds to the central pool) that reflect the opportunity cost of money to the parent firm. These are called internal interest rates.

**Payment netting in international cash management**

There are significant interaffiliate cash flows representing cross-border fund transfers. There are costs associated with these cash flows, including the cost of buying foreign exchange (the bid–offer spread), the opportunity cost of the time in transit (the time taken to receive the funds) and other costs such as cable charges. Minimising the volume of these cash flows leads to a reduction in these costs. The flows can be reduced via payment netting, which can be done on a bilateral or multilateral basis.

Figure 9.2 illustrates how bilateral netting works. In this case there are two companies, A and B, selling goods and services to each other. Without netting there are two payments: from A to B, \(X(A \rightarrow B)\), and from B to A, \(X(B \rightarrow A)\). With netting, only one payment is made, which is equal to the net payment, \(N\):

\[
N = X(A \rightarrow B) - X(B \rightarrow A) \tag{9.1}
\]

This payment will be made by A to B if \(X(A \rightarrow B) > X(B \rightarrow A)\), and vice versa.

Consider now multilateral netting involving four companies (A, B, C and D), as shown in Figure 9.3. In this case there is greater scope for reducing fund transfers by netting out each company’s inflows against its outflows. To execute this function properly, a centre is needed to collect and record detailed information on intercompany accounts. As we can see from Figure 9.3, 12 payments have to be made without netting, and these are also shown in Table 9.1 (payments are recorded in columns and receipts are record in rows). Each company has to make three payments and receive three payments to and from other companies, as shown in Table 9.1.

If we calculate \(N\) as the difference between receipts and payments, we get

![FIGURE 9.2 Bilateral netting.](image-url)
Without netting

With netting

\[ N(A) = X(B \rightarrow A) + X(C \rightarrow A) + X(D \rightarrow A) - [X(A \rightarrow B) + X(A \rightarrow C) + X(A \rightarrow D)] \] (9.2)

\[ N(B) = X(A \rightarrow B) + X(C \rightarrow B) + X(D \rightarrow B) - [X(B \rightarrow A) + X(B \rightarrow C) + X(B \rightarrow D)] \] (9.3)

\[ N(C) = X(A \rightarrow C) + X(B \rightarrow C) + X(D \rightarrow C) - [X(C \rightarrow A) + X(C \rightarrow B) + X(C \rightarrow D)] \] (9.4)

\[ N(D) = X(A \rightarrow D) + X(B \rightarrow D) + X(C \rightarrow D) - [X(D \rightarrow A) + X(D \rightarrow B) + X(D \rightarrow C)] \] (9.5)

Since

\[ N(A) + N(B) + N(C) + N(D) = 0 \] (9.6)

it follows that multilateral netting will result in three payments only, as shown in Figure 9.3. The circles representing the companies are blank because it could be any combination, depending on the actual payments and receipts. In general, if \( n \) companies are involved, the maximum number of payments without netting is \( n(n - 1) \). With netting, the number of payments reduces to \( n - 1 \). Figure 9.4 shows the effect of multilateral netting. If 36 companies are
involved, the maximum number of payments without netting is 1260, which would be the case if every company makes a payment to every other company. With netting, the number declines to 35 payments only. Obviously, as the number of companies increases the advantage of multilateral netting becomes more pronounced.

**Decentralised cash management**

There are at least two reasons why decentralised cash management may be preferable. It is preferred when delays are expected in transferring funds to countries where the banking system is inefficient. Having the funds ready may be required to settle a transaction exposure with unknown timing. Decentralised cash management is also preferred when it is felt that local representation is necessary in order to maintain on-the-spot links with clients and banks. This is why this issue is not as much about the choice between centralised and decentralised cash management systems, but about determining the extent of centralisation and decentralisation.

If decentralised cash management is the case, what are the factors that determine where and in which currency cash balances are held? The following are some guidelines:

1. If it is anticipated that the funds received in a particular currency will be needed in the future, then transaction costs make it sensible to keep these funds in the same currency.
2. The same reasoning applies if there is no forward market in the underlying currency.
3. If political risk in one country is high, then funds should be kept in the home country rather than in the country in whose currency the funds are denominated.
4. Liquidity considerations make it sensible to keep funds in the currency in which they are most likely to be needed in the future.

5. Taxes are also important. In the presence of withholding taxes, funds should not be kept in countries with high tax rates.

Obviously, these factors may conflict with each other. Therefore a trade-off will be involved in the decision-making process.

9.4 THE EFFECTIVE FINANCING RATE AND THE EFFECTIVE RATE OF RETURN

In the absence of bid–offer spreads in interest rates, there is no difference between the lending and deposit rates in any currency, so the formula used to calculate the effective financing rate would be similar to that used to calculate the effective rate of return. We shall, therefore, consider the case without bid–offer spreads first.

No bid–offer spreads
Since there is no difference, let us just consider financing in currency y when the base currency is x and the exchange rate is expressed as $S(x/y)$. The effective financing rate in currency y depends on the nominal interest rate on y, $i_y$, and the percentage change in the exchange rate, $\Delta S$.

Suppose that an amount, $K$, of currency y is borrowed at time $t$ at a nominal interest rate $i_y$. The base currency value of the amount borrowed, $L_t$, is given by

$$L_t = KS_t$$

(9.6)

where $S_t$ is the spot exchange rate prevailing at time $t$. When the loan matures at time $t + 1$, the y currency amount to be repaid (principal plus interest), $L_{t+1}$, is given by

$$L_{t+1} = KS_{t+1}(1 + i_y)$$

(9.7)

where $S_{t+1}$ is the spot exchange rate prevailing at time $t + 1$. The effective financing rate, $e_y$, is therefore given by

$$(1 + e_y) = \frac{L_{t+1}}{L_t}$$

(9.8)

or

$$(1 + e_y) = \frac{KS_{t+1}(1 + i_y)}{KS_t}$$

(9.9)

which reduces to
\[(1 + e_y) = (1 + i_y)(1 + \hat{S})\]  
\hspace{2cm} (9.10)

Thus
\[e_y = (1 + i_y)(1 + \hat{S}) - 1\]  
\hspace{2cm} (9.11)

By ignoring the term \(i_y \hat{S}\), an approximate formula for the effective financing rate would be
\[e_y \approx i_y + \hat{S}\]  
\hspace{2cm} (9.12)

which tells us that the effective financing rate is approximately equal to the foreign nominal interest rate plus the rate of change of the exchange rate. So, if \(e_y < i_y\), foreign currency financing would be cheaper than domestic currency financing and vice versa. The effective financing rate may be negative, implying that the borrower pays back fewer units of the base currency than the amount actually borrowed.

The effective financing rate can be looked upon from an \textit{ex ante} perspective (before the fact) or from an \textit{ex post} perspective (after the fact). At time \(t\), the borrower does not know what the effective financing rate will be, because it depends on the percentage change in the spot exchange rate between \(t\) and \(t+1\): this is unknown at time \(t\). Decisions taken at time \(t\) then have to be based on the expected or \textit{ex ante} effective financing rate. At time \(t+1\), however, the change in the spot exchange rate is known and so is the effective financing rate. Hence, the actual or \textit{ex post} effective financing rate is realised at time \(t+1\). The \textit{ex post} rate tells the borrower whether or not his or her decision at time \(t\) was the right decision. The right decision is indicated by an effective financing rate that is lower than the domestic interest rate, which is the cost of financing in the base currency.

Changes in the spot exchange rate cause the effective financing rate, \(e\), to be different from the nominal interest rate on the foreign currency, \(i_y\). By observing equation (9.11), we can consider the following possibilities:

1. If the foreign currency appreciates against the domestic currency (that is, the exchange rate rises, \(\hat{S} > 0\)), then the effective financing rate will be higher than the nominal foreign interest rate (\(e_y > i_y\)).
2. If the exchange rate at time \(t+1\) is the same as at time \(t\) (that is, there is no change in the exchange rate, \(\hat{S} = 0\)), the effective financing rate and the nominal foreign interest rate will be equal (\(e_y = i_y\)).
3. If the foreign currency depreciates against the domestic currency (that is, the exchange rate declines, \(\hat{S} < 0\)), then the effective financing rate will be lower than the nominal foreign interest rate (\(e_y < i_y\)). If the (absolute) rate of change of the exchange rate is equal to the foreign interest rate (\(|\hat{S}| = i_y\) or \(\hat{S} = -i_y\)), the effective financing rate will be zero (\(e_y = 0\)). And if the (absolute) rate of change of the exchange rate is less than the foreign interest rate (\(|\hat{S}| < i_y\) or \(\hat{S} < -i_y\)), the effective financing rate will be negative (\(e_y < 0\)).
All of the above-mentioned possibilities can be derived from equation (9.11) or equation (9.12). For example, for the effective financing rate to be zero, we require \( i_y + \dot{S} = 0 \), which implies that \( \dot{S} = -i_y \) or \( |\dot{S}| = i_y \). The relationship between the effective financing rate and the nominal foreign interest rate can be represented diagrammatically. Figure 9.5 is a diagrammatic representation of equation (9.10), which plots \((1 + e)\) on the vertical axis and \((1 + i_y)\) on the horizontal axis, so the term \((1 + \dot{S})\) would represent the slope of the straight line relating the two variables. So, if \( \dot{S} = 0 \), the straight line representing the relationship would be OB, which has a slope of 1 and an equation given by \(1 + e = 1 + i_y\). On this line, \(e\) and \(i_y\) are equal. If \( \dot{S} > 0 \) then the straight line would be OA, which is steep since it has a slope greater than 1. On this line, \(e\) is greater than \(i_y\). If the exchange rate declines by more than the foreign interest rate (that is, \( \dot{S} < -i_y \)), then the line would be OE, which has a negative slope, implying a negative \(e\).

If \(i_y\) is taken to be the deposit rather than the borrowing rate, then the effective rate of return may be written as

\[
    r_y = (1 + i_y)(1 + \dot{S}) - 1
\]

(9.13)

or

\[
    r_y \approx i_y + \dot{S}
\]

(9.14)
which means that the effective rate of return on a short-term investment in a y-denominated asset is approximately equal to the sum of the interest rate on the currency and the percentage change in the exchange rate.

9.5 INTRODUCING THE BID–OFFER SPREADS

If we allow for the bid–offer spreads in exchange and interest rates, the equations used to calculate the effective financing rate and the effective rate of return will be different. Let us start with the effective financing rate. Suppose that an amount, $K$, of currency $y$ is borrowed at time $t$ at the offer foreign interest rate, $i_{y,a}$. The foreign currency amount is converted into the base currency at the bid exchange rate prevailing at time $t$, $S_{b,t}$. Thus, the base currency value of the amount borrowed, $L_t$, is given by

$$L_t = KS_{b,t}$$

(9.15)

When the loan matures at time $t+1$, the $y$ currency amount to be repaid (principal plus interest), $L_{t+1}$, is given by

$$L_{t+1} = KS_{a,t+1}(1+i_{y,a})$$

(9.16)

where $S_{a,t+1}$ is the offer spot exchange rate prevailing at time $t+1$. The effective financing rate is therefore given by

$$(1+e_y) = \frac{KS_{a,t+1}(1+i_{y,a})}{KS_{b,t}}$$

(9.17)

or

$$e_y = (1+i_{y,a}) \left[ \frac{S_{a,t+1}}{S_{b,t}} \right] - 1$$

(9.18)

Since $S_{a,t+1} = (1 + m)S_{b,t+1}$, it follows that

$$e_y = (1+i_{y,a}) \left[ \frac{S_{b,t+1}(1+m)}{S_{b,t}} \right] - 1$$

(9.19)

or

$$e_y = (1+i_{y,a})(1+S_b)(1+m) - 1$$

(9.20)

where $S_b$ is the percentage change in the bid exchange rate between $t$ and $t+1$. Obviously, the effective financing rate as calculated from equation (9.20) is greater than that calculated from equation (9.11). This is because the bid–offer spread is a transaction cost that is added to the cost of borrowing, making it higher than otherwise. An approximate formula can also be obtained by working out equation (9.20) and ignoring the small terms to obtain
\[ e \approx i_{y,a} + \hat{S}_b + m \]  

(9.21)

In this case the choice between foreign currency financing and base currency financing depends on a comparison between the effective financing rate and the base currency offer rate.

Likewise, the formula for the effective rate of return will differ if allowance is made for the bid–offer spreads in interest and exchange rates. The process from which the formula is derived can be modified as follows. Suppose that the foreign currency equivalent of an amount, \( K \), of the domestic currency is available for foreign currency investment at time \( t \) at the bid foreign interest rate, \( i_{y,b} \). The base currency amount is converted into foreign currency at the offer exchange rate prevailing at time \( t \), \( S_{a,t} \). Thus, the foreign currency value of the amount available for investment is \( K/S_{a,t} \). When the investment matures at time \( t+1 \), the base currency equivalent of the foreign currency amount received is given by

\[ I_{t+1} = \frac{K}{S_{a,t}} (1 + i_{y,b}) S_{b,t+1} \]  

(9.22)

This is because the amount of the foreign currency realised from the investment is sold at the lower bid rate. The effective rate of return is therefore given by

\[ (1 + r_y) = \frac{(K/S_{a,t})(1 + i_{y,b}) S_{b,t+1}}{K} \]  

(9.23)

or

\[ r_y = (1 + i_{y,b}) \left[ \frac{S_{b,t+1}}{S_{a,t}} \right] - 1 \]  

(9.24)

which means that

\[ r_y = (1 + i_{y,b}) \left[ \frac{S_{b,t+1}}{S_{b,t}(1+m)} \right] - 1 \]  

(9.25)

or

\[ r_y = \frac{(1 + i_{y,b})(1 + \hat{S}_b)}{1+m} - 1 \]  

(9.26)

The effective rate of return calculated from equation (9.25) is lower than that calculated from equation (9.13). This is because the bid–offer spread is a transaction cost that reduces the net return on investment, making the effective rate of return lower than otherwise. In this case the choice between foreign currency and base currency investment depends on a comparison between the effective rate of return and the domestic bid rate.
**9.6 IMPLICATIONS OF CIP AND UIP**

Two international parity conditions have some implications for short-term financing decisions: covered interest parity (CIP) and uncovered interest parity (UIP). The implications of the two parity conditions are discussed in turn.

**Covered interest parity**

To assess the implications of CIP for short-term financing decisions, we will again modify the process used to derive an expression for the effective financing rate. This process is modified by assuming that the borrowing firm wishes to avoid foreign exchange risk. To do this, the foreign currency exposure can be covered in the forward market. In this case, the foreign currency is bought forward at time $t$, which means that the amount to be repaid is converted into foreign currency at the forward rate prevailing at time $t$, $F_t$. Thus

$$L_{t+1} = KF_t(1+i_y)$$  \hspace{1cm} (9.27)

The effective financing rate is, therefore, given by

$$(1+e_y) = \frac{KF_t(1+i_y)}{Ks_t}$$  \hspace{1cm} (9.28)

which gives

$$e_y = (1+i_y)(1+f) - 1$$  \hspace{1cm} (9.29)

where $f$ is the forward spread. If CIP holds then

$$1 + f = \frac{1+i_x}{1+i_y}$$  \hspace{1cm} (9.30)

By substituting equation (9.30) into equation (9.29) we obtain

$$e_y = i_x$$  \hspace{1cm} (9.31)

Thus, if CIP holds, the effective financing rate will be equal to the interest rate on the base currency. Hence foreign currency financing will be useless in the sense that it will not be cheaper than domestic currency financing. If, however, CIP is violated such that

$$1 + f > \frac{1+i_x}{1+i_y}$$  \hspace{1cm} (9.32)

which means that the forward spread is larger than the interest rate differential, it follows that
\[(1 + e_y) > (1 + i_y) \left( \frac{1 + i_x}{1 + i_y} \right) \quad (9.33)\]

or

\[e_y > i_x \quad (9.34)\]

which means that foreign currency financing is not desirable, since it is more expensive than base currency financing. If, on the other hand, CIP is violated such that the forward spread is smaller than the interest rate differential, that is

\[1 + f < \frac{1 + i_x}{1 + i_y} \quad (9.35)\]

then

\[(1 + e_y) < (1 + i_y) \left( \frac{1 + i_x}{1 + i_y} \right) \quad (9.36)\]

or

\[e_y < i_x \quad (9.37)\]

which means that foreign currency financing is desirable because it is cheaper than base currency financing. The implications of CIP for short-term financing decisions are listed in Table 9.2.

The same results can be derived by allowing for the bid–offer spreads in interest and exchange rates. Depending on the direction of covered arbitrage, CIP may be written (approximately) as

\[(1 + i_{x,a}) = \frac{(1 + i_{y,b})(1 + f)}{(1 + m)} \quad (9.38)\]

<table>
<thead>
<tr>
<th>The CIP condition</th>
<th>Direction of violation</th>
<th>Financing decision</th>
<th>Investment decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>holds</td>
<td>–</td>
<td>Indifference between foreign and base currency financing</td>
<td>Indifference between foreign and base currency investment</td>
</tr>
<tr>
<td>Violated</td>
<td>Forward spread is larger than interest differential</td>
<td>Base currency financing is more desirable</td>
<td>Foreign currency investment is more desirable</td>
</tr>
<tr>
<td>Violated</td>
<td>Forward spread is smaller than interest differential</td>
<td>Foreign currency financing is more desirable</td>
<td>Base currency investment is more desirable</td>
</tr>
</tbody>
</table>
or
\[ (1+i_{y,a}) = \frac{(1+i_{x,b})}{(1+f)(1+m)} \]  
(9.39)

By combining equations (9.20) and (9.39), we obtain
\[ (1+e_y)(1+i_{x,b}) = \frac{1+\hat{S}_b}{(1+f)} \]  
(9.40)

If \( \hat{S}_b \approx f \), it follows that \( e_y \approx i_{x,b} \). Likewise, if we combine equation (9.26) and (9.38) we obtain
\[ (1+r_y)(1+i_{x,a}) = \frac{1+\hat{S}_b}{(1+f)} \]  
(9.41)

which gives \( r_y \approx i_{x,a} \).

Uncovered interest parity
If UIP holds, then the percentage change in the spot exchange rate will be equal to the interest rate differential. This may be represented by
\[ 1+\hat{S} = \frac{1+i_x}{1+i_y} \]  
(9.42)

By substituting equation (9.42) into equation (9.11) we again obtain \( e_y = i_x \). The implications of UIP for short-term financing and investment are listed in Table 9.3.

9.7 THE PROBABILITY DISTRIBUTION OF THE EFFECTIVE FINANCING RATE AND THE EFFECTIVE RATE OF RETURN

Using the UIP condition as a criterion for foreign currency financing and investment decisions implies that the decision is based on the expected change in the exchange rate such that foreign currency financing will be preferred if the exchange rate is expected to change by less than what is implied by UIP. As we have seen, it is sometimes difficult to obtain a point forecast for the exchange rate. In this case, we use a probability distribution for the expected change in the exchange rate to arrive at a probability distribution for the effective financing rate (the same analysis is valid for the effective rate of return).

Suppose that the expected (percentage) change in the exchange rate assumes the values \( \hat{S}_1, \hat{S}_2, ..., \hat{S}_n \) with probabilities \( p_1, p_2, ..., p_n \). In this case the expected value of the (percentage) change in the exchange rate is given by
**TABLE 9.3** UIP as a criterion for foreign currency financing and investment.

<table>
<thead>
<tr>
<th>The UIP condition</th>
<th>Direction of violation</th>
<th>Financing decision</th>
<th>Investment decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Holds</td>
<td>–</td>
<td>Indifference between foreign and base currency financing</td>
<td>Indifference between foreign and base currency investment</td>
</tr>
<tr>
<td>Violated</td>
<td>Expected change in the spot rate is larger than interest differential</td>
<td>Base currency financing is more desirable</td>
<td>Foreign currency investment is more desirable</td>
</tr>
<tr>
<td>Violated</td>
<td>Expected change in the spot rate is smaller than interest differential</td>
<td>Foreign currency financing is more desirable</td>
<td>Base currency investment is more desirable</td>
</tr>
</tbody>
</table>

\[ E(\dot{S}) = \sum_{i=1}^{n} \dot{S}_i p_i \]  
(9.43)

The effective financing rate would have a similar probability distribution in the sense that it assumes the values \( e_{y,1}, e_{y,2}, ..., e_{y,n} \) with probabilities \( p_1, p_2, ..., p_n \). The expected value of the effective financing rate is therefore given by

\[ E(e_y) = \sum_{i=1}^{n} e_{y,i} p_i \]  
(9.44)

The measure of risk in this case is the variance or the standard deviation of the effective financing rate, both of which represent the variability of the effective financing rate or the dispersion around its expected value. The variance is given by

\[ \sigma^2(e) = \sum_{i=1}^{n} p_i [e_{y,i} - E(e_y)]^2 \]  
(9.45)

whereas the standard deviation is measured as the square root of the variance as

\[ \sigma(e_y) = \sqrt{\sum_{i=1}^{n} p_i [e_{y,i} - E(e_y)]^2} \]  
(9.46)

Alternatively, the expected value of the effective financing rate can be calculated directly from equation (9.11) as

\[ E(e_y) = (1+i_y)[1+E(\dot{S})] - 1 \]  
(9.47)

Expected values, variances and standard deviations can be calculated if probability distributions are available. If not, one can calculate these statistics
from historical data, in which case the expected value is replaced by the mean or average value. Therefore, if we have a sample of observations on the effective financing rate: \(e_{y,1}, e_{y,2}, ..., e_{y,n}\), the mean or average value, \(\bar{e}_y\), the variance and the standard deviation are given respectively by

\[
\bar{e}_y = \frac{\sum_{t=1}^{n} e_{y,t}}{n} \quad (9.48)
\]

\[
\sigma^2 (e_y) = \frac{\sum_{t=1}^{n} (e_{y,t} - \bar{e}_y)^2}{n-1} \quad (9.49)
\]

\[
\sigma (e_y) = \sqrt{\frac{\sum_{t=1}^{n} (e_{y,t} - \bar{e}_y)^2}{n-1}} \quad (9.50)
\]

It is important to remember that financing with the base currency involves no foreign exchange risk, since the cost of financing (which is the interest rate on the base currency) is known with certainty. When foreign currency financing is chosen, the effective financing rate is at best probabilistic, in the sense that it assumes a range of values with some probabilities. This, naturally, implies the presence of risk. The choice between foreign currency financing and domestic currency financing, therefore, depends on the attitude towards risk. The decision maker may in this case want to minimise (maximise) the effective financing rate (the effective rate of return) per unit of risk as measured by the standard deviation. Alternatively, we may assume that the decision maker is a mean–variance expected utility maximiser such that the utility function is given by

\[
E[U(e_y)] = E(e_y) - \gamma \sigma^2 (e_y) \quad (9.51)
\]

where \(\gamma\) is the coefficient of risk aversion. The difference between financing and investment decisions is that \(U'(e_y) < 0\), whereas \(U'(r_y) > 0\), respectively.

Since \(e_y = i_y + \dot{S}\), it follows that

\[
\sigma^2 (e_y) = \sigma^2 (i_y) + \sigma^2 (\dot{S}) + 2\sigma(i_y, \dot{S}) \quad (9.52)
\]

where \(\sigma(i_y, \dot{S})\) is the covariance of \(i_y\) and \(\dot{S}\). Equation (9.52) means that over a long period of time, the variability of the effective financing rate is determined by the variability of the interest rate and that of the percentage change in the exchange rate.

9.8 USING CURRENCY PORTFOLIOS FOR SHORT-TERM FINANCING AND INVESTMENT

There is no reason why a firm cannot choose to finance with (or invest in) a portfolio of currencies, which may or may not include the base currency. In
fact, such an operation may be beneficial because it is a very well-known principle in finance that diversification reduces risk.

Let us look at the matter in general terms, using financing operations for the purpose of illustration. Assume that there are two foreign currencies, $y$ and $z$. The effective financing rate in currency $y$ assumes the values $e_{y,i}$ with probabilities $p_{y,i}$ for $i = 1, 2, ..., n$, such that $e_{y,n} > e_{y,n-1} > ... > e_{y,1}$, which means that the $n$th value of the effective financing rate is the greatest, whereas the first value is the smallest. Similarly, the effective financing rate of currency $z$ assumes the values $e_{z,j}$ with probabilities $p_{z,j}$ for $j = 1, 2, ..., m$ such that $e_{z,m} > e_{z,m-1} > ... > e_{z,1}$. Let us assume that the financing portfolio is formed by assigning weights $w_y$ and $w_z$ to currencies $y$ and $z$ respectively such that $w_y + w_z = 1$.

The worst thing that can happen is that the exchange rates of the base currency against the two foreign currencies move in such a way as to produce the highest values of the individual currencies’ effective financing rates, $e_{y,n}$ and $e_{z,m}$. In this case the highest effective financing rate of the portfolio, $e_{p,n,m}$, will be given by

$$e_{p,n,m} = w_y e_{y,n} + w_z e_{z,m} \tag{9.53}$$

Since $e_{y,n}$ is realised with a probability $p_{y,n}$ and $p_{z,m}$ is realised with a probability $p_{y,m}$, then $e_{p,n,m}$ is realised with a joint probability calculated as the product of the individual probabilities (that is, $p_{y,n}p_{z,m}$). This probability is naturally smaller than any of the two individual probabilities. If the effective financing rates of the individual currencies can assume any value, the effective financing rate of the portfolio will be given by

$$e_{p,i,j} = w_y e_{y,i} + w_z e_{z,j} \tag{9.54}$$

which materialises with a joint probability $p_{y,i}p_{z,j}$. Table 9.4 lists all of the possible combinations (that is, for various values of $i$ and $j$).

The expected value of the effective financing rate of the portfolio can be obtained by multiplying each entry in the third column by the corresponding entry in the fourth column and then adding up the products. This can be represented by

$$E(e_p) = \sum_{i=1}^{n} \sum_{j=1}^{m} (w_y e_{y,i} + w_z e_{z,j}) p_{y,i}p_{z,j} \tag{9.55}$$

where

$$\sum_{i=1}^{n} \sum_{j=1}^{m} p_{x,i} p_{y,j} = 1 \tag{9.56}$$

The variance and the standard deviation of the portfolio’s effective financing rate are given by
TABLE 9.4 Possible values for the effective financing rate of a two-currency portfolio.

<table>
<thead>
<tr>
<th>Effective financing rate of $y$ ($e_{y,i}$)</th>
<th>Effective financing rate of $z$ ($e_{z,j}$)</th>
<th>Effective financing rate of portfolio ($e_{p,i,j}$)</th>
<th>Joint probability ($P_{x,i}P_{y,j}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{y,1}$</td>
<td>$e_{z,1}$</td>
<td>$w_{y}e_{y,1} + w_{z}e_{z,1}$</td>
<td>$P_{y,1}P_{z,1}$</td>
</tr>
<tr>
<td>$e_{y,1}$</td>
<td>$e_{z,2}$</td>
<td>$w_{y}e_{y,1} + w_{z}e_{z,2}$</td>
<td>$P_{y,1}P_{z,2}$</td>
</tr>
<tr>
<td>$e_{y,1}$</td>
<td>$e_{z,m}$</td>
<td>$w_{y}e_{y,1} + w_{z}e_{z,m}$</td>
<td>$P_{y,1}P_{z,m}$</td>
</tr>
<tr>
<td>$e_{y,2}$</td>
<td>$e_{z,1}$</td>
<td>$w_{y}e_{y,2} + w_{z}e_{z,1}$</td>
<td>$P_{y,2}P_{z,1}$</td>
</tr>
<tr>
<td>$e_{y,2}$</td>
<td>$e_{z,2}$</td>
<td>$w_{y}e_{y,2} + w_{z}e_{z,2}$</td>
<td>$P_{y,2}P_{z,2}$</td>
</tr>
<tr>
<td>$e_{y,2}$</td>
<td>$e_{z,m}$</td>
<td>$w_{y}e_{y,2} + w_{z}e_{z,m}$</td>
<td>$P_{y,2}P_{z,m}$</td>
</tr>
<tr>
<td>$e_{y,n}$</td>
<td>$e_{z,1}$</td>
<td>$w_{y}e_{y,n} + w_{z}e_{z,1}$</td>
<td>$P_{y,n}P_{z,1}$</td>
</tr>
<tr>
<td>$e_{y,n}$</td>
<td>$e_{z,2}$</td>
<td>$w_{y}e_{y,n} + w_{z}e_{z,2}$</td>
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<tr>
<td>$e_{y,n}$</td>
<td>$e_{z,m}$</td>
<td>$w_{y}e_{y,n} + w_{z}e_{z,m}$</td>
<td>$P_{y,n}P_{z,m}$</td>
</tr>
</tbody>
</table>

$$
\sigma^2 (e_p) = \sum_{i=1}^{n} \sum_{j=1}^{m} \left[ (w_y e_{y,i} + w_z e_{z,j}) - E(e_p) \right]^2 p_{y,i} p_{z,j} 
$$

(9.57)

$$
\sigma (e_p) = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{m} \left[ (w_y e_{y,i} + w_z e_{z,j}) - E(e_p) \right]^2 p_{y,i} p_{z,j}}
$$

(9.58)

Alternatively, the expected value of the portfolio’s effective financing rate can be calculated from the effective financing rates of the individual currencies as

$$
E(e_p) = w_y E(e_y) + w_z E(e_z)
$$

(9.59)

or in terms of historical data

$$
e_p = w_y \bar{e}_y + w_z \bar{e}_z
$$

(9.60)

whereas the variance is measured as

$$
\sigma^2 (e_p) = w_y^2 \sigma^2 (e_y) + w_z^2 \sigma^2 (e_z) + 2w_y w_z \sigma(e_y, e_z)
$$

(9.61)

where $\sigma(e_y, e_z)$ is the covariance of the individual effective financing rates. The variance as given by equation (9.61) can also be expressed in terms of the correlation coefficient between the individual effective financing rates, $\rho(e_y, e_z)$, as
As we can see from equation (9.62), the correlation coefficient plays an important role in determining the standard deviation of the effective financing rate of the portfolio. Figure 9.6 shows the relationship between the standard deviation and the correlation coefficient as represented by equation (9.62). It is drawn for the special case when $\sigma(e_y) = \sigma(e_z) = 5$ and $w_y = w_z = 0.5$. If $\rho = -1$, the standard deviation of the effective financing rate of the portfolio is zero. In this case, risk is eliminated completely by choosing an equally-weighted portfolio. The highest value of the standard deviation of 5 is obtained when $\rho = 1$. In this case, there is no reduction in risk. The rate at which risk is reduced increases as $\rho \to -1$, and this is why the relationship is nonlinear.

Notice also that correlation between $e_y$ and $e_z$ is determined by the correlation between the percentage changes in the exchange rates, $\delta(x/y)$ and $\delta(x/z)$. If

$$e_y = a + b(e_z)$$

then

$$i_y + \delta(x/y) = a + b[i_z + \delta(x/z)]$$

which gives

$$\delta(x/y) = [a + bi_z - i_y] + \delta(x/z)$$

which shows that the correlation between $\delta(x/y)$ and $\delta(x/z)$ determines the correlation between $e_y$ and $e_z$.  

**FIGURE 9.6** The standard deviation of the effective financing rate of a currency portfolio as a function of the correlation coefficient.

\[
\sigma^2(e_p) = w_y^2 \sigma^2(e_y) + w_z^2 \sigma^2(e_z) + 2w_y w_z \sigma(e_y) \sigma(e_z) \rho(e_y, e_z)
\] (9.62)
International Long-Term Financing, Capital Structure and the Cost of Capital

10.1 INTERNATIONAL BANK LOAN FINANCING

Definition and classification
International bank loans are classified into two categories: foreign loans and Euroloans. Foreign loans are raised by borrowers who are foreign to the country where the loans are raised. International loans, however, mostly take the form of Euroloans or Eurocredits, which are denominated in a currency other than the currency of the country where the loans are raised. Euroloans and foreign loans are also distinguished as follows. While Euroloans are financed wholly out of Eurocurrency funds, irrespective of whether the borrower is a resident or a non-resident of the country in question, foreign loans are domestic currency credits extended to non-resident borrowers.

Syndicated loans (Euro or foreign) are characterised by being so large in size that it becomes necessary to form a syndicate or a group of lending banks to finance the loan. The advantage of syndication is that it enables banks to spread the risk of very large loans amongst themselves. This is important because a large multinational firm may need credit in excess of what a single bank can offer. A syndicated loan is arranged by a lead bank on behalf of a client such as a multinational firm. The lead bank seeks the participation of a group (syndicate) of banks, each providing a portion of the loan. One or more banks are designated as reference banks, which is necessary to establish the reference interest rate.

Pricing international loans
Pricing an international loan, which amounts to the determination of the interest charged to the borrower, depends on a number of factors. It is also influenced by a combination of risk evaluation, market conditions, and shifts
in the demand for and supply of loans. The following factors are important for determining the interest rate charged to the borrower.

**The spread and the reference rate**
The interest paid on syndicated loans is usually computed by adding a spread to the London interbank offer rate (LIBOR) or another reference rate such as the US prime rate or the Singapore interbank offer rate (SIBOR). The following factors determine the spread:

1. The availability of liquidity or loanable funds relative to demand. Spreads are likely to be higher in a market that is characterised by a shortage of liquidity and excess demand for loanable funds.
2. The creditworthiness of the borrower, as high-quality borrowers are charged lower spreads than low-quality borrowers. High-quality borrowers include OECD governments, multinational firms, major OECD companies and international organisations such as the World Bank.
3. The maturity of the loan, with higher spreads charged on long-maturity loans.

In domestic bank lending, interest rates allow for the narrowest loan spread when the central government is the borrower because there is no credit risk (the government can print money or tax people to meet its debt obligations). When non-government borrowers are involved the spreads increase as a function of credit risk. In international lending, central government borrowers pay varying interest rates because of differences in perceived country risk. Other borrowers pay higher interest rates because of (i) differences in country risk, (ii) differences in credit risk, and (iii) differences in the currency of loan denomination.

**The fees charged**
In addition to interest payments, the borrower is expected to pay management fees, participation fees, commitment fees and taxes. Management fees are charged by managing banks for their services. These are one-time charges levied when the loan agreement is signed. Participation fees are divided among all banks in relation to their share of the loan. Commitment fees are charged to the borrower as a percentage of the undrawn portion of the loan. There is normally a trade-off between fees and the spread over the reference rate. Banks prefer a combination of high fees and low spreads because they can advertise a low spread (which is good for business) without losing revenue.

**Currency of denomination**
The currency of denomination affects loan pricing in two ways: (i) interest rates vary from one currency to another; and (ii) the potential gain/loss on a currency must be considered when the currency of denomination is different from the base currency of the borrower.
The empirical evidence
Sargen (1976) attempted to explain differences in the loan spreads that borrowing countries pay. He used two estimated equations, the first of which shows how the loan spread varies between developing and developed countries. This equation includes the following explanatory variables: (i) type of borrower, (ii) year of loan commitment and (iii) length of loan commitment. The findings indicate that borrowers from developing countries pay an average spread of 140 basis points over LIBOR, while developed countries pay on average 25 or less. Since the maturity of the loan exerts a small influence on borrowing costs, the second equation attempts to consider variations in loan spreads within the groups of developing countries. The variables used to explain loan spreads include: (i) an income effect (high-income versus low-income LDCs); (ii) a Mexico effect, a dummy variable related to Mexico’s long experience as an international borrower; (iii) the debt–services ratio, or the ratio of debt service payments to export receipts; (iv) inflation; (v) general increase in interest costs from 1974 to 1975; and (vi) loan maturity. All six variables turned out to be statistically significant.

Brittain (1977) carried out similar analysis, using a single variable to explain LIBOR spreads, the ratio of external debt to GNP. He found that LDC borrowers with higher debt to GNP ratios pay higher LIBOR spreads for medium-term funds. In an analysis by Angelini et al. (1979) the authors produced an equation in which three country risk variables proved helpful in accounting for differences in LIBOR spreads: export growth, growth in per capita GNP and the ratio of GNP to external debt. Other variables of importance were found to be the debt service ratio and the ratio of reserves to imports.

Loan documentation
Loan agreements specify the rights and obligations of the borrower and lender. They specify the timing, calculation and the method of paying interest, principal, and fees. Several clauses have been developed for international lending, including the following:

1. Changes in circumstances. If any law or regulation changes such that the loan becomes illegal, the borrower must repay the loan. A second part of the clause protects against changes in reserve requirements or changes that may lead to an increase in the bank’s cost of funding the loan.
2. Sovereign immunity. Under most laws, a sovereign state is immune from the seizure of assets.
3. Governing law and jurisdiction. This clause specifies the law that governs the contract.
4. Cross default. A default by the borrower under any loan agreement constitutes an automatic default under the bank’s agreement.
5. Negative pledge. This clause forbids any secured borrowing unless the bank is secured equally. It prevents a bank’s position from deteriorating relative to other lenders.

**Risk sharing and reduction**

International banks use the following techniques to reduce and shift risks involved in international lending:

1. Loan selection and structuring. This process includes an analysis of credits to screen out inferior loans, application of loan limits, emphasis on booking higher quality credits and adherence to country, customer and currency limits.
2. Participation in loans. Many banks participate in large loans, each taking a small portion of the amount.
3. Use of guarantees and insurance. Central government agencies, central banks, and commercial banks provide loan guarantees. A variety of agencies provide insurance, such as the US Import–Export Bank and those involved in providing insurance for FDI.
4. Floating rate loans. These loans provide protection for the lender bank against interest rate risk (risk is shifted to the borrower).

### 10.2 INTERNATIONAL BOND FINANCING

**Eurobonds and foreign bonds**

International bonds can be Eurobonds or foreign bonds. Again, the distinction depends on whether the borrower is a domestic or a foreign resident, and whether the issue is denominated in the domestic or a foreign currency. A Eurobond issue is underwritten by an international syndicate of banks and other financial institutions and placed (that is, sold) in countries other than the country in whose currency the issue is denominated.

The Eurobond market, which is an extension of the offshore or external financial markets, has emerged because of some of the same factors that have led to the emergence of the Eurocurrency market, or the market for short-term Eurocurrency funds. These include: (i) the absence of regulatory interference; (ii) less stringent disclosure requirements; and (iii) favourable tax status, as interest income is not subject to income and withholding tax.

A foreign bond is underwritten by a syndicate consisting of members from a single country, sold primarily within that country and denominated in its currency. The issuer (or borrower), however, is foreign. Even if the borrower is not particularly interested in the currency of the issue, the funds raised can be swapped for other currencies. Foreign bonds have nicknames: foreign bonds sold in the USA are Yankee bonds, in Japan they are Samurai bonds, and in the UK they are bulldogs.
Types of international bond

International bonds can be of several types. These are briefly described in turn.

*Straight fixed-rate bonds*

The most common type of bonds are straight bonds on which coupons are normally paid annually or semiannually. Interest payments are a fixed percentage (determined by the coupon rate) of the par or the face value of the bond. On maturity, the bondholder receives the face value and the last interest payment.

*Floating rate notes (FRNs)*

Unlike straight bonds, FRN holders receive a variable semiannual coupon payment. The variable coupon rate is determined with reference to (by adding a margin to) a variable reference rate, such as LIBOR. FRNs came to the market as a natural outcome of the increase in interest rate volatility, which caused investors to be reluctant to hold long-maturity straight bonds.

*Convertible bonds*

Convertible bonds are equity-related bonds. They resemble straight bonds with the added feature that they are convertible to equity prior to maturity at a specified price per share. This feature enables the borrower to issue debt instruments with lower coupon rates than the corresponding straight bonds. In this case, the lender is willing to accept a lower coupon rate than on a comparable straight bond because of the attractiveness of the feature of convertibility.

*Bonds with equity warrants*

These are another type of equity-related bonds. They give the holder the extra privilege of having the right (which may or may not be exercised) to buy the shares of the same company issuing the bonds. Thus warrants are like equity options except that the issuers of the bond and the warrant are the same party, unlike the case of options, in which the two issuers are different.

*Zero coupon bonds*

The holder of a zero coupon bond does not receive coupon payments prior to the maturity date. Upon maturity, the holder receives the full face value of the bond, which is initially purchased at a discount. The attractive feature of zero coupon bonds is that they are not subject to reinvestment risk, which is encountered in the case of straight bonds. With the changing level of market interest rates, the holder of a straight bond is exposed to reinvestment risk because the holder does not know at what rate the coupon payments received prior to the maturity of the bond can be reinvested. Since zero coupon bonds offer no coupon payments prior to maturity, this kind of risk does not arise.
Multicurrency bonds
A multicurrency bond holder receives payments in more than one currency. One variant is a dual currency bond, which has different currency denominations for coupon payments and face value payments.

Global bonds
The concept of global bonds was introduced by the World Bank in 1989. Global bonds are defined as very large issues that are sold simultaneously in the world’s major capital markets. Global bonds may be held and cleared through several different systems in the major geographical regions, and the securities can move freely from one system to another. The implication of these characteristics is that these bonds are highly liquid.

The primary market
A borrower wanting to raise funds by issuing Eurobonds to the investing public will invite an investment banker to serve as a lead manager of an underwriting syndicate that will bring the bonds to market. The underwriting syndicate is a group of investment banks, merchant banks and the merchant banking arms of commercial banks that specialise in some phase of a public issue. The lead manager usually invites co-managers to form a managing group to help negotiate terms with the borrower, assess market conditions and manage the issue. The managing group along with other banks serve as underwriters for the issue in the sense that they will commit their own capital to buy the issue from the borrower at a discount from the issue price. The discount or the underwriting spread is typically in the 2% to 2.5% range. Most of the underwriters, along with other banks, will be part of a selling group that sells the bonds to the investing public. Members of the underwriting syndicate receive a portion of the spread, depending on the number and type of function they perform. The lead manager gets the full spread, but a bank serving only as a member of the selling group receives a smaller portion.

The choice of the currency of denomination
Just like the case with short-term financing, the decision concerning the choice of currency depends on the relative cost of borrowing in various currencies. This is certainly true for a risk-neutral issuer, but for an issuer that is risk-averse, risk becomes another decision variable. For the purpose of the following discussion, we will assume that the issuer is risk neutral. However, it is straightforward to modify the decision rules for a risk-averse issuer by introducing a risk premium.

Suppose that a firm wants to raise an amount, \( K \), of the base currency, \( x \), by issuing either base currency bonds or foreign currency (\( y \) bonds (which can be Eurobonds or foreign bonds). For simplicity let us assume that these are zero coupon bonds, such that all of the payments are made on maturity, which we
will assume to be \( n \) years. If a domestic currency bond is chosen, then the amount to be paid by the firm will be

\[
L_{x,t+n} = K(1+i_x)^n
\]  
(10.1)

where \( i_x \) is the base currency interest rate. If foreign currency denomination is used, the foreign currency amount raised is \( K/S_t \) where \( S_t \) is the initial exchange rate. On maturity, the base currency equivalent of the foreign currency amount due is

\[
L^*_{x,t+n} = \frac{KS_{t+n}}{S_t}(1+i_y)^n
\]  
(10.2)

where \( S_t \) is the exchange rate prevailing on maturity when the borrowed funds are to be repaid. Equation (10.2) can be rewritten as

\[
L^*_{x,t+n} = K(1+\hat{S})^n(1+i_y)^n
\]  
(10.3)

where \( \hat{S} \) is the annual percentage rate of change of the exchange rate between 0 and \( n \).

Assuming risk neutrality, foreign currency denomination will be preferred if \( L^*_{x,t+n} < L_{x,t+n} \) or if

\[
K(1+\hat{S})^n(1+i_y)^n < K(1+i_x)^n
\]  
(10.4)

which can be simplified to give the condition

\[
i_y + \hat{S} < i_x
\]  
(10.5)

If this condition is satisfied the bond issue should be denominated in currency \( y \) rather than base currency \( x \). Let us assume that, from a British perspective, currency \( y \) is the Australian dollar. A British firm should, if the condition represented by (10.5) is satisfied, choose an Australian dollar issue rather than a pound issue. If the firm sells the bonds in Australia, this would be a foreign bond issue. If the bonds are sold in any other country, this would be a Eurobond issue. On the other hand, if

\[
i_y + \hat{S} > i_x
\]  
(10.6)

the firm should use base currency denomination. In the case of the British firm, if the bonds are sold in the UK, it would be a domestic (rather than an international) bond issue. If the bonds are sold outside the UK, it would be a Eurobond issue.

### 10.3 INTERNATIONAL EQUITY FINANCING

Unlike the case of loans and bonds, the internationalisation of equities did not take off until about 1983. International equity markets encompass primary
market functions (underwriting of new equity issues) and secondary market functions (trading) of equities outside the issuer’s home country. The London International Stock Exchange is the best example of an international secondary equity market, accounting for a large portion of international equity trading.

International business firms operate in international equity markets by listing their shares on foreign stock exchanges (the secondary market function) and by selling new shares to foreigners (the primary market function). We will deal with these functions in turn.

**Listing on foreign stock exchanges**

Cross listing refers to a firm having shares listed on one or more foreign exchanges in addition to the home country stock exchange. Although cross listing is not a new concept, the increased globalisation of world equity markets has caused the amount of listing to explode in recent years. A firm may decide to cross list for the following reasons:

1. Cross listing provides a means of expanding the investor base for a firm’s stock, thus potentially increasing the demand for the stock. Increased demand may boost the price and improve liquidity.
2. It establishes name recognition of the company in a new capital market, thus paving the way for the firm to source new equity or debt capital from local investors.
3. It brings the firm’s name before more investors and customers. International diversification is validated if investors can trade the security on their own stock exchanges.
4. Cross listing may mitigate the possibility of a hostile takeover of the firm through the broader investor base created for the firm’s shares.
5. It may be helpful in supporting a new equity issue.
6. Broadening ownership outside the national frontiers, which can be accomplished via cross listing, may help reduce price fluctuations.

Cross listing obligates the firm to adhere to the securities regulations of its home country as well as the regulations of the countries in which it is cross listed. Thus the benefits of listing on foreign stock exchanges must be balanced against the cost of the implied commitment to full disclosure. Because the disclosure guidelines of the US Securities and Exchange Commission (SEC) are more stringent than in other countries, US firms find it easier to list on foreign stock exchanges than non-US firms wishing to list on US stock exchanges. But having decided to list on a foreign stock exchange, the question arises as to where to list. The choice depends on the motive behind foreign listing. If the motive is to support a new equity issue, the target market should be the listing market. If it is to boost the firm’s commercial and political visibility, the market should be the one in which the firm has significant physical operations. If the motive is to improve the liquidity of existing shares, then
the market should be a major liquid market such as New York, London and Tokyo.

**Selling new shares in international markets**

A firm may sell its newly issued shares to foreign investors in one of the following ways:

1. Selling shares in a particular foreign stock market underwritten in whole or in part by institutions from the host country. It may take the form of a private placement, in which the whole issue is sold to one or a few investors, typically institutional investors such as pension funds and insurance companies.

2. Selling Euro-equity issues to foreign investors in more than one country simultaneously. The prefix “Euro” has the same meaning we came across earlier. The integration of national capital markets has led to the emergence of a Euro-equity market.

3. Selling a foreign subsidiary’s shares to investors in the host country. This can lower a firm’s cost of capital if investors in the host country award a higher capitalisation rate on the subsidiary’s earnings than on the firm’s earnings.

4. Selling shares to a foreign firm as part of a strategic alliance. This may involve the sharing of the cost of developing new technology or pursuing complimentary marketing activities.

### 10.4 OTHER SOURCES OF FINANCING

There are some other means of long-term financing. These are: (i) parallel loans, (ii) credit swaps, (iii) government lending and (iv) development institution lending. These will be discussed in turn.

**Parallel loans**

A parallel loan involves an initial exchange of funds between firms in different countries, such that the transaction is reversed some time in the future. For example, suppose that a subsidiary of a US company in Japan needs some yen funds while the subsidiary of a Japanese company in the US needs funds in US dollar. The US company then lends the Japanese subsidiary US dollar funds, while the Japanese company lends the US subsidiary an approximately equivalent amount in yen. After an agreed period of time, the US subsidiary pays the yen loan (principal plus interest) while the Japanese subsidiary simultaneously pays off the US dollar loan. Obviously, this operation does not involve any foreign exchange risk because no currency conversion takes place.

Parallel loans were specifically designed to circumvent foreign exchange controls. However, they have the added advantage over bank lending that the
two firms can avoid transaction costs in the form of bid–offer spread in both interest and exchange rates. The problem with parallel loans is that it is difficult to find two counterparties with exactly matching needs.

Credit swaps
A credit swap makes it possible to acquire a loan for a foreign subsidiary without having to send funds abroad. It involves the exchange of currencies between a bank and a firm, not between two firms. The procedure is best illustrated with the help of an example. A British company could place euro-denominated funds (deposited) with a German bank in Frankfurt for a certain period of time. The bank can then instruct its branch, corresponding bank or subsidiary in London to grant a British subsidiary of the German company a pound-denominated loan. On the maturity of the loan and the deposit, the bank’s branch will receive the loan repayment while the British company receives the deposit.

Government lending
Host governments of foreign investments provide financing when they believe that the underlying projects will generate jobs, provide some transfer of technology, or train local workers. Countries acting as hosts for foreign investment often provide financial incentives to foreign investors including loans, subsidies, grants and loan guarantees.

Lending by international development institutions
There are a number of development institutions that grant developing countries loans to finance some infrastructure projects. While these loans are granted to the host governments, the companies working there are financed indirectly.

10.5 THE COST OF CAPITAL
A firm’s capital consists of equity (retained earnings and funds obtained by issuing equity) and debt (borrowed funds). The firm’s cost of retained earnings reflects an opportunity cost representing what the existing shareholders could have earned if they had received the earnings as dividends and invested the funds themselves. The firm’s cost of new equity capital that is obtained by issuing new equity reflects the opportunity cost of what the new shareholders could have earned if they had invested their funds elsewhere. The cost of new equity capital is higher than the cost of retained earnings because it also includes the expenses associated with selling the new equity. The cost of debt is easier to measure because interest expenses are incurred by the firm as a result of borrowing funds.
The cost of capital has a major impact on a firm’s value. To fund its operations, a firm uses a capital structure (that is, a combination of equity and debt) that minimises its cost of capital, and therefore maximises its value. The lower the firm’s cost of capital, the lower is its required rate of return on a given proposed project. Therefore estimating the cost of capital is a step that must be taken before indulging in the capital budgeting exercise, as we shall see in Chapter 12.

For a multinational firm, financing can take a much wider range of forms than for a purely domestic firm. Generally speaking, a multinational firm can raise financing from either internal or external sources. Internal sources include retained earnings and funds provided by subsidiaries. External sources include national and international capital markets. Faced with such a lucrative menu, the multinational firm’s choice depends on several factors, including the following: (i) the need to maintain or strengthen the extent of control over subsidiaries; (ii) the need to receive regular cash inflow from subsidiaries; (iii) the purpose for which financing is needed; (iv) other aspects of the overall business strategy, such as the objective of minimising global tax liabilities; (v) expectations concerning the future path of exchange and interest rates; and (vi) the desire to minimise exposure to various types of risk, such as foreign exchange risk and country risk.

The weighted average cost of capital
A firm’s weighted average cost of capital, $k$, is calculated as a weighted average of the cost of debt capital, $k_d$, and equity capital, $k_e$, with the weights determined by the proportions of debt and equity in the capital structure. Thus, if the capital of a firm consists of $D$ debt and $E$ equity, then the cost of capital is given by

$$k = \left( \frac{D}{D+E} \right)k_d (1-\tau) + \left( \frac{D}{D+E} \right)k_e$$

(10.7)

where $\tau$ is the tax rate. The first term, which reflects the weighted cost of debt, $[D/(D + E)]k_d$, is multiplied by the term $(1-\tau)$, which is less than unity. This is because debt financing involves tax saving, given that interest expenses are tax deductible.

The cost of equity capital, $k_e$, is the equity market’s expected rate of return on the firm’s equity, based upon the equity market’s opportunity cost of forgoing investment in other stocks with the same risk. In addition to the business risk of the firm’s operations, the cost of equity capital depends on the firm’s relative debt level, $D/(D + E)$, since the degree of financial leverage influences the risk of equity.

The cost of capital of a multinational firm invariably differs from the cost of capital of a domestic firm because of the differences between domestic firms and multinational firms. If the multinational firm raises capital in more than
one country, then its overall cost of capital will be a weighted average of the weighted average cost of capital corresponding to each country with the weights being the proportions of capital raised in each country. If the amounts of debt capital and equity capital raised in country $i$ are $D_i$ and $E_i$ respectively, where $i = 1, 2, ..., n$ (assuming that the amounts are measured in domestic currency terms) then the total amounts of debt and equity capital raised are

$$D = \sum_{i=1}^{n} D_i$$  \hspace{1cm} (10.8)

$$E = \sum_{i=1}^{n} E_i$$  \hspace{1cm} (10.9)

The weighted average cost of capital raised in country $i$ is

$$k_i = \left[ \frac{D_i}{D_i + E_i} \right] (1 - \tau_i) + \left[ \frac{E_i}{D_i + E_i} \right] k_{ei}$$  \hspace{1cm} (10.10)

Hence the overall cost of capital is given by

$$k = \sum_{i=1}^{n} \left[ \frac{D_i + E_i}{D + E} \right] k_i$$  \hspace{1cm} (10.11)

Otherwise, the overall cost of capital can be calculated from the overall debt and equity costs of capital, which are weighted averages of the individual countries’ costs of debt and equity capital.

A number of factors can explain the observed difference between the cost of capital of a multinational firm and that of a domestic firm. Some of these factors make the cost of capital of a multinational firm lower than that of a domestic firm, whereas others do the reverse. Because of the diversity of these factors, one cannot reach a general conclusion as to whether the cost of capital of a domestic firm is higher or lower than that of a multinational firm: it all depends on the particular case of each firm.

To start with, size does matter. Indulging in international operations leads to growth beyond what is available from domestic operations only. Because of this factor, multinational firms tend (on average) to be larger in size than domestic firms. Larger sizes reduce the cost of both debt and equity capital. Large multinational firms borrow substantial amounts of funds, and in the process they receive preferential borrowing rates from creditors such as banks. Moreover, multinational firms exploit economies of scale through their relatively large issues of bonds and equity. Economies of scale lead to lower costs (as a percentage of the amount raised) of new bond or equity issues.

Access to international capital markets is another factor. Multinational firms source funds from domestic as well as international capital markets. Access to international capital markets allows multinational firms to obtain lower
borrowing rates. They could also obtain funds at lower costs through their subsidiaries. Or they could indulge in cross-border financing operations, such as parallel loans and swaps.

The cost of capital is also affected by the probability of bankruptcy, such that the higher this probability the higher will be the cost of capital, as creditors and shareholders demand higher rates of return on their funds. A basic principle in finance is that diversification reduces risk. In this case international diversification leads to stability of cash flows and hence to lower probability of bankruptcy.

Exposure to foreign exchange risk is yet another factor. Funds remitted to a multinational firm from its subsidiaries are converted into the firm’s base currency. Because of the exchange rate factor, the base currency value of foreign currency cash flows will be highly volatile. The variability of the base currency value of foreign currency cash flows leads to a higher probability that the firm may go bankrupt, thus raising the cost of capital.

While international operations imply international diversification, which in turn implies stability of cash flows, they also lead to exposure to country risk. Extreme exposure to country risk (particularly political risk) may lead to an outright confiscation of the project, and hence big losses for the multinational firm. By following the same argument concerning the probability of bankruptcy, it is easy to conclude that exposure to country risk, just like exposure to foreign exchange risk, leads to a higher cost of capital.

### 10.6 Variations in the Cost of Capital and Capital Structure

In this section we discuss the reasons for inter-country differences in the cost of capital and inter-firm differences in capital structure. We start with the first issue of cross-country differences in the cost of capital, which can explain why multinational firms based in some countries may have a competitive advantage over others. Furthermore, understanding the differences between the cost of debt capital and the cost of equity capital can explain why multinational firms based in certain countries have more debt-intensive capital structures.

#### Differences in the cost of debt capital

The cost of debt capital is determined by the risk-free interest rate and the risk premium. The cost of debt capital is higher in some countries than in others because the risk-free rate and/or the risk premium is higher. Differences in the risk-free rate are due to several factors that affect the supply of, and demand for, loanable funds and hence the level of the interest rate. For example, the interest rate varies with the state of the economy, tending to rise when the
economy is booming and to decline when the economy is in a slump. There is also a direct positive relationship between the level of the nominal interest rate and expected inflation. Other factors that affect the level of the interest rate are the stance of monetary policy (tight or expansionary), tax laws (whether or not these laws encourage saving), and demographic factors (younger households tend to save less).

The risk premium is meant to compensate creditors for the risk of default by the borrower. This risk varies across countries because of differences in economic conditions, relationships between companies and creditors, government intervention, and the degree of financial leverage. The risk premium tends to be lower in countries where economic conditions are more stable. The risk of default increases as the economy moves into recession, so we should expect the risk premium to increase as economic activity slows down. The risk premium is also lower in countries where the relationship between creditors and companies is so close that creditors stand ready to extend credit in the event of financial distress.

**Differences in the cost of equity capital**

Now we turn to cross-country differences in the cost of equity capital. Recall that the cost of equity capital is an opportunity cost: what shareholders can earn on investments with similar risk if the equity funds were distributed to them. This return consists of the risk-free interest rate and a risk premium. Differences in the risk-free rates lead to differences in the cost of equity capital, which also depends on investment opportunities in the underlying country. Countries with abundant investment opportunities will have a higher cost of equity capital than countries with limited investment opportunities.

**Choosing the capital structure**

Choosing the capital structure is the choice of the debt–equity ratio. Debt is useful because interest payments are tax-deductible, but too much debt gives the impression that the company is financially vulnerable, which raises the cost of equity. The firm should aim at the level of debt–equity ratio that minimises the cost of capital and hence maximises the value of the firm. The advantages of using debt as opposed to equity vary from one company to another because of two sets of factors. The first set of factors pertain to the specific characteristics of the company, whereas the other factors pertain to the characteristics of the countries where the subsidiaries and the underlying projects are located.

There are three corporate characteristics that affect the capital structure: the stability of the firm’s cash flows, its credit risk and access to earnings. Firms with more stable cash flows can handle more debt because these cash flows can be used to cover periodic interest payments. One way to achieve stability of cash flows is to diversify across countries, which means that more
geographically diversified firms tend to have more stable cash flows and more debt-intensive capital structure (that is, a higher debt–equity ratio). Likewise, firms with lower credit risk (lower probability of default on loans) can handle a more debt-intensive capital structure. Lower credit risk would be the case if the firm has a strong and competent management and if it has marketable assets that can serve as a collateral. Finally, firms that are more profitable can use retained earnings to finance their operations, in which case they would use equity-intensive capital structures. This is why growth-oriented multinational firms have a higher debt–equity ratio than those with less growth.

There are factors that pertain to the host country, and this is why we can observe country differences in the capital structure. For example, firms in Japan and Germany have traditionally used a higher debt–equity ratio than those in the USA or the UK. This is because the probability of bankruptcy in Japan and Germany is lower, given that the governments in these countries are more willing to step in and rescue troubled firms. It is also a tradition that banks in these countries are not only creditors but also shareholders, in which case they have a vested interest in rescuing troubled firms.

The factors influencing the capital structure that pertain to the host country include the following. The first of these factors arises when there are restrictions on stock ownership in the host country. If, for one reason or another, investors in the host country are not allowed or are unwilling to invest in foreign shares, a multinational firm operating in that country will find it easier to raise equity capital there. In this case the multinational firm will have a more capital-intensive structure. Conversely, the multinational firm will have a more debt-intensive capital structure if it operates in a country where interest rates are low. The multinational firm will also opt for debt-intensive capital structure if it operates in a country with a weak currency. In this case most of the cash flows arising from the project are used to meet interest payments. A similar situation arises when country risk is high. A multinational firm that operates in a country where the risk of confiscation or blocked funds is high tends to borrow intensively in that country, and hence it will have a debt-intensive capital structure. Local debt financing will also be used when there is a withholding tax on remittances.

In general, therefore, multinational firms prefer to have a debt-intensive capital structure when their subsidiaries are subject to low local interest rates, weak currencies, a high degree of country risk and high taxes. A multinational firm may deviate from its target capital structure in each country where financing is obtained, while achieving its target capital structure on a consolidated basis. This policy of ignoring the local target capital structure in favour of a global capital structure can be justified under certain circumstances. For example, a multinational firm operating in a country that does not allow the listing of its shares on the local stock exchange will have a higher debt–equity ratio than desired otherwise. This high debt–equity ratio can be counterbalanced by using lower ratios in other countries.
Research findings on capital structure

Some recent research casts doubt on the validity of the traditional corporate finance model, suggesting that firms select optimal capital structures by trading off various tax and incentive benefits of debt financing against financial distress costs. Hovakimian et al. (2001) argue that while there is support for the trade-off models in the empirical literature, recent evidence suggests that a firm’s history may play a more important role in determining its capital structure. Titman and Wessels (1988), for example, show that highly profitable firms often use their earnings to pay back debt, which makes them less levered than less profitable firms. Moreover, Masulis and Korwar (1986) and Asquith and Mullins (1986) show that firms tend to issue equity following an increase in stock prices. The implication of this observation is that firms that perform well tend to reduce their debt–equity ratio subsequently.

Some researchers argue that the negative correlation between profits and leverage is consistent with Donaldson’s (1961) pecking order, which is used to describe how firms make their financing decisions. Donaldson argued that firms prefer to fund new investment with retained earnings (as opposed to borrowed funds), but they prefer debt to equity financing. If this is the case then firms accumulate retained earnings, becoming less levered when they are profitable and accumulate debt, becoming more levered, when they are unprofitable. If firms are indifferent about their capital structures, as suggested by Miller (1977), then they will not make future capital structure choices that offset their earnings history. Shyam-Sunder and Myers (1999) argue that the pecking order story provides a better empirical description of capital structure than do traditional trade-off models.

There are also dynamic models of capital structure, such as those of Fischer et al. (1989) and Leland (1998). In these models transaction costs are introduced to generate short-run pecking order behaviour. These models suggest that firms periodically readjust their capital structures towards a target ratio that reflects the costs and benefits of debt financing that are found in static trade-off models. The models also suggest that firms repurchase equity after an increase in share prices to adjust towards an optimal capital structure. However, this prediction is inconsistent with the observation that firms tend to issue equity following stock price increases.

Hovakimian et al. (2001) test the hypothesis that firms tend to move towards a target debt–equity ratio when they either raise new capital or repurchase existing capital. Their results suggest that although past profits are an important predictor of observed capital structures, firms often make financing and repurchase decisions that offset these earnings-driven changes in their capital structures. The results also suggest that stock prices play an important role in determining a firm’s financing choice. Firms that experience large stock price increases are more likely to issue equity and retire debt than are firms that experience stock price decline.
10.7 DEBT AND EQUITY EXPOSURE

In Chapter 4 we came across the definition of operating exposure. In this chapter we examine debt and equity exposure and show how they are related to operating and net cash flow exposures. To start with, let us define net cash flows as

\[ N = \pi - I \]  

(10.12)

where \( I \) is the interest payment on debt capital, \( D \), such that \( I = iD \), where \( i \) is the interest rate. Thus, net cash flow exposure, which measures the sensitivity of base currency net cash flows to changes in exchange rate, which gives

\[ E_N = \frac{\dot{N}_x}{S} \]  

(10.13)

We will use these definitions to illustrate the concepts of leverage and hedging effects of debt financing.

The leverage and hedging effects of debt financing

By using a two-period model, we can write equation (10.13) as

\[ E_N = \frac{(N_{x,t+1}/N_{x,t}) - 1}{S} \]  

(10.14)

which gives

\[ E_N = \frac{(\pi_{x,t+1} - I_{x,t+1})/(\pi_{x,t} - I_{x,t}) - 1}{S} \]  

(10.15)

Consider first the case of a firm that only raises base currency debt capital. In this case, \( E_I = l_x/S = 0 \) and \( I_{x,t+1} = I_{x,t} \), where \( E_I \) is the interest exposure. It follows that the net cash flow exposure of such a firm is given by

\[ E'_N = \frac{(\pi_{x,t+1} - I_{x,t})/(\pi_{x,t} - I_{x,t}) - 1}{S} \]  

(10.16)

Hence

\[ E_N' - E_N = \frac{[(\pi_{x,t+1} - I_{x,t})(\pi_{x,t} - I_{x,t}) - 1 - (\pi_{x,t+1}/\pi_t - 1)]/S}{S_\pi_{x,t}(\pi_{x,t} - I_{x,t})} > 0 \]  

(10.17)

which can be simplified to

\[ E_N' - E_N = \frac{I_{x,t}(\pi_{x,t+1} - \pi_{x,t})}{S_\pi_{x,t}(\pi_{x,t} - I_{x,t})} > 0 \]  

(10.18)

Obviously, equation (10.18) means that \( E_N' > E_N \), which means that the net cash flow exposure of a firm that raises base currency debt capital is greater than its operating exposure.
Now, consider the case of a firm that raises foreign currency debt capital. In this case, \( E_I > 0 \), such that \( I_{x,t+1} > I_{x,t} \) for \( S > 0 \), even if \( I_{y,t+1} = I_{y,t} \). Hence

\[
E_N' - E_N = \frac{(\pi_{x,t+1} - I_{x,t})/(\pi_{x,t} - I_{x,t}) - 1 - [(\pi_{x,t+1} - I_{x,t+1})/(\pi_{x,t} - I_{x,t}) - 1]}{S}
\]

which can be simplified to

\[
E_N' - E_N = \frac{I_{x,t+1} - I_{x,t}}{S(\pi_{x,t} - I_{x,t})} > 0
\]

Equation (10.20) means that \( E_N < E_N' \). This is the currency exposure hedging effect of debt financing when operating cash flows are positively related to changes in exchange rates. It tells us that foreign currency financing reduces net cash flow exposure.

Figures 10.1–10.3 illustrate these possibilities when \( \pi_y = 100 \) and \( S \) assumes values ranging between 1.00 and 2.00. Figure 10.1 illustrates the case when there is no foreign currency borrowing, in which case \( I_x \) is unchanged at 40. In this case, net cash flows increase with the exchange rate. We can see from Figure 10.1(b) that the operating exposure is constant at 1, the interest rate exposure is constant at 0, and the net cash flow exposure is always higher than the operating exposure (exposures are plotted against \( S \)). In Figure 10.2 there is foreign currency borrowing in which case all exposures turn out to be equal to one (pure conversion exposures). We can also see that the net cash flow exposure is lower than in the previous case. Figure 10.3 illustrates the case when the interest rate on the foreign currency, and hence the interest payments, increases with the exchange rate. In this case the operating exposure is constant at 1, whereas the net cash flow exposure can be positive or negative, depending on the rate of change of the exchange rate.

**Equity exposure**

Equity exposure is the sensitivity of base-currency equity value to changes in the exchange rate. The total value of a firm in base currency terms is equal to the value of debt plus equity. Hence

\[
V_x = E_x + D_x
\]

Then we can define total value exposure, equity exposure and debt exposure respectively as

\[
E_V = \frac{V_x}{S}
\]
FIGURE 10.1 Cash flows and exposures when base currency interest payments do not change with the exchange rate.

\[
E_E = \frac{\dot{E}_x}{S} \quad (10.23)
\]

\[
E_D = \frac{\dot{D}_x}{S} \quad (10.24)
\]

\[V_x \text{ may be regarded as the present value of the operating cash flows, which gives}
\]

\[V_x = \frac{\pi_x}{k} \quad (10.25)
\]

where \( k \) is the cost of capital. If \( k \) is independent of the exchange rate, it follows that \( E_V = E_{\pi} \). Hence, \( E_{\pi} = E_E \) in the case of all-equity financing.

Now, consider the case of base currency debt financing, such that \( E_D = 0 \) and \( D_{x,t+1} = D_{x,t} \). The percentage change in the firm’s total value is a weighted average of the percentage changes in debt and equity, which gives
If we divide equation (10.26) by \( \hat{S} \), we obtain

\[
\frac{\hat{V}_x}{\hat{S}} = \frac{\hat{D}_x}{\hat{S}} \left( \frac{D_{x,t}}{V_{x,t}} \right) + \frac{\hat{E}_x}{\hat{S}} \left( \frac{E_{x,t}}{V_{x,t}} \right) \tag{10.27}
\]

or

\[
E_{\pi} = E_D \left( \frac{D_{x,t}}{V_{x,t}} \right) + E_E \left( \frac{E_{x,t}}{V_{x,t}} \right) \tag{10.28}
\]

Since \( E_D = 0 \), it follows that
FIGURE 10.3 Cash flows and exposures when foreign currency interest payments change with the exchange rate.

\[ E_E = \frac{E_\pi}{1 - (D_{x,t} / V_{x,t})} \]  

(10.29)

Hence there is a nonlinear relationship between equity exposure and the debt ratio. Figure 10.4 shows this relationship for \( E_\pi = 1 \). We can also see that equity exposure is related positively to both the operating exposure and the debt ratio if the operating exposure is positive because

\[ \frac{\partial E_E}{\partial E_\pi} = \frac{1}{1 - (D_{x,t} / V_{x,t})} > 0 \]  

(10.30)

and

\[ \frac{\partial E_E}{\partial (D_{x,t} / V_{x,t})} = \frac{E_\pi}{[1 - (D_{x,t} / V_{x,t})]^2} > 0 \]  

(10.31)
Consider now the case of debt financing in the currency of exposure ($y$). We will for this purpose assume that the value of debt in currency $y$ remains unchanged as the exchange rate changes, which means that $E_D = 1$. Thus, equation (10.28) becomes

$$E_x = \frac{D_{x,t}}{V_{x,t}} + E \left( \frac{E_{x,t}}{V_{x,t}} \right)$$

which gives

$$E_E = \frac{E_x - (D_{x,t}/V_{x,t})}{1 - (D_{x,t}/V_{x,t})}$$

FIGURE 10.4 Equity exposure as a function of the debt ratio for a zero debt exposure.

FIGURE 10.5 Equity exposure as a function of the debt ratio for a unit debt exposure.
in which case $\partial E_E / \partial E_\pi > 0$, as in equation (10.29), and

$$
\frac{\partial E_E}{\partial (D_{x,t}/V_{x,t})} = \frac{-1 + E_\pi}{[1-(D_{x,t}/V_{x,t})]^2} 
$$

(10.34)

which can be anything, depending on the value of $E_\pi$. Equation (10.33) embodies two effects on equity exposure: (i) the currency exposure hedging effect (the use of more debt reduces equity exposure); and (ii) the financial leverage effect (higher exposure for higher debt ratio). The ultimate effect on equity exposure depends on how the two effects are combined. Figure 10.5 shows how equity exposure is related to the debt ratio for various values of the operating exposure.
11.1 OVERVIEW

International long-term portfolio investment is investment in long-term securities (bonds and equity) denominated in various currencies. The secondary equity markets of the world (also called stock markets or share markets) serve two purposes by providing marketability and equity valuation. Investors or traders who buy shares from the issuing firm in the primary market may not want to hold them indefinitely: the secondary market allows share owners to reduce their holdings of unwanted shares and purchasers to get the shares. Firms will have a difficult time attracting buyers in the primary market without the marketability provided through the secondary market. Competitive trading in the secondary market establishes fair market prices for existing issues.

International portfolio investment has, since the 1980s, been going through spectacular growth, which can be attributed to the following reasons:

1. Deregulation of financial markets.
2. The desire of international investors to improve performance, which can be accomplished via international diversification that enables them to obtain a better risk–return combination.
3. The advent of floating exchange rates. While floating rates produce more risk, they also provide an opportunity for a better risk–return combination. The exchange rate factor can contribute significantly to the overall base currency rate of return.
4. The incorporation of the academic literature on international portfolio diversification in the strategies of portfolio managers. Most of the academic research has become more concerned with practical issues, as academics strive to win consultancy work from financial institutions.
5. Modernisation and increased competitiveness of stock exchanges. For example, the New York Stock Exchange switched from fixed to flexible
commissions in 1975. In London there was the Big Bang in the 1980s that made the London Stock Exchange truly international. In general, there has been competition between stock exchanges to attract listings.

6. The development of technology and online trading has made it easier to buy and sell foreign securities.

7. The trend towards more disclosure and provision of information.

8. Expanded pool of liquidity. The growth of the Eurocurrency market and national money markets has led to an expanded international pool of liquidity that is available for international investors, enhancing their ability to finance portfolios.

A question may arise here as to why international investment would need such a diversified menu of factors to grow, in the sense that the diversification benefit of international portfolio investment could be adequate to propel growth. However, international portfolio investment is different from domestic portfolio investment in many respects. The following are some of these differences:

1. Differences in risk and perceptions of risk, including business risk, economic risk and liquidity risk. Furthermore, international investment is subject to foreign exchange risk.

2. Differences in market mechanisms, which can be an obstacle to investment. For example, some markets are illiquid, whereas others are characterised by a concentration of activity. Furthermore, some stock exchanges do not have a trading monopoly, and there are invariably differences in the settlement and how long it takes. Typically, there are some differences in transaction costs and government regulations.

3. Differences in the availability and quality of available information, including disclosure.

4. Differences in accounting standards, which arise from (i) lack of agreement on the objectives of financial standards; (ii) different requirements under the company laws of individual countries; (iii) differences in tax laws; and (iv) differences in the development of the local professional bodies. Specific differences in accounting standards pertain to consolidation practices, disclosure, foreign exchange accounting, auditing and accounting for inflation.

11.2 INVESTMENT IN BONDS

The secondary bond market
The secondary bond market comprises market makers and brokers connected by an array of telecommunication equipment. Market makers stand ready to buy and sell for their own accounts by quoting two-way bid and ask (offer) prices. Market makers trade directly with one another, through a broker, or
with retail customers, but electronic trading (online trading) is making
grounds at the expense of more traditional trading.

Market makers tend to be the same investment banks, merchant banks and
commercial banks that serve as lead managers in an underwriting of bond
issues. Brokers accept buy or sell orders from market makers, attempting to
find a matching party for the other side (they may also trade for their
accounts). Brokers charge a small commission for their services to the market
maker that engages them. They do not deal directly with retail clients.

Secondary market transactions require a system for transferring ownership
and payment from one party to another. A clearing system would have a group
of depository banks that physically store bond certificates. When a transaction
is conducted, book entries are made to transfer the ownership of the bond
certificates from the seller to the buyer and transfer funds from the buyer’s cash
account to the seller’s account. Physical transfer of the bond seldom takes place.

Bonds are fixed-income securities that are regarded to be attractive invest­
ment vehicles by international investors. The following are some features of
the bond market:

1. The world bond market is larger than the world stock market, offering
   sought-after opportunities for large institutional investors.
2. Bond markets are driven by a somewhat different mix of factors than stock
   markets. Therefore there are different timing patterns.
3. Bond markets offer opportunities for selective risk reduction (currency risk,
   sectoral risk), as well as broader portfolio risk reduction.
4. Investment in bonds can be combined with programmes aimed at currency
   risk neutralisation via currency derivatives.
5. Bond markets offer investors favourable liquidity due to high volume.

Currency denomination of bond investments
Let us for simplicity of exposition assume that we are dealing with investment
in zero coupon bonds. If a bond with a face value $V_{t+n}$ and maturity of $n$ years
is bought at time $t$ at a price $P_t$ and held until maturity, then the bondholder
will at year $n$ receive the face value $V_{t+n}$. The annual compound rate of return, $r$, on this bond investment can be calculated from the equation

$$P_t(1+r)^n = V_{t+n}$$  \hspace{1cm} (11.1)

in which case $r$ is given by

$$r = \left( \frac{V_{t+n}}{P_t} \right)^{1/n} - 1$$  \hspace{1cm} (11.2)

In general, if a base currency amount, $K$, is invested for $n$ years in a domestic
bond (or a domestic bond portfolio) offering an implicit rate of return, $r_x$, then
the value of the investment after $n$ years is given by
\[ I_{x,t+n} = K(1 + r_x)^n \] (11.3)

We could derive a similar expression for a foreign currency bond investment. The base currency amount, \( K \), is converted into the foreign currency at the spot exchange rate prevailing at time \( t \), \( S_t \), to obtain \( K/S_t \) units of the foreign currency. This amount is then used to buy foreign currency-denominated bonds offering an annual rate of return of \( r_y \). The foreign currency amount accumulated after \( n \) years is obtained by compounding the initial amount at \( r_y \) to obtain \( K/S_t(1 + r_y)^n \). The base currency value of this investment at year \( n \), \( I^*_{x,t+n} \), is obtained by converting the foreign currency amount into the base currency at the spot exchange rate prevailing at year \( t + n \), \( S_{t+n} \), to obtain

\[ I^*_{x,t+n} = K(1 + r_y)^n \left[ \frac{S_{t+n}}{S_t} \right] \] (11.4)

We could express the ratio of the exchange rate at \( t + n \) to the exchange rate at year \( t \), \( S_{t+n}/S_t \), in terms of the average annual rate of change in the exchange rate, \( \dot{S} \), as follows

\[ \frac{S_{t+n}}{S_t} = (1 + \dot{S})^n \] (11.5)

Therefore

\[ I^*_{x,t+n} = K(1 + r_y)^n (1 + \dot{S})^n \] (11.6)

Now we are in a position to make a choice between investing in base currency-denominated bonds and foreign currency-denominated bonds. Assuming risk neutrality (in the sense that the investor is indifferent between the two investments if they offer the same return) foreign currency-denominated bonds will be preferred if \( I^*_{x,t+n} > I_{x,t+n} \), or if

\[ K(1 + r_y)^n (1 + \dot{S})^n > K(1 + r_x)^n \] (11.7)

which can be approximated, by working out the expression and ignoring the small cross products, to

\[ \dot{S} > r_x - r_y \] (11.8)

Equation (11.8) says that investment in foreign currency-denominated bonds will be preferred if the foreign currency is expected to appreciate by more than the difference between the rates of return on base currency-denominated bonds and foreign currency-denominated bonds. The equation also implies that foreign currency-denominated bonds will be preferred even if they offer a lower rate of return than domestic currency-denominated bonds, provided that the foreign currency appreciates by more than the differential return, \( r_x - r_y \).

It is important to bear in mind that the two rates of return (\( r_x \) and \( r_y \)) are known at time \( t \) if the bonds are held until maturity, since the coupon
payments and the face values are known in advance. However, the change in the exchange rate is not known in advance, in which case the investor has to act on the basis of the expected change in the exchange rate. At \( t + n \), the value of the change in the exchange rate is realised, at which time the investor can determine, ex post, whether or not the right decision was made at time \( t \).

The effect of taxes

We now compare the after-tax returns on domestic currency and foreign currency bond investments. Two kinds of taxes are relevant to the return on bond investment: income tax and capital gains tax. Income tax is applied to interest income, whereas capital gains tax is applied to capital gains. If the bonds are held until maturity, then the return on base currency-denominated bonds takes the form of interest income, which makes this return subject to income tax only. The return on foreign currency-denominated bonds consists of interest income and the appreciation of the foreign currency, which occurs when the exchange rate rises (\( S_{t+n} > S_t \)). Hence the return on foreign-currency denominated bonds is subject to capital gains tax in addition to income tax. The condition represented by (11.8) can be modified to take into account the effect of taxes, to the following:

\[
(1 - \tau_g) S > (1 - \tau_n) r_x - (1 - \tau_n) r_y
\]  
(11.9)

where \( \tau_g \) is the capital gains tax applicable to foreign exchange gains and \( \tau_n \) is the income tax rate applicable to interest income. For the time being, we will assume that the domestic and foreign tax rates are identical. By manipulating equation (11.9) we obtain the following rule: foreign currency-denominated bonds will be preferred if

\[
S > \frac{1 - \tau_n}{1 - \tau_g} (r_x - r_y)
\]  
(11.10)

If \( \tau_g < \tau_n \), then \((1 - \tau_n)/(1 - \tau_g) < 1\), which means that the right-hand side of (11.10) is smaller than the right-hand side of (11.8). Thus, the condition required to prefer foreign currency-denominated bonds can be more easily satisfied on an after-tax basis. For given rates of return on base currency and foreign currency-denominated bonds, a lower rate of appreciation of the foreign currency is required to prefer foreign currency-denominated bonds on an after-tax basis than on a before-tax basis.

If, on the other hand, \( \tau_g > \tau_n \), then \((1 - \tau_n)/(1 - \tau_g) > 1\), which means that, for given rates of return on base currency and foreign currency bonds, a higher rate of appreciation of the foreign currency is required to prefer foreign currency-denominated bonds on an after-tax basis than on a before-tax basis. The reason for this result is simple and intuitive. Foreign currency appreciation is taxed at the capital gains tax rate, not at the income tax rate. Thus, a lower capital gains tax rate encourages investing in bonds denominated in currencies that are expected to appreciate.
Let us now consider the case when there are different tax rates and assume that tax is applied only in the country where the investment is undertaken. We will assume that there are two different income tax rates, but no capital gains tax. In this case, the interest income derived from base currency bonds is taxed at the domestic income tax rate, \( t_n \), whereas the interest income derived from foreign currency-denominated bonds is taxed at the foreign income tax rate, \( t^*_n \). Thus, the condition given by (11.9) will change to

\[
S > (1 - t_n) r - (1 - t^*_n) r_y
\]

which says that foreign currency denominated bonds will be preferred to domestic currency denominated bonds if the foreign currency is expected to appreciate by more than the after-tax rate of return differential.

11.3 INVESTMENT IN EQUITIES

An investor buying equities in a company becomes a shareholder of that company who is entitled to dividend payments. These payments, however, are not contractual in the sense that they may or may not be paid even if the company makes profit. The company may simply decide, at the discretion of the board of directors, not to distribute any dividends and opt for retained or undistributed profit to finance further expansion. But even with this in mind, investors buy shares in anticipation of making profit through capital appreciation resulting from the rise in the price or the market value of the equity.

Investment in equity therefore provides returns in two forms: dividends and capital appreciation. Thus

\[
R = d + a
\]

where \( R \) is the total rate of return on equity investment, \( d \) is the dividend yield and \( a \) is the rate of capital appreciation (the rate of change in equity prices), both of which are measured in percentage terms. If a base currency amount, \( K \), is invested in domestic equities for \( n \) periods (years), and assuming that dividend payments can be reinvested at the same underlying rate of return, then the value of the invested capital at year \( t + n \), \( I_{x,t+n} \), will be given by

\[
I_{x,t+n} = K(1 + R_x)^n
\]

or

\[
I_{x,t+n} = K(1 + d_x + a_x)^n
\]

Let us now consider what happens if the same amount is invested in foreign equities. The amount \( K \) is converted at the spot exchange rate, \( S_t \), to obtain \( K/S_t \) units of the foreign currency. The foreign currency value of the investment after \( n \) years is obtained by compounding the foreign currency amount invested at the rate of return on foreign equities. This amount is then re-
converted into the base currency at the exchange rate prevailing then, $S_{t+n}$. Thus, we obtain

$$I_{x,t+n}^* = \frac{KS_{t+n}}{S_t}(1 + d_y + a_y)^n$$

(11.15)

where $d_y$ and $a_y$ are the dividend yield and the rate of capital appreciation associated with foreign equity investment. Equation (11.15) can be rewritten as

$$I_{x,t+n}^* = K(1 + \hat{S})(1 + d_y + a_y)^n$$

(11.16)

Assuming risk neutrality, foreign investment will be preferred if

$$I_{x,t+n}^* > I_{x,t+n}$$

or

$$K(1 + \hat{S})^n(1 + d_y + a_y)^n > K(1 + d_x + a_x)^n$$

(11.17)

which can be approximated by working out the expressions and ignoring the small cross product terms to obtain

$$\hat{S} + d_y + a_y > d_x + a_x$$

(11.18)

or

$$\hat{S} > (d_x - d_y) + (a_x - a_y)$$

(11.19)

which means that foreign equity investment would be preferred even if it offers lower dividend yield and rate of capital appreciation than domestic equity investment. This would be the case if the foreign currency appreciates by more than the sum of the dividend yield differential and the capital appreciation rate differential. Notice, however, that at the time when the decision concerning the choice is made, the values of these variables are unknown, so the decision should be made on their expected or ex ante values. At $t + n$, however, the values of the variables are realised, which enables the investor to find out whether or not the right decision was made at time $t$.

**The effect of taxes**

We now compare the after-tax returns on domestic and foreign equity investments. In this case, income tax is applied to dividends, whereas capital gains tax is applied to capital gains. In the presence of taxes, the after-tax rate of return on domestic equity investment is given by

$$R_x = (1 - \tau_n)d_x + (1 - \tau_g)a_x$$

(11.20)

The after-tax rate of return on foreign equity investment is given by

$$R_x^* = (1 - \tau_n)d_y + (1 - \tau_g)(a_y + \hat{S})$$

(11.21)

which shows that the capital gains tax applies to the appreciation component and the foreign exchange gains. Thus, foreign equity investment is preferred on an after-tax basis if $R_x^* > R_x$ or if
(1 − \tau_n)dy + (1 − \tau_g)(ay + \delta) > (1 − \tau_n)dx + (1 − \tau_g)ax \quad (11.22)

which can be modified to

\delta > \frac{1 − \tau_n}{1 − \tau_g}(dx − dy) + (ax − ay) \quad (11.23)

Let us now consider what happens if \tau_g < \tau_n. In this case, \frac{(1 − \tau_n)}{(1 − \tau_g)} < 1, so the right-hand side of equation (11.23) will be smaller than the right-hand side of equation (11.19). Thus, the condition required to prefer foreign equity investment can be more easily satisfied on an after-tax basis. For given domestic and foreign dividend yields and rates of capital appreciation, a lower rate of appreciation of the foreign currency is required to prefer foreign equity investment on an after-tax basis than on a before-tax basis. If, on the other hand, \tau_g > \tau_n, then a higher rate of appreciation of the foreign currency is required for the investor to prefer foreign equity on an after-tax basis.

Assume now that there are different income tax rates and that the capital gains tax applies to capital appreciation in the same currency only (that is, it does not apply to foreign exchange gains). Equation (11.21) becomes

\begin{align*}
R^*_x &= (1 − \tau_n^*)dy + (1 − \tau_g^*)ax + \delta \\
\delta > &\frac{1 − \tau_n}{1 − \tau_g}(dx − dy) + (ax − ay) \quad (11.24)
\end{align*}

in which case, foreign equity investment is preferred if

(1 − \tau_n^*)dy + (1 − \tau_g^*)ax + \delta > (1 − \tau_n)dx + (1 − \tau_g)ax \quad (11.25)

which can be rearranged to produce the condition

\delta > (1 − \tau_n)dx − (1 − \tau_n^*)dy + (1 − \tau_g)ax − (1 − \tau_g^*)ay \quad (11.26)

Equation (11.26) says that foreign equity investment is preferred to domestic equity investment if the foreign currency is expected to appreciate by more than the sum of the after tax dividend yield differential and the after-tax capital appreciation rate differential.

**Investment vehicles and portfolio management styles**

There are five alternative investment vehicles for foreign equity investment, which will be described in turn.

**Direct purchase of securities in overseas markets**

The firm carrying out the purchase or sale of foreign securities must have a branch or correspondent securities firm authorised to deal on the foreign stock exchange. When a firm buys individual foreign stocks, it would be exposed to four different kinds of risk: (i) country risk, which relates to economic and political events that have adverse effects on prices or the liquidity of the market; (ii) foreign exchange risk, which results from fluctuations in exchange rates; (iii) systematic risk, or market risk, which is the risk
that cannot be eliminated via diversification; and (iv) unsystematic risk, which refers to the variability of the return on a single security.

The use of American Depository Receipts (ADRs)
An ADR is a receipt issued by a US bank certifying that the bank holds an equivalent number of shares issued by a foreign company. Securities brokers provide the market-making function in ADRs. The market price of an ADR (in US dollar terms) is the price of the underlying security in foreign currency terms multiplied by the exchange rate adjusted for the number of shares included in one ADR (say, ten). ADRs have two advantages over the direct purchase of equities: (i) avoidance of foreign exchange transaction costs; and (ii) ability to trade the foreign securities during regular stock market trading hours of the investor’s stock market.

Single country funds
Single country funds trade the shares of companies of a single country. The majority of country funds have a closed end status. A closed end fund issues a given number of shares that are traded on the stock exchange as if the fund were an individual stock by itself. Unlike the shares of an open-end fund, shares of a closed-end country fund cannot be redeemed, and the underlying net asset value is set at the home market of the fund. The share value of a fund may very well diverge from the underlying net asset value in the fund’s home market. The difference is known as the premium/discount.

International funds
International funds trade the shares of foreign companies belonging to several countries. The risk related to international funds is called “modified systematic risk” (international diversification reduces risk below the level of risk experienced in most or all national markets).

Global funds
Unlike international funds, global funds also trade domestic shares. In the USA, international funds are defined to include no more than 24% in US securities, whereas global funds have 50%. These funds are issued by large organisations. They tend to be cost-effective in that they sell and redeem shares at net asset value.

International portfolio management may be conducted by following one of three approaches. The first is passive indexation, whereby international fund managers concentrate on a large number of large capitalisation share issues belonging to leading market indices. The second is the tactical asset management strategy, which is based on a broad economic analysis of the leading market indices. The third is the global approach, which is based on measuring risk, return and correlation in terms of the investor’s base currency.

Passive management has its theoretical support in the efficient market hypothesis. If the market has priced securities according to all available
information, and if the market responds to newly arriving information in a rational manner, then it may be difficult, if not impossible, to outperform the market. Active management, on the other hand, requires considerable analysis and forecasting. The forecaster must provide detailed forecasts of the (i) national economic trends and their influence on each national market in terms of risk and return; and (ii) industrial trends on a worldwide basis, together with their implications for risk and return of specific company shares on a worldwide basis.

There is also modified active management, which requires a basic amount of forecasting for risk return and correlation of the securities or the national indices. Based on this information set, an efficient frontier can be estimated and used as a basis for portfolio selection and construction.

11.4 INTERNATIONAL EQUITY RETURNS AND DIVERSIFICATION

Returns on equity investment vary across markets because of cross-country differences, the first set of which pertain to macroeconomic factors. Solnik (1984) examined the effect on equity returns of exchange rate changes, interest rate differentials, the level of domestic interest rates, and changes in domestic inflationary expectations. He found that international monetary variables had only a weak influence on equity returns in comparison with domestic variables. In another study, Asprem (1989) found that industrial production, employment, imports, interest rates and inflation explained only a small portion of the variability of equity returns for 10 European countries. However, he also found that substantially more of the variation was explained by an international market index.

The role of exchange rates is also important. Adler and Simon (1986) examined the exposure of a sample of foreign equity and bond index returns to exchange rate changes. They found that changes in exchange rates generally explained a larger portion of the variability of foreign bond indexes than foreign equity indexes, but that some equity markets were more exposed to exchange rate changes than were the respective foreign bond markets. In another study, Eun and Resnick (1988) found that the cross-correlations among major stock markets and exchange markets are relatively low, but positive. The result implies that exchange rate changes in a given country reinforce the stock market movements in that country as well as in the other countries examined.

Differences in industrial structures also play a role, although the studies examining this factor are inconclusive. Roll (1992) concluded that the industrial structure of a country is important in explaining a significant part of the correlation structure of international equity index returns. He also found that
industry factors explained a larger portion of stock market variability than did exchange rate changes. In contrast, Eun and Resnick (1984) found, for a sample of 160 stocks from eight countries and 12 industries, that the pairwise correlation structure of international security returns could better be estimated from models that recognise country factors rather than industry factors. Similarly, using individual stock return data for 829 firms from 12 countries and representing seven broad industry groups, Heston and Rouwenhorst (1994) concluded that “industrial structure explains very little of the cross section differences in country return volatility”. Overall, these empirical studies imply that there are unique country factors (the level of domestic interest rate and expected inflation) that explain national equity returns. Because of these unique factors there are benefits to be gained from international diversification.

International portfolio managers do not consider return only but also risk. It is a well-known principle in portfolio theory that diversification reduces risk if the rates of return on the assets from which a portfolio is composed are less than perfectly correlated. Because the economies of various countries differ in many respects, they are likely to be passing through different phases of the business cycle at the same point in time. The implication of these differences for the issue under discussion here is that rates of return in different countries are likely to be less positively correlated than those from different sectors within the same economy. This point has been established by Lessard (1976), Levy and Sarnat (1970) and Solnik (1974). Table 11.1 contains a correlation matrix of the quarterly rates of return on equity investment in the USA, Japan, the UK and Australia over the period January 1988–2001, with and without the exchange rate factor. When the exchange rate factor is taken into account the correlation matrix is reported from four national perspectives, in which case the national currency of the underlying country is taken to be the base currency of the investor. Although the rates of return are predominantly positively correlated, the correlation coefficients are adequately low to allow for some benefits from diversification.

We could write the rate of return on foreign equity investment measured in base currency terms as

$$R_x = A_y + \hat{S}$$

(11.27)

where $A_y = d_y + a_y$ is the total rate of return in foreign currency terms. The variance of $R_x$ is given by

$$\sigma^2 (R_x) = \sigma^2 (A_y) + \sigma^2 (\hat{S}) + 2\sigma (A_y, \hat{S})$$

(11.28)

which means that the variance of the base currency rate of return can be decomposed into three components: the variance of the rate of return in foreign currency terms, the variance of the percentage change in the exchange rate and (twice) the covariance of these two components. Table 11.2 reports the results of this decomposition of the variance of quarterly base currency
rates of return on equity investment from various national perspectives over the period 1988–2001. It is obvious that in all cases the variability of the rate of return in foreign currency terms is a major contributor to the variability of the base currency rate of return.

Let us see what happens when we combine a domestic security or portfolio, which gives a rate of return $R$, with a foreign security or portfolio that gives a rate of return $R^*$, both of which are measured in base currency terms (the subscript $x$ is not shown for convenience). If the weights assigned to the two securities are $w$ and $w^*$, then the expected (or the average) value of the rate of return on the portfolio and its variance are given respectively by

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### TABLE 11.2 Decomposition of the variances of base currency rates of return on foreign equity investment.

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<tr>
<th></th>
<th>$\bar{A}_y$</th>
<th>$\bar{S}$</th>
<th>$\bar{R}_x$</th>
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<th>$\sigma^2(S)$</th>
<th>$\sigma^2(R_x)$</th>
<th>$\sigma(A_y, S)$</th>
<th>$\rho(A_y, S)$</th>
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<td>33.1</td>
<td>–</td>
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<td>–0.78</td>
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<td>45.0</td>
<td>145.0</td>
<td>3.24</td>
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<td>1.61</td>
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<td>49.0</td>
<td>–11.91</td>
<td>–0.35</td>
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<td>0.63</td>
<td>39.3</td>
<td>21.8</td>
<td>74.0</td>
<td>6.44</td>
<td>0.22</td>
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<td>81.6</td>
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<td>–</td>
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<tr>
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<td>94.3</td>
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<td>39.3</td>
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\[ E(R_p) = wR + w^* R^* \]  
\[ \sigma^2(R_p) = w^2 \sigma^2(R) + w^{*2} \sigma^2(R^*) + 2ww^* \sigma(R)\sigma(R^*)\rho(R, R^*) \]  

where \( \rho(R, R^*) \) is the correlation coefficient between the base currency and foreign currency rates of return. The measure of risk is the standard deviation of the rate of return, which is the square root of the variance. Hence

\[ \sigma(R_p) = \sqrt{w^2 \sigma^2(R) + w^{*2} \sigma^2(R^*) + 2ww^* \sigma(R)\sigma(R^*)\rho(R, R^*)} \]  

By using equation (11.31), we can demonstrate that the reduction in risk via diversification depends upon the correlation coefficient of the rates of return. If the rates of return are perfectly correlated (that is, \( \rho(R, R^*) = 1 \)), then the standard deviation of the rate of return on the portfolio is given by

\[ \sigma(R_p) = \sqrt{w\sigma(R) + w^* \sigma(R^*)}^2 = w\sigma(R) + w^* \sigma(R^*) \]  

which is a weighted average of the standard deviations of the individual securities or portfolios. Hence diversification does not lead to a reduction in risk if
11.4 INTERNATIONAL EQUITY RETURNS AND DIVERSIFICATION

the rates of return are perfectly correlated. This case is unlikely to arise in practice, even more so if one of the securities is domestic and the other is foreign. It is more likely the case that the rates of return are less than perfectly correlated (that is, \( \rho(R, R^*) < 1 \)). If the correlation coefficient is zero we have

\[
\sigma(R_p) = \sqrt{w^2 \sigma^2(R) + w^{*2} \sigma^2(R^*)} < w\sigma(R) + w^* \sigma(R^*)
\] (11.33)

which shows that the standard deviation of the rate of return on the portfolio is less than the weighted average of the two standard deviations. This means that the portfolio risk is lower when the securities are uncorrelated.

International diversification is most beneficial in terms of risk reduction if the domestic and foreign securities are negatively correlated. In the extreme case when they are perfectly negatively correlated (that is, \( \rho(R, R^*) = -1 \)), the standard deviation of the rate of return on the portfolio is given by

\[
\sigma(R_p) = |w\sigma(R) - w^* \sigma(R^*)| = w\sigma(R) - w^* \sigma(R^*)
\] (11.34)

which is the lowest value that can be assumed by the standard deviation of the rate of return on the portfolio.

For a given value of the correlation coefficient, it is possible to construct a large number of portfolios from the domestic and foreign securities by assigning different values to the weights \( w \) and \( w^* \). For each portfolio there is a combination of the expected rate of return as given by equation (11.29) and the standard deviation as given by equation (11.31). International diversification leads to a situation in which the same level of return can be achieved at a lower level of risk, or a higher rate of return can be achieved at the same level of risk. This is more so than what results from diversification within the same market.

**Empirical evidence on international diversification**

Various researchers have documented evidence on the extent to which portfolio investment is concentrated in domestic equities. These include French and Porteba (1991) and Cooper and Kaplains (1994). This runs counter to the strand of research that collectively established a strong case for international diversification (including Grubel (1968), Solnik (1974), Lessard (1976) and Eun and Resnick (1988)). Several explanations can be put forward to resolve this inconsistency, including the following:

1. Domestic securities may provide investors with certain extra services, such as hedging against domestic inflation. This may not be convincing, given that equities are not a good inflation hedge (bonds may do a better job in this respect). Cooper and Kaplains (1994) rule out inflation hedging as a primary cause for home bias.
2. There may be barriers, formal or informal, to investing in foreign securities. For example, there may be some restrictions on share holding by foreigners.
3. Some investors may not invest more than a certain percentage of their funds in foreign securities.
4. The presence of taxes and transaction/information costs.
5. Investors tend not to hold securities with which they are not familiar.

Although the risk reduction available from international diversification in equity is well documented, much less research has been devoted to diversification in bonds. Levy and Lerman (1988) have investigated this issue in an attempt to answer three questions pertaining to: (i) the extent to which international diversification among bonds can produce returns in excess of those available only in domestic bonds; (ii) the possibility of constructing international diversified portfolios, despite the relatively low mean returns of bonds compared to equity; and (iii) the impact of diversification on portfolios made up of bonds and equities from various markets. They found that US bond investors were in a position to improve their performance by 3 to 5 percentage points a year by diversifying internationally rather than restricting their investments to domestic bonds. This result is attributed to the low correlations between the bond markets in various countries compared with correlations among equity markets. They also found a very large potential for international diversification in equities and bonds.

Another related issue is diversification by including emerging equity markets. By using 24 years of data, Conover et al. (2002) suggest that emerging equity markets are a worthy addition to a US investor’s portfolio of developed market equities. Specifically, they found that portfolio returns increased by approximately 1.5 percentage points a year when emerging country equities were included in the portfolio. They also found that the benefits of investing in emerging markets accrued almost exclusively during periods of restrictive US monetary policy (otherwise they were trivial). They further suggested that evaluating monetary conditions is a necessary prerequisite for identifying an optimal allocation of assets to international equities.

### 11.5 INTERNATIONAL CAPITAL ASSET PRICING MODEL

The conventional or domestic capital asset pricing model (CAPM) postulates that the expected return on an asset or portfolio is positively related to its systematic risk, the component of risk that cannot be eliminated by diversification. The relationship can be written as

\[ R_j = i + \beta (R_m - i) \]  

(11.35)

where \( R_j \) is the equilibrium or required expected rate of return on a security or a portfolio \( j \), \( i \) is the risk-free interest rate and \( R_m \) is the expected rate of return on the market portfolio, such as the portfolio implied by a stock market index. \( \beta \) can be calculated as a regression coefficient from the equation
\[
\beta = \frac{\sigma(R_j, R_m)}{\sigma^2(R_m)}
\]  

(11.36)

where \(\sigma(R_j, R_m)\) is the covariance of the rates of return on the portfolio and the market, whereas \(\sigma^2(R_m)\) is the variance of the rate of return on the market. Equation (11.35) tells us that the expected return on a security or a portfolio is equal to the risk-free rate plus a risk premium that is linearly related to a measure of systematic risk, \(\beta\), the latter being the risk that the security or the portfolio contributes to the market as a whole. Investors are, therefore, compensated for bearing systematic risk only. If the expected return is greater than is implied by this equation, the underlying security would be very attractive and investors would rush to buy it, raising its price and lowering its return.

From equation (11.35) we can see the following:

1. A security with a zero \(\beta\) has a rate of return that is equal to the risk-free interest rate.
2. A security with a \(\beta\) of less than one (less risky than the market portfolio) has an expected return higher than the risk-free rate but lower than the expected return on the market portfolio.
3. A security with a systematic risk that is equal to that of the market portfolio (\(\beta = 1\)) has an expected rate of return that is equal to the return on the market portfolio.
4. If the security is more risky than the market portfolio (\(\beta > 1\)) then it will offer an expected rate of return that is higher than what is offered by the market portfolio.

The **CAPM in a global setting**

To apply the CAPM in a global setting, several issues arise. The first issue concerns the definition of the market portfolio: should it be the market portfolio in the base currency, the market portfolio in a foreign currency, a combination of the two portfolios, or a global market portfolio? The second issue concerns changes in exchange rates: should these changes be included in the market portfolio’s rate of return? Then there is the choice of the appropriate risk-free rate, which differs from one country to another. Finally, there is the issue of exposure to foreign exchange risk: should currency exposure be taken into account as a risk factor that requires an adjustment in the expected rate of return?

These issues have been addressed by the global asset pricing model (GAPM) (see for example, Dumas, 1993). This model is based on the idea of international diversification of portfolios, which is what is practised in reality. Given relatively thorough international diversification, the model is used to determine the risk-adjusted required rate of return from the perspective of a certain currency. The model may be written as
\[ k_j = i + \beta_g (k_g - i) + \beta_q (k_q - i) \]  \hspace{1cm} (11.37)

where \( \beta_g \) is the measure of the sensitivity of the asset’s rate of return relative to the rate of return on the global market portfolio, \( \beta_q \) is the sensitivity to the rate of change in wealth-weighted index of exchange rates, \( k_g \) is the expected rate of return on the global market portfolio measured in base currency terms and \( k_q \) is the expected rate of change in the domestic currency value of the wealth-weighted portfolio of other currencies.

Equation (11.37) is applicable to any particular currency regardless of the currency denomination of the asset. This is because \( k_j \) in this case is the expected rate of return expressed in base currency terms. The risk-free rate is the rate of return on base currency risk-free assets. There are two differences between equations (11.35) and (11.37). The first difference is that the market portfolio in equation (11.35) becomes the global market portfolio in equation (11.37). The second and more important difference is that equation (11.37) embodies two risk factors: one for market risk and the other for currency exposure risk. Thus, the GAPM shows that an asset has two betas, one measuring market risk and the other measuring currency risk. These two betas must be measured simultaneously by using multiple regression analysis.

As we have seen, the rate of return on a single asset in terms of a particular currency (that is different from the currency of denomination of the asset) consists of two components: the rate of return in terms of the currency of denomination and the rate of appreciation (or depreciation) of this currency against the base currency (the currency in which \( k_j \) is measured). The global market portfolio is some sort of weighted average of these returns. In practice, the global rate of return is calculated from the value of a world stock index measured in a particular currency. Similarly, the rate of change in the base currency can be calculated from the effective exchange rate of the base currency. The GAPM can be used to calculate the expected rate of return on a foreign project. It can be used to find out the rate for either domestic currency or foreign currency analysis.

The GAPM can also be used to forecast the expected change of the exchange rate of the foreign currency in terms of the base currency. For two currencies, \( x \) and \( y \), such that the base currency is \( x \), we have

\[ \dot{S}^e(x/y) = i_x - i_y + \beta_g (k_g - i_x) + \beta_q (k_q - i_x) \]  \hspace{1cm} (11.38)

Equation (11.38) says the following. The expected change in the exchange rate depends on three factors: (i) the interest rate differential, \( i_x - i_y \); (ii) the difference between the rate of return on the global market portfolio and the interest rate on currency \( x \), \( k_g - i_x \); and (iii) the difference between the rate of change in the index of the exchange rate (the effective exchange rate of currency \( x \)) and the interest rate on currency \( x \), \( k_q - i_x \). If \( \beta_g = \beta = 0 \), then

\[ \dot{S}^e(x/y) = i_x - i_y \]  \hspace{1cm} (11.39)
which is uncovered interest parity (UIP).

Sometimes, the basic CAPM representation of equation (11.35) is modified by including an exchange rate factor. The modified model can be written in a stochastic time series form as

$$R_{j,t} = \beta_{0,j} + \beta_{1,j} R_{m,t} + \beta_{2,j} S_t + \varepsilon_{j,t}$$  \hspace{2cm} (11.40)

in which case $\beta_{2,j}$ measures a firm’s exposure to exchange rate movements after taking into account the overall market’s exposure to changes in the underlying exchange rate. If $\beta_{2,j} = 0$, this means that firm $j$ has the same exchange rate exposure as the market portfolio. If $\beta_{2,j} \neq 0$, then the conventional CAPM is misspecified. Dominguez and Tesar (2001) used this modified model to investigate exposure to foreign exchange risk. They obtained results that they found to be consistent with high degrees of exchange rate exposure at both the firm and industry level in eight countries.

**Market segmentation**

The implication of this analysis is that international diversification can produce abnormal returns if markets are segmented and the securities are priced according to the domestic CAPM using a domestic market portfolio as a benchmark. If, on the other hand, markets are integrated, securities are priced according to the international CAPM using an internationally diversified portfolio as a benchmark.

There are several reasons for market segmentation:

1. Legal barriers to foreign investment. These barriers may take the form of an outright restriction on investment by foreigners, or other forms such as the imposition of higher tax rates on foreigners investing in domestic assets.
2. The difficulty of finding and interpreting information about foreign securities, which may be due to the use of different accounting standards.
3. Foreign exchange risk. The problem with foreign exchange risk is that it may not be possible to hedge, perhaps because of the unavailability of forward contracts with long maturities or in certain currencies. Moreover, foreign exchange risk does not conform to the positive risk–return trade-off. Bearing more foreign exchange risk does not necessarily mean expecting higher return.
4. Purchasing power risk. Segmentation can arise because prices of what investors consume relative to the returns they earn change differently in different countries.
5. Political and country risk. Political risk pertains to changes in the rules governing foreign investment by the host government. Country risk encompasses all of the factors that can adversely affect its economic performance.
6. Transaction costs. These costs are higher when they involve foreign transactions. Some of these costs include the bid–offer spread in foreign
exchange, settlement costs and the custodial costs associated with buying and selling foreign securities.

7. Taxation. In the absence of double taxation agreements, investment returns may be taxed twice in the foreign and home countries.

8. Domestic regulations. Some countries impose restrictions on the ability of their citizens to invest in foreign securities. Recently, however, measures of financial deregulation have included the dismantling of these restrictions.

Two observations related to market segmentation may also be made here. First, although strong linkages have developed between the three largest equity markets (the USA, Japan and the UK), these linkages do not necessarily mean that they are fully integrated. They are at best partially integrated, primarily because of foreign exchange risk. The second observation is that leading bond markets display strong independent performance and variability due to the tendency for interest rates in national markets to move according to domestic trends (for example, internal and external balances).

11.6 MANAGING FOREIGN EXCHANGE RISK IN INTERNATIONAL PORTFOLIOS

International portfolio managers pay significant attention to the management of foreign exchange risk arising from investing in international portfolios. In this section we discuss issues related to the management of foreign exchange risk arising from international portfolio investment.

Approaches to hedging foreign exchange risk in international portfolios

Believing that internationalportfolio managers do not have sufficient expertise in the field of foreign exchange, some institutional investors have turned to specialised “overlay managers” to manage the foreign exchange risk separately. In general, there are three approaches to the management of foreign exchange risk in securities portfolios. The first is joint or full-blown optimisation over the assets and currencies. The second is partial optimisation over the currencies, given a predetermined position in the core portfolio. The third is separate optimisation over currencies. The first approach is based on the assumption that the portfolio manager has expertise in many asset classes and can structure a portfolio to account for correlations between asset prices and exchange rates. Approaches 2 and 3, on the other hand, are based on the assumption of foreign exchange risk management via an “overlay programme”. In approach 2, currencies are managed separately from the core portfolio, but the manager still controls the total portfolio risk. In approach 3, currencies are managed completely independently of the rest of the portfolio,
and their performance is measured against a separate benchmark (for example, money market instruments).

In terms of mean–variance optimisation, investors choose portfolio weights that maximise an objective utility function, \( U(\mu, \sigma^2) \), where \( \mu \) and \( \sigma^2 \) are respectively the mean and the variance of the underlying rate of return (on an asset, a portfolio or a hedging instrument), such that \( \partial U/\partial \mu > 0 \) and \( \partial U/\partial \sigma^2 < 0 \).

In approach 1, positions in assets and currencies are determined simultaneously in order to optimise the risk–return combination for the whole portfolio. Thus the decision problem can be written formally as

\[
\max_{w, w_j} [U(\bar{R}_p, \sigma^2(R_p))] \tag{11.41}
\]

where \( w_i \) is the weight assigned to asset \( i \), and \( w_j \) is the weight assigned to the hedging instrument in currency \( j \). In this case the arguments of the utility function are the mean and variance of the overall rate of return on the portfolio, \( R_p \). This would also be the case if the foreign exchange risk is not hedged.

Optimisation in approach 2 is conditioned on predetermined underlying asset positions. The asset weights are initially determined without regard to the hedging instrument, and then the currency weights of the hedging instrument are determined. Thus the two-step optimisation problem can be written as

\[
\begin{align*}
\max_{w_i} & \left[ U(\bar{R}_A, \sigma^2(R_A)) \right] \\
\max_{w_j} & \left[ U(\bar{R}_p, \sigma^2(R_p)|w_i) \right] 
\end{align*} \tag{11.42}
\]

where \( R_A \) is the rate of return on the asset.

In the third approach of separate optimisation, the asset and hedge weights are determined independently of each other. The weights assigned to assets and hedging instruments are found by solving two independent optimisation problems given by

\[
\begin{align*}
\max_{w_i} & \left[ U(\bar{R}_A, \sigma^2(R_A)) \right] \\
\max_{w_j} & \left[ U(\bar{R}_H, \sigma^2(R_H)) \right] 
\end{align*} \tag{11.43}
\]

where \( R_H \) is the rate of return on the hedging instrument. Jorion (1994) uses this type of analysis to conclude that the overlay structure is inherently suboptimal because it ignores interactions between the asset prices and exchange rates. On the basis of historical data, he estimated the efficiency loss to be something like 40 basis points.

**Matched hedging and basket hedging**

Hedging the foreign exchange risk arising from internationally diversified portfolios can be done by taking opposite positions in hedging instruments
such as futures, forwards and options. Let us concentrate on forward contracts. The manager of an internationally diversified portfolio with a base currency $x$ and long positions on securities denominated in currencies $y_1, y_2, ..., y_n$ can hedge his portfolio by selling these currencies forward against $x$ in amounts determined by their weights in the portfolio. This operation would lock in the future base currency value of the securities. Hedging single currency exposures separately is called matched hedging. This hedging strategy would be effective if the portfolio is denominated in a few major currencies. If a large number of currencies are involved, this operation will be costly, particularly if some exotic currencies are involved. Forward contracts on these currencies are typically characterised by wide bid–offer spreads. The alternative would be basket hedging, which consists of selling a few major currencies that can adequately represent the movements of the exchange rates of all of the currencies in the portfolio. Tucker et al. (2001, p. 98) believe that selling three major currencies would be adequate. The objective would then be to determine the weights $w_1, w_2$ and $w_3$ that minimise the variance of the exchange rate factor of the basket minus the exchange rate factor of the portfolio. Formally, the constrained optimisation problem is formulated as follows:

Minimise \[ \sum_{i=1}^{4} \sum_{j=1}^{4} w_i w_j \sigma(\hat{S}_i, \hat{S}_j) \] (11.45)

subject to

\[ \sum_{i=1}^{3} w_i = 1 \] (11.46)

\[ w_4 = -1 \] (11.47)

where $w_4$ is the weight of the portfolio. To solve this optimisation problem, we construct the Lagrange equation

\[ L = \sum_{i=1}^{4} \sum_{j=1}^{4} w_i w_j \sigma(\hat{S}_i, \hat{S}_j) + \lambda_1 (w_1 + w_2 + w_3 - 1) + \lambda_2 (w_4 + 1) \] (11.48)

Thus, we have

\[ \frac{\partial L}{\partial w_1} = 2w_1 \sigma^2 (\hat{S}_1) + 2\sigma(\hat{S}_1, \hat{S}_2) w_2 + 2\sigma(\hat{S}_1, \hat{S}_3) w_3 + 2\sigma(\hat{S}_1, \hat{S}_4) w_4 + 2\sigma(\hat{S}_1, \hat{S}_4) w_4 = 0 \] (11.49)

\[ \frac{\partial L}{\partial w_2} = 2w_2 \sigma^2 (\hat{S}_2) + 2\sigma(\hat{S}_2, \hat{S}_3) w_3 + 2\sigma(\hat{S}_2, \hat{S}_4) w_4 + 2\sigma(\hat{S}_2, \hat{S}_4) w_4 = 0 \] (11.50)
\[
\frac{\partial L}{\partial w_3} = 2w_3\sigma^2(\hat{S}_3) + 2\sigma(\hat{S}_1, \hat{S}_3)w_1 + 2\sigma(\hat{S}_2, \hat{S}_3)w_2 + 2\sigma(\hat{S}_3, \hat{S}_4)w_4 + \lambda_1 = 0
\]  
(11.51)

\[
\frac{\partial L}{\partial w_4} = 2w_4\sigma^2(\hat{S}_4) + 2\sigma(\hat{S}_1, \hat{S}_4)w_1 + 2\sigma(\hat{S}_2, \hat{S}_4)w_2 + 2\sigma(\hat{S}_3, \hat{S}_4)w_4 + \lambda_2 = 0
\]  
(11.52)

\[
\frac{\partial L}{\partial \lambda_1} = w_1 + w_2 + w_3 - 1 = 0
\]  
(11.53)

\[
\frac{\partial L}{\partial \lambda_2} = w_4 + 1 = 0
\]  
(11.54)

which gives the matrix equation
\[
Bw = C
\]  
(11.55)

where

\[
B = \begin{bmatrix}
2\sigma^2(\hat{S}_1) & 2\sigma(\hat{S}_1, \hat{S}_2) & 2\sigma(\hat{S}_1, \hat{S}_3) & 2\sigma(\hat{S}_1, \hat{S}_4) & 1 & 0 \\
2\sigma(\hat{S}_2, \hat{S}_1) & 2\sigma^2(\hat{S}_2) & 2\sigma(\hat{S}_2, \hat{S}_3) & 2\sigma(\hat{S}_2, \hat{S}_4) & 1 & 0 \\
2\sigma(\hat{S}_3, \hat{S}_1) & 2\sigma(\hat{S}_3, \hat{S}_2) & 2\sigma^2(\hat{S}_3) & 2\sigma(\hat{S}_3, \hat{S}_4) & 1 & 0 \\
2\sigma(\hat{S}_4, \hat{S}_1) & 2\sigma(\hat{S}_4, \hat{S}_2) & 2\sigma(\hat{S}_4, \hat{S}_3) & 2\sigma^2(\hat{S}_4) & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\]  
(11.56)

\[
w' = [w_1 \ w_2 \ w_3 \ w_4 \ \lambda_1 \ \lambda_2]
\]  
(11.57)

and

\[
C' = [0 \ 0 \ 0 \ 0 \ 1 \ -1]
\]  
(11.58)

Therefore, the weights for a basket hedge are calculated by solving the matrix equation (11.55) to obtain
\[
w = B^{-1}C
\]  
(11.59)

Thus, the amount of currency \(y_i\) (\(i = 1, 2, 3\)) sold forward is given by
\[
M_i = \frac{w_iK}{F(x/y_i)}
\]  
(11.60)

where \(K\) is the \(x\) currency value of the portfolio. For more detail, see DeRosa (1991).
Hedging currency risk in international portfolios when the base currency is pegged to a basket

Suppose that the base currency, $x$, is pegged to a basket. Recall from Chapter 3 that a portfolio manager with securities denominated in $k$ currencies will in this case give a total base currency rate of return that is given by

$$R_x = \sum_{j=1}^{k} A_j + C$$

(11.61)

where $A_j$ is the rate of return on a $y_j$-denominated currency and $C$ is the currency factor (the foreign exchange gain/loss). If the portfolio is chosen such that: (i) the currencies denominating the securities are identical to those of which the basket consists ($k = n$); and (ii) the weights in the portfolio are identical to the weights in the basket ($\alpha_j = \beta_j$ for $j = 1, 2, ..., n$), it follows that $C = 0$ (see Chapter 3). In this case, the variance of $R_x$ in equation (11.61) is determined entirely by the variance and covariances of the $A_j$s, which makes it independent of the currency factor. Thus, by constructing the international portfolio such that the weight of the security denominated in a particular currency is equal to the weight of the same currency in the basket, foreign exchange risk can be eliminated completely. Needless to say, this operation requires knowledge of the structure of the basket to which the base currency is pegged.

Empirical evidence

Figure 11.1 demonstrates the effect of international diversification and that of currency hedging. In general, as the number of securities from which the

![Figure 11.1](image-url)
portfolio consists increases, the return–risk ratio improves, though at a decreasing rate. The effect of international diversification is to make it possible to obtain a higher return–risk ratio for any given number of securities. The effect of currency hedging is to boost the effect of international diversification, allowing the portfolio manager to obtain even higher return–risk ratios for the same number of securities. This is in theory, but what does the empirical evidence tells us?

Several studies have suggested that hedging foreign exchange risk can reduce the variability of returns on international portfolios while having little impact on or even enhancing returns. These studies include Madura and Reiff (1985), Eun and Resnick (1988), Perold and Schulman (1988), Eaker and Grant (1991), Glen and Jorion (1993) and Levich and Thomas (1993a). For example, Madura and Reiff (1985) estimated the returns on country stick indices to determine the degree of risk reduction resulting from currency hedging. They found that the hedged portfolio had half the variance of a corresponding unhedged portfolio for the same level of return. However, Abken and Shrikhande (1997) have produced results indicating that the dominance of hedged portfolios during the period 1980–85 was reversed during the period 1986–96, when the dollar was generally depreciating.
12.1 DEFINITION AND CLASSIFICATION

Foreign direct investment (FDI) is the process whereby residents of one country (the source country) acquire ownership of assets for the purpose of controlling the production, distribution and other activities of a firm in another country (the host country). The International Monetary Fund’s *Balance of Payments Manual* defines FDI as “an investment that is made to acquire a lasting interest in an enterprise operating in an economy other than that of the investor, the investor’s purpose being to have an effective voice in the management of the enterprise”. The United Nation’s 1999 *World Investment Report* (UNCTAD, 1999) defines FDI as “an investment involving a long-term relationship and reflecting a lasting interest and control of a resident entity in one economy (foreign direct investor or parent enterprise) in an enterprise resident in an economy other than that of the foreign direct investor (FDI enterprise, affiliate enterprise or foreign affiliate)”. The term “long-term” is used in the last definition in order to distinguish FDI from portfolio investment, which we dealt with in the previous chapter. FDI does not have the portfolio investment characteristic of being short term in nature, involving a high turnover of securities.

The common feature of these definitions lies in words like “control” and “controlling interest”, which represent the most important feature that distinguishes FDI from portfolio investment, since a portfolio investor does not seek control or lasting interest. There is no agreement, however, on what constitutes a controlling interest, but most commonly a minimum of 10% shareholding is regarded as allowing the foreign firm to exert significant influence, either potentially or actually exercised, over the key policies of the underlying project. Many firms are unwilling to carry out foreign investment unless they have 100% equity ownership and control. Others refuse to make such investments unless they have at least majority control (that is, a 51% stake). In recent years, however, there has been a tendency for indulging in FDI cooperative arrangements where several firms participate and no single party holds majority control (for example, joint ventures).
Classification of FDI

FDI can be classified from the perspective of the investing firm and from the perspective of the host country (the recipient of FDI). From the perspective of the investor, it is possible to distinguish among horizontal FDI, vertical FDI and conglomerate FDI. Horizontal FDI is undertaken for the purpose of horizontal expansion to produce the same or similar kinds of goods abroad (in the host country) as in the home country. Hence product differentiation is the critical element of market structure for horizontal FDI. More generally, horizontal FDI is undertaken to exploit more fully certain monopolistic or oligopolistic advantages such as patents or differentiated products, particularly if expansion at home were to violate anti-trust laws. Vertical FDI, on the other hand, is undertaken for the purpose of exploiting raw materials (backward vertical FDI) or to be nearer to consumers through the acquisition of distribution outlets (forward vertical FDI).

From the perspective of the host country, FDI can be classified into (i) import-substituting FDI, (ii) export-increasing FDI and (iii) government-initiated FDI. Import-substituting FDI involves the production of goods previously imported by the host country, necessarily implying that imports by the host country and exports by the investing firm will decline. This type of FDI is likely to be determined by the size of the host country’s market, transportation costs and trade barriers. Export-increasing FDI, on the other hand, is motivated by the desire to seek new sources of input, such as raw materials and intermediate goods. This kind of FDI is export-increasing in the sense that the host country will increase its exports of raw materials and intermediate products to other countries, where the investing firm and its subsidiaries are located. Government-initiated FDI may be triggered, for example, when a government offers incentives to foreign investors in an attempt to eliminate a balance of payments deficit.

FDI may also be classified into expansionary and defensive types. Expansionary FDI seeks to exploit firm-specific advantages in the host country. This type of FDI has the additional benefit of contributing to sales growth of the investing firm at home and abroad. On the other hand, defensive FDI seeks cheap labour in the host country with the objective of reducing the cost of production.

FDI may take one of three forms: greenfield investment, cross-border mergers and acquisitions (M&As), and joint ventures. Greenfield investment occurs when the investing firm establishes new production, distribution or other facilities in the host country. Typically, host countries prefer greenfield investment because of the job-creating potential and value-added output. FDI may occur via an acquisition of, or a merger with, an established firm in the host country (the vast majority of M&As are indeed acquisitions rather than mergers). This mode of FDI has two advantages over greenfield investment: (i) it is cheaper, particularly if the acquired project is a loss-making operation that can be bought at a low price; and (ii) it allows the investor to get quick
access to the market. Firms may be motivated to engage in cross-border acquisi­
tions to bolster their competitive positions in the world market by acquiring
special assets from other firms or by using their own assets on a larger scale. A
large number of M&As fail in the sense that the firms engaging in this activity
do not produce better results in terms of share prices and profitability than
those firms that do not indulge in this activity.

FDI can also take the form of joint ventures either with a host country firm
or with a government institution, as well as with another company that is
foreign to the host country. One side normally provides the technical expert­
tise and its ability to raise finance while the other side provides valuable input
through its local knowledge of the bureaucracy as well as local laws and
regulations.

What is a multinational firm?
Most FDI is carried out by multinational firms, but it is difficult to pinpoint
what constitutes a multinational firm. The 1999 World Investment Report
defines multinational firms (which it calls transnational corporations) as
“incorporated or unincorporated enterprises comprising parent enterprises
and their foreign affiliates”. A parent enterprise or firm is defined as “an enter­
prise that controls assets of other entities in countries other than its home
country, usually by owning a certain equity capital stake”. A foreign affiliate is
defined as “an incorporated or unincorporated enterprise in which an
investor, who is resident in another economy, owns a stake that permits a
lasting interest in the management of that enterprise”. Foreign affiliates may
be subsidiaries, associates or branches. A subsidiary is an incorporated enter­
pire in the host country in which another entity directly owns more than half
of the shareholders’ voting power and has the right to appoint or remove a
majority of the members of the administrative, management or supervisory
body. An associate is an incorporated enterprise in the host country in which
an investor owns a total of at least 10%, but not more than half, of the share­
holders’ voting power. A branch is a wholly or jointly owned unincorporated
enterprise in the host country, which may take the form of a permanent office
of the foreign investor or an unincorporated partnership or a joint venture. A
branch may also refer to land, structures, immovable equipment and mobile
equipment (such as oil drilling rigs and ships) operating in a country other
than the investor’s country. Here, the term “subsidiary” is used as a generic
term for the “foreign establishment” or “foreign arm” of a multinational firm.

Some attempts have been made to measure the extent of being “multina­
tional” according to a set of indicators. Dorrenbacher (2000) proposes a
measure based on the following indicators: (i) structural indicators, (ii) perfor­
mance indicators and (iii) attitudinal indicators. Structural indicators include
the number of countries where the firm is active, the number of foreign
subsidiaries, the number of foreign employees and the number of stock
markets on which the firm’s shares are listed. Performance indicators include
foreign sales and operating income of foreign subsidiaries. The attitudinal indicators include management style and the international experience of top management.

12.2 EXPLAINING FOREIGN DIRECT INVESTMENT

A number of theories have been put forward to explain FDI. In this section, we present these theories as a series of hypotheses.

Hypothesis 1: FDI depends on the rate of return on the underlying project

The differential rates of return hypothesis represents one of the first attempts to explain FDI flows. This hypothesis postulates that capital flows from countries with low rates of return to countries with high rates of return move in a process that eventually leads to the equality of \textit{ex ante} real rates of return. The hypothesis obviously assumes risk neutrality, making the rate of return the only variable upon which the investment decision depends. Risk neutrality in this sense implies that the investor considers domestic and foreign direct investments to be perfect substitutes, or in general that direct investment in any country, including the home country, is a perfect substitute for direct investment in any other country.

One problem with the differential rates of return hypothesis is that it is not consistent with the observation that countries simultaneously experience inflows and outflows of FDI. This is because a rate of return differential implies capital flows in one direction only, from the low-rate country to the high-rate country, and not vice versa. There is, obviously, something missing in this hypothesis. Furthermore, the validity of the differential rates of return hypothesis can be questioned on theoretical grounds. First of all, multinational firms may indulge in FDI for reasons other than profit, particularly in the short run and medium run. For example, the objective may be to maximise sales revenue in accordance with the market penetration objective. Or the objective may not be purely financial, but rather logistical and operational, such as the desire to circumvent trade barriers. In general, multinational firms are faced with a multiplicity of objectives for their international operations, and these objectives are likely to change with the passage of time. More importantly, however, risk aversion implies that the FDI decision does not only depend on return but also on risk. Finally, the differential rates of return hypothesis does not explain why a firm indulges in FDI rather than portfolio investment.

Hypothesis 2: FDI depends on return and risk

When the assumption of risk neutrality is relaxed, risk becomes another variable upon which the FDI decision is made. If this proposition is accepted, then
the differential rates of return hypothesis becomes inadequate, in which case we resort to the diversification (or portfolio) hypothesis to explain FDI. The choice among various projects is therefore guided not only by the expected rate of return, but also by risk. The same idea of reducing risk via diversification that is relevant to portfolio investment is used here. Because of risk aversion, a rate of return differential will not induce capital flows in one direction until the differential disappears via arbitrage. Rather, capital mobility will be constrained by the desire to minimise or reduce risk, which is achieved via diversification.

**Hypothesis 3: FDI depends on market size**

The volume of FDI in a host country depends on its market size. This hypothesis is particularly valid for the case of import-substituting FDI. As soon as the size of the market of a particular country has grown to a level warranting the exploitation of economies of scale, this country becomes a potential target for FDI inflows. The rationale for the hypothesis that firms increase their investment in response to their sales is based on neoclassical domestic investment theories.

One way to test the market size hypothesis is to find out whether or not the share of FDI of a given country going to a group of host countries is correlated with the individual income level of the host country. The empirical studies using this testing approach seem to support the hypothesis that higher levels of sales and the host country’s income are positively related to FDI.

Size does matter, according to a survey by A.T. Kearney, the results of which were summarised in the 17 February 2000 issue of *The Economist*. This survey is based on the views of 135 executives of the world’s 1000 largest companies who gave marks on a scale of 0–3 for the likelihood of investing in a particular country. The top three countries favoured for investment turned out to be the USA, China and Brazil. Mexico was ranked sixth, whereas India was ranked seventh. Needless to say, the ranking of countries from the top to the bottom of the list did not exactly match the ranking of countries in terms of size, because of the influence of other determinants of FDI. For example, Japan was twentieth, whereas the UK was fourth.

**Hypothesis 4: FDI can be explained in terms of firm-specific advantages**

When a firm establishes a subsidiary in another country it faces several disadvantages emanating from differences in language, culture and the legal system as well as other inter-country differences. If, in spite of these disadvantages, a firm engages in FDI, it must have some advantages arising from intangible assets such as a well-known brand name, patent-protected technology, managerial skills and other firm-specific factors. It is, therefore, firm-specific advantages that explain why a firm can compete successfully in a foreign market. This hypothesis, however, does not explain why a firm chooses to invest in country A rather than country B.
Hypothesis 5: FDI is triggered by the need for internalisation

According to the internalisation hypothesis, FDI arises from efforts by firms to replace market transactions with internal transactions. For example, if there are problems associated with buying oil products on the market, a firm may decide to buy a foreign refinery. These problems arise from imperfections and failure of markets for intermediate goods, including human capital, knowledge, marketing and management expertise. The advantages of internalisation are the avoidance of time lags, bargaining and buyer uncertainty. The internalisation hypothesis explains why firms use FDI in preference to exporting and importing from foreign countries. It also explains why they may shy away from licensing. Because of the significant time lags and transaction costs associated with market purchases and sales, firms replace some of the market functions with internal processes (that is, with intra-firm transactions).

Hypothesis 6: FDI exists because of the international immobility of factors of production

According to the location hypothesis, FDI exists because of the international immobility of some factors of production such as labour and natural resources. This immobility leads to location-related differences in the costs of factors of production. One form of location-related differences in the costs of factors of production is the locational advantage of low wages. Thus, the level of wages in the host country relative to wages in the home country is an important determinant of FDI. This is why countries like India attract labour-intensive production (such as footwear and textiles) from high-wage countries. It is also why multinational firms wanting to establish production facilities in North America choose Mexico in preference to Canada.

Locational advantages do not only take the form of low wages, as they are also applicable to other factors of production. For example, a firm may indulge in FDI by building a factory in a country where it is cheap to generate hydroelectric power. Similarly, a factory could be located near a copper mine in the host country if copper is an important input in the production process. This is a locational advantage because significant savings can be made on the cost of shipping copper from where it is produced to where it is used. Apart from these savings, the firm can avoid delays in the delivery of copper shipments arising from the time it takes to ship the metal and the red tape that may be involved in this operation. In general, the location hypothesis emphasises the importance of unavoidable government constraints such as trade barriers. Capital may also be the underlying factor of production, particularly if capital markets are segmented. The idea here is that FDI flows to countries where the cost of capital is low.

Hypothesis 7: The eclectic theory

The eclectic theory results from the integration of the industrial organisation hypothesis, the internalisation hypothesis and the location hypothesis
without being too precise about how they interrelate. The eclectic theory aims at answering the following two questions. First, if there is demand for a particular commodity in a particular country why is it not met by a local firm producing in the same country or by a foreign firm exporting from another country? Second, suppose that a firm wants to expand its operations, why does it not do so via other means, which include (i) producing in the home country and exporting to the foreign country; (ii) expanding into a new line of business within the home country; (iii) indulging in portfolio investment in the foreign country; and (iv) licensing its technology to foreign firms that can carry out the production.

Suppose that there is demand for a particular product in which a particular domestic firm has an ownership advantage. What happens depends on the internalisation and locational advantages. If there are no internalisation gains, the firm will license its ownership advantage to another firm, particularly if locational factors favour expansion abroad. If there are internalisation gains and if locational factors favour home expansion, the firm expands at home and exports. And, if there are internalisation gains and if locational factors favour foreign expansion, FDI will take place.

Hypothesis 8: The role of the product life cycle
According to the product life cycle hypothesis, products go through a cycle of initiation, exponential growth, slowdown and decline. The hypothesis postulates that firms indulge in FDI at a particular stage in the life cycle of the products that they initially produce as innovations.

FDI takes place as the cost of production becomes an important consideration, which is the case when the product reaches maturity and standardisation. FDI is thus a defensive move to maintain the firm’s competitive position against its domestic and foreign rivals. The product life cycle hypothesis predicts that the home country where the innovative product first appears switches over time from an exporting to an importing country.

Hypothesis 9: The role of oligopolistic reaction
The oligopolistic reaction hypothesis postulates that FDI by one firm triggers a similar action by other leading firms in the industry in an attempt to maintain their market shares. An implication of the oligopolistic reaction hypothesis, which is incompatible with some stylised facts, is that the process of FDI is self-limiting, since the invasion of each other’s home market leads to an increase in competition and a decline in the intensity of oligopolistic reaction. Moreover, this hypothesis also fails to identify the factors that trigger the initial investment.

Hypothesis 10: The role of internal financing
Internal financing refers to the utilisation of profit generated by a subsidiary to finance the expansion of FDI by a multinational firm in the same country.
where the subsidiary operates. The internal financing hypothesis postulates that multinational firms commit a modest amount of their resources to their initial direct investment, whereas subsequent expansions are financed by reinvesting the profits obtained from operations in the host country. It therefore implies the existence of a positive relationship between internal cash flows and investment outlays, which is plausible because the cost of internal financing is lower. The hypothesis seems to be more appropriate for explaining FDI in developing countries for at least two reasons: (i) the presence of restrictions on the movement of funds, and (ii) the rudimentary state and inefficiency of financial markets.

Hypothesis 11: The effect of exchange rates
The currency areas hypothesis postulates that firms belonging to a country with a strong currency tend to invest abroad, whereas firms belonging to a country with a weak currency do not have such a tendency. In other words, countries with strong currencies tend to be sources of FDI, whereas countries with weak currencies tend to be host countries or recipients of FDI. This hypothesis is based on capital market relationships, foreign exchange risk and the market’s preference for holding assets denominated in strong currencies. It is arguable that a multinational firm in a hard currency area is able, based on reputation, to borrow at lower rates than local firms in a country that has a soft currency. In essence, the crucial assumption is that there is bias in capital markets, which arises because an income stream located in a country with a weak currency is associated with foreign exchange risk. Hence the view arises that a strong currency firm may be more efficient in hedging foreign exchange risk.

Froot and Stein (1991) have come up with a more elaborate theory based on market imperfections to explain the effect of exchange rates. They argue that a weak currency may be associated with FDI inflows owing to informational imperfections in the capital market and that these imperfections make the cost of external financing higher than the cost of internal financing.

Changes in exchange rates are theoretically bound to have an effect on FDI. First of all, depreciation of the domestic currency makes domestic assets more attractive for foreigners while foreign assets become more expensive for residents in the home country. Thus FDI inflows will increase. This, according to Froot and Stein (1991), explains the increase in FDI in the US as a result of the depreciation of the US dollar that started in March 1985. But this argument can be dismissed as follows. In a world with mobile capital, risk-adjusted expected returns on all international assets will be equalised. For this equality to hold, depreciation of the domestic currency will result in a rise in the prices of domestic assets. The question that arises here is that if foreigners can buy domestic assets with an appreciating currency, why can’t domestic residents with access to global capital markets borrow the foreign currency and take advantage of the situation just like the foreign investors? Still, Froot and Stein
(1991) argue that “the view that exchange rates are irrelevant to FDI is at odds with more than just casual empiricism”.

The effect of exchange rates on FDI may be ambiguous because the latter is affected by both the level and variability of exchange rates. Moreover, the effect of the level of the exchange rate depends on the destination of the goods produced. If the investor aims at serving the local market then FDI and trade are substitutes, in which case appreciation of the currency of the host country attracts FDI inflows. Alternatively, if FDI is aimed at re-exports then FDI and trade are complements. In this case appreciation of the currency of the host country reduces FDI inflows through lower competitiveness. The effect of exchange rate variability also depends on the objective of FDI. If the investor aims at serving the local market then exchange rate variability encourages FDI. If, however, the objective is to re-export then this benefit vanishes.

**Hypothesis 12: Diversification with barriers to international capital flows**

For international diversification to be carried out through firms, two conditions must hold: (i) there must exist barriers or costs to portfolio flows that are greater than those associated with direct investment, and (ii) investors must recognise that multinational firms provide diversification opportunities that are unavailable otherwise. It has been found that there is a systematic relationship between the extent of international involvement and excess market value. This relationship gets stronger in periods characterised by the presence of barriers to capital flows.

**Hypothesis 13: The role of political risk and country risk**

Lack of political stability discourages inflows of FDI. Political risk arises because unexpected modifications of the legal and fiscal frameworks in the host country may change the economic outcome of a given investment in a drastic manner. For example, a decision by the host government to impose restrictions on capital repatriation to the investor’s home country will adversely affect the cash flows received by the parent company.

**Hypothesis 14: The role of tax policies**

Domestic and foreign tax policies affect the incentive to engage in FDI and the means by which it is financed. Tax policies affect the decisions taken by multinational firms via three channels. First, the tax treatment of income generated abroad has a direct effect on the net return on FDI. Second, the tax treatment of income generated at home affects the net profitability of domestic investment and the relative profitability of domestic and foreign investment. Third, tax policies affect the relative cost of capital of domestic and foreign investment.

Swenson (1994) empirically examined how taxes shape FDI and found that increased taxes boost inward FDI. While simple intuition might suggest that
higher taxes should discourage both foreign and domestic investments, Scholes and Wolfson (1990) have shown that the general equilibrium effects on asset returns combined with a careful consideration of foreign tax systems reveals reasons for foreign investors to increase their investments in response to high taxes in the host country.

**Hypothesis 15: The effect of trade barriers**

FDI may be undertaken to circumvent trade barriers, such as tariffs, because FDI can be viewed as an alternative to trade. This means that open economies without many restrictions on international trade should receive less FDI flow. The surge in FDI in countries like Mexico and Spain is partly attributed to the desire of multinational firms to circumvent the trade barriers imposed by NAFTA and the EU. Sometimes, the threat of protectionism by the host government triggers FDI. Blonigen and Feenstra (1996) argue that the literature on *quid pro quo* FDI suggests that FDI may be induced by the threat of protection and that FDI may be used as an instrument to defuse protectionist threats.

**Hypothesis 16: The effect of government regulations**

Most governments adopt policies aimed at both encouraging and discouraging inward FDI by offering incentives on the one hand and disincentives (taking the form of restrictions on the activities of multinational firms) on the other. Typically, they offer incentives (such as financial and tax incentives as well as market preferences) while simultaneously placing restrictions on the activities of multinational firms.

One particular case of using incentives to offset disincentives is when the host government uses a package that includes trade-related investment performance (TRIP) requirements, which are seen by some as a significant obstacle to FDI. TRIP requirements include two components: (i) local content and (ii) export minima. These requirements should be viewed as disincentives to FDI because the local content requirements may lead to increased cost and decreased earnings, which makes the underlying project less competitive. Likewise, export minima may lead to lower earnings, adversely affecting the attractiveness of FDI. Normally, TRIP requirements are combined with incentives such as preferential tax status, access to foreign exchange and import protection.

**Hypothesis 17: The role of strategic and long-term factors**

Some strategic and long-term factors have been put forward to explain FDI. These factors include the following:

1. The desire on the part of the investor to defend existing foreign markets and foreign investments against competitors.
2. The desire to gain and maintain a foothold in a protected market or to gain and maintain a source of supply that may prove useful in the long run.
3. The need to develop and sustain a parent–subsidiary relationship.
4. The desire to induce the host country into a long commitment to a particular type of technology.
5. The advantage of complementing another type of investment.
6. The economies of new product development.
7. Competition for market shares among oligopolists and the concern for strengthening bargaining positions.

12.3 INTERNATIONAL CAPITAL BUDGETING

Although the decision to invest abroad may be taken for non-financial reasons, it is imperative that the underlying project is financially viable. Capital budgeting (also called investment appraisal and project evaluation) is the technique (or techniques) used for evaluating the financial viability of a project.

International capital budgeting is more complicated than domestic capital budgeting because multinational firms are typically large and capital-intensive, and because the process involves a larger number of parameters and decision variables. In general, international capital budgeting involves a consideration of more risk than domestic capital budgeting. But like domestic capital budgeting, international capital budgeting involves the estimation of some measures or criteria that indicate the feasibility or otherwise of a project (a capital budgeting evaluation measure) such as the net present value (NPV). However, certain factors that are not considered in domestic capital budgeting should be taken into account in international capital budgeting because of the special nature of FDI projects. The estimation of NPV and similar criteria requires (i) the identification of the relevant expected cash flows to be used for the analysis of the proposed project; and (ii) the determination of the proper discount rate for finding the present value of the cash flows.

International capital budgeting involves substantial spending (capital investment) in projects that are located in foreign (host) countries, rather than in the home country of the multinational firm. Foreign projects differ from purely domestic projects with respect to a number of factors: the foreign currency dimension, different economic indicators (such as inflation) in different countries, and different risk characteristics with which the multinational firm is not as familiar as those pertaining to domestic projects. All of these differences lead to a higher level of risk in international capital budgeting than in domestic capital budgeting.

International capital budgeting involves a number of issues that do not appear in domestic capital budgeting. For example, a project may be feasible from the perspective of a subsidiary but not from the perspective of the
parent firm. Measures of project feasibility, such as the NPV, can be calculated from either perspective depending on whether the calculation is based on the cash flows received by the subsidiary or those remitted by the subsidiary to the parent firm. One view is that it should be considered from the subsidiary’s perspective, since the subsidiary will eventually be in charge of administering the project. It is also argued that projects should be evaluated from the subsidiary’s perspective because the subsidiary belongs to the parent firm, which means that what is good for the subsidiary should be good for the parent firm.

One counterargument is that since the multinational firm is financing the project, it should be assessed from its perspective. This is more the case if the subsidiary is wholly owned by the parent firm. After all, the parent firm’s objective is to enhance its net worth, which is what is expected by its shareholders. Hence, for a project (domestic or foreign) to be accepted by the parent firm, it must have a positive net present value from its perspective. An exception would be when the subsidiary is not wholly owned by the parent firm. In this case the subsidiary would also have the objective of increasing its net worth as expected by the shareholders who are not shareholders of the multinational firm. Hence the acceptability, or otherwise, of a project is determined by negotiation between the parent firm and the subsidiary.

Another argument for evaluating projects from the subsidiary’s perspective is that there is a tendency for the subsidiaries not to appreciate fully the ways in which a project may benefit the parent firm. This tendency is reinforced by the practice of rewarding the subsidiary’s management on the basis of its net income, not on its contribution to the consolidated performance of the parent firm.

What makes this matter important is that the net after-tax cash flows that accrue to the parent firm can be substantially different from those accruing to the subsidiary. The earnings of the subsidiary are subject to corporate income tax and withholding tax in the host country, and part of the after-tax earnings is kept by the subsidiary as retained earnings. Sometimes the whole amount is retained, in which case the parent firm gets nothing. The amount remitted to the parent firm is then converted to its base currency, and these earnings are subsequently taxed once more by the home government. Hence, what may look like an attractive project from the point of view of the subsidiary may be utterly unattractive to the multinational firm. Several reasons explain the difference between the cash flows accruing to the multinational firm and to the subsidiary. Remember that the net present value and other measures of the feasibility of projects are calculated from the net after-tax cash flows.

The first reason for the difference between the cash flows received by the subsidiary and those received by the parent firm is tax differentials, arising when there is a difference between the tax rates in the host country and the home country. If the host government imposes a lower tax rate on earnings
than the home government then the project may be feasible from the perspective of the subsidiary but not from the perspective of the parent firm.

The second reason is restricted remittances. This occurs when the host government requires a certain percentage of the subsidiary’s earnings to remain in the host country. Sometimes the earnings generated by the subsidiary are required to be reinvested in the host country for some years before they can be remitted to the multinational firm, giving rise to so-called “blocked funds”. If there are restrictions on remittances, then the parent firm will not have access to these funds, and hence its after-tax cash flows will be lower than those of the subsidiary. Again, the project may not be feasible from the parent firm’s perspective but feasible from the subsidiary’s perspective.

The third reason is excessive remittances. This occurs when the multinational firm charges the subsidiary high administrative fees, making the cash flows accruing to the subsidiary lower than those accruing to it. In this case, the project may be feasible from the parent firm’s perspective but not from the perspective of the subsidiary. There are obviously differences between the revenue/cost configurations of the parent firm and the subsidiary: what is regarded as revenue by the parent firm is regarded as cost by the subsidiary. The same conclusion would be valid if the parent firm charged the subsidiary high transfer prices.

The fourth reason for differences in cash flows is exchange rate movements. If the base currency appreciates against the foreign currency, then the cash flows received by the parent firm will be reduced in value measured in terms of the base currency. Remember that the subsidiary and the parent firm calculate the net present value and other measures of the feasibility of a project on the basis of cash flows denominated in terms of two different currencies. Fluctuations in exchange rates lead to fluctuations in the cash flows received by the parent firm in terms of the base currency for a given amount received from the subsidiary denominated in the foreign currency.

The fifth reason is differences in the discount rates used by the parent firm and the subsidiary to calculate the present value of future cash flows arising from the project. From the subsidiary’s perspective, the appropriate discount rate should relate to the cost of funding facing the subsidiary’s local competitors. For the parent firm, the discount rate should be related to the cost of capital pertaining to its worldwide operations. Obviously, the two rates can diverge significantly.

Figure 12.1 shows that a cash flow, $X_t$, arising from an FDI project is not exactly what is received by the subsidiary or the parent firm. The differences arise from the foreign tax rate, $\tau^*$; the domestic tax rate, $\tau$; the percentage of blocked funds that the subsidiary cannot remit to the parent firm, $\theta$; and the exchange rate, $S$. What the parent firm receives also depends on whether or not there is tax credit, and whether the tax credit is full or partial.
The accounting rate of return
The accounting rate of return is the percentage return on capital invested in the project, normally (but not necessarily) the average annual percentage profit before taxation relative to the average amount of capital invested in the project. This method is criticised because it is based on profit, which is an accounting concept, rather than on cash flows, which are more appropriate for a resource allocation decision-making problem like capital investment. Another problem with this technique is that it takes no account of the size of the project or the time value of money. Moreover, the accounting rate of return is affected by accounting conventions such as the choice of the depreciation method. Simply stated, there is no “correct way” of measuring the accounting rate of return.

The payback period
The payback period measures how quickly the initial outlay in an investment is paid back from after-tax cash flows that are generated from the investment. Only the projects that are paid back within a period of time that is acceptable
to the investor will be undertaken. The payback period method is preferable to
the accounting rate of return method because it is based on cash flows. However, it also ignores the time value of money as well as the cash flows that occur after the initial investment has been paid back. Moreover, setting a maximum acceptable payback period is essentially an arbitrary decision: there is no easy way to relate the payback period to more general criteria such as profit maximisation or value maximisation.

Because the risk associated with international projects increases as years into the future are considered, the payback period is often used as a “short-hand” way of incorporating risk. Although this method is incomplete from the standpoint of financial theory, it has the advantage of providing a concrete cash-oriented measure of risk because it helps answer the question: how long will the investor’s funds be tied up in the project under consideration? It is because of this “risk-bounding interpretation” of the payback period that this criterion is used as a constraint that needs to be satisfied before accepting the project. Of course, it is possible to adjust this method to take into account the time value of money by discounting the cash flows. However, the discounted payback criterion still takes no account of the cash flows arising after the cut-off date.

The net present value

The net present value (NPV) is a project evaluation criterion that is based on cash flows, taking account of the time value of money by discounting future cash flows at an appropriate discount rate. NPV measures the absolute financial benefit of a project, consistently leading to an absolute financial gain by adding to the shareholders’ net worth. A project is found acceptable if the NPV is positive. To choose between two mutually exclusive projects the one that is picked must have a higher NPV.

The NPV is the difference between the initial investment outlay (the capital cost) and the sum of the discounted cash flows realised from the project. If the initial investment is $X_0$ and the cash flows resulting from the project over years $1, 2, \ldots, n$, are $X_1, X_2, \ldots, X_n$, then the NPV is given by the equation

$$NPV = -X_0 + \sum_{t=1}^{n} \frac{X_t}{(1+k)^t} + \frac{V_n}{(1+k)^n}$$

(12.1)

where $k$ is the discount rate, $n$ is its lifetime and $V_n$ is its salvage value (also called the terminal or liquidation value). The discount rate is normally taken to be (or is closely related to) the cost of capital or the required rate of return on the investment. The underlying idea here is that for a project to be financially acceptable it must attract a rate of return that is at least equal to the cost of obtaining the funds required to finance it.

Equation (12.1) is the net present value from the perspective of the subsidiary when there are no taxes. It may also represent the NPV from the
perspective of the parent firm if the exchange rate is fixed at parity and there are no taxes or blocked funds (that is, \( S_t = 1, \tau^* = \tau = 0 \)). Otherwise the NPV from the perspective of the parent firm if there is full tax credit can be calculated from the equation

\[
NPV = -S_0 X_0 + [(1-\tau^*)(1-\theta)(1-\tau+\tau^*)] \sum_{t=1}^{n} \frac{S_t X_t}{(1+k)^t} + \frac{S^n V_n}{(1+k)^n} \tag{12.2}
\]

Figure 12.2 shows the relationship between the NPV and the discount rate for various values of the other parameters. In 1, it is assumed that \( S_t = 1 \) and \( \tau^* = \tau = 0 \). In 2, the assumption of a depreciating foreign currency is introduced, whereas 3 is based on the assumption of constantly appreciating foreign currency. In 4 and 5, there is no change in the exchange rate, but \( \theta \) takes the values 0.2 and 0.5 respectively. Finally, 6 assumes that \( \theta = 0.3, \tau^* = 0.4, \tau = 0.5 \), and that there is a full tax credit.

It is cash flows, and not accounting profits, that should be discounted when NPV is calculated. Income statements, in which accounting profits are reported, are used to show how well the firm has performed, but they do not track cash flows. In principle, the profit figure is intended to measure changes in value, albeit imperfectly. (Operating) cash flows are the funds generated by the firm’s operations and are available for spending. The funds may be used to expand investment in fixed assets, to pay dividends, to expand working capital, to retire debt or for a variety of other purposes.

Consider a project that costs \( X_0 \), producing \( X_t \) in year \( t \), where \( t = 1, \ldots, n \). Assuming a zero salvage value, the project’s NPV is given by

\[
NPV = -X_0 + \sum_{t=1}^{n} \frac{X_t}{(1+k)^t} \tag{12.3}
\]

If we calculate the NPV on the basis of accounting profits, the capital cost is depreciated over the life of the project. Hence accounting profit for year \( t \) is \( X_t - X_0/n \). In this case the NPV will be given by the equation

\[
NPV = \sum_{t=1}^{n} \frac{X_t - X_0/n}{(1+k)^t} \tag{12.4}
\]

which can be manipulated to produce

\[
NPV = -X_0 \sum_{t=1}^{n} \frac{1}{n(1+k)^t} + \sum_{t=1}^{n} \frac{X_t}{(1+k)^t} \tag{12.5}
\]

which is obviously larger than in the original case (equation 12.3). Hence, a misleading picture would emerge if project evaluation is based on accounting profits. This difference is due to whether investment expenditure is recognised when it occurs (the first case) or when it shows up as depreciation (the second case). The first case is more logical: projects are financially viable
because the cash flows they generate are used either for distribution to shareholders or for reinvestment. One reason for the difference between cash flows and accounting profits is that accountants recognise profit when it is earned, not when it is realised.

In order to calculate the present value of the future cash flows, a discount rate, \( k \), must be used. This discount rate is normally the required rate of return on the project, which may or may not be equal to the parent firm’s cost of capital. Whether or not the required rate of return is equal to the project’s cost of capital depends on the riskiness of the project.

**The internal rate of return**

The internal rate of return (IRR) is the discount rate that makes the NPV of a project equal to zero. It is calculated by solving for \( r \) the equation

\[
-X_0 + \sum_{t=1}^{n} \frac{X_t}{(1+r)^t} + \frac{V_n}{(1+r)^n} = 0
\]  

(12.6)

Thus, the IRR on a project with a zero NPV is equal to the discount rate or the cost of capital. A project will have a positive NPV if the IRR is greater than the cost of capital and vice versa. The problem with the IRR, however, is that its calculation is based on the assumption that cash flows can be reinvested at the same rate, and this is not necessarily the case. Hence, it may be in conflict with NPV when competing projects have different sizes or time horizons. Moreover, a project may, under certain circumstances, have multiple IRRs, which creates difficulty in interpreting the simple decision rule whereby a project is selected if the IRR is greater than the cost of capital. For example, how can we implement this rule if one of the values of the IRR is lower than the cost of capital?

In general, the NPV and IRR criteria may lead to conflicting conclusions when mutually exclusive projects have different scales or when the time patterns of the cash flows are different. If such a conflict arises, the decision should be based on the NPV criterion.
The profitability index
The profitability index (PI) is calculated by dividing the present value of cash flows by the initial investment, $X_0$. Hence the profitability index (ignoring the salvage value) is calculated as

$$PI = \frac{\sum_{t=1}^{n} X_t/(1+k)^t}{X_0}$$

(12.7)

A project is undertaken if its profitability index is greater than one. To choose between two mutually exclusive projects we pick the project that has a higher profitability index. A conflict may arise between the NPV and the PI in the case of mutually exclusive projects due to differences in the project size. In this case the conflict should be resolved in favour of the NPV if the parent firm is not under a capital rationing constraint (see Ross et al., 1996, Chapter 6).

The adjusted present value
The adjusted present value (APV) equation takes the form

$$APV = -X_0 + \sum_{t=1}^{n} \frac{O_t(1-\tau)}{(1+k_u)^t} + \sum_{t=1}^{n} \frac{\tau D_t}{(1+i)^t} + \sum_{t=1}^{n} \frac{\tau I_t}{(1+i)^t} + \frac{V_n}{(1+k_u)^n}$$

(12.8)

where $O$ represents operating cash flows, $D$ is depreciation, $I$ is interest payments, $k_u$ is the cost of equity for an all-equity financed firm and $i$ is the borrowing rate. Just like the NPV, a project is accepted if the APV is positive, and a project is chosen from two mutually exclusive projects if it has a higher APV.

It is obvious from equation (12.8) that each cash flow that is a source of value is considered individually and discounted at a discount rate consistent with the risk inherent in that cash flow. Operating cash flows and the salvage value are discounted at $k_u$ because the firm would receive these cash flows irrespective of its capital structure. The tax savings resulting from interest payments, $\tau I_t$, are discounted at the borrowing rate, $i$. The tax savings due to depreciation, $\tau D_t$, are also discounted at $i$ because these cash flows are less risky than the operating cash flows.

Lessard (1985) developed an APV formula that explicitly recognises currency conversion and special cash flows that are encountered in the analysis of foreign projects. This formula takes the form

$$APV = \sum_{t=1}^{n} \frac{E_0(S_t)O_t(1-\tau)}{(1+k_u)^t} + \sum_{t=1}^{n} \frac{E_0(S_t)\tau D_t}{(1+i)^t} + \sum_{t=1}^{n} \frac{E_0(S_t)\tau I_t}{(1+i)^t} + \frac{E_0(S_t)V_n}{(1+k_u)^n}$$

$$-S_0X_0 + S_0R_0 + S_0L_0 - \sum_{t=1}^{n} \frac{E_0(S_t)P_t}{(1+i)^t}$$

(12.9)
where \( E_0(S_t) \) is the expected value of the exchange rate at time \( t \) when the expectation is made at time 0, \( R \) represents the restricted funds freed by the project, \( L \) represents concessionary loans and \( P \) is the payment needed to cover the concessionary loan. Thus, the APV of a foreign project can be estimated as the capital cost (cash outflow) plus the present values of the following items: (i) remittable operating cash flows; (ii) tax saving from depreciation and capital allowances; (iii) subsidies to the project; (iv) other tax savings; (v) the project’s effect on corporate debt capacity; (vi) other cash inflows and outflows that result directly from the project. Booth (1982) shows that under certain circumstances the NPV and APV are equivalent.

12.5 ADJUSTING PROJECT ASSESSMENT FOR RISK

International capital budgeting involves a consideration of more risk than domestic capital budgeting. Both domestic and international projects are subject to market risk, which is specific to industries and may involve the likely evolution of markets and competitor behaviour. The parent firm may perceive the market risk for international projects as exceeding that associated with domestic projects. This may be attributed to the relative lack of knowledge about foreign markets.

International capital budgeting also involves a consideration of country risk, which is the risk of an adverse outcome arising from economic and political factors in the host country. For example, inflationary policies in the host country are an adverse factor for a subsidiary that depends on local supplies while exporting its products if there is no offsetting depreciation of the currency. Country risk would also be present if the government of the host country imposes import controls when a subsidiary depends on imported raw materials. It also involves changing the “rules of the game” such as changes in tax laws and the regulations governing the repatriation of capital.

From the previous discussion, it is obvious that a typical project evaluation process consists of the following steps:

1. Estimating the incremental cash flows arising in the host country, taking into account any tax effects.
2. Estimating remittable cash flows to the parent firm and translating these cash flows into the base currency at the spot exchange rates expected to prevail in each future time period.
3. Incorporating into the remitted cash flows any indirect costs and benefits that arise as a result of undertaking the project. All tax effects applicable to the parent firm must be considered at this stage.
4. Discounting the parent firm’s incremental cash flows at a rate that reflects the risk associated with the project or the particular cash flow.
When a parent firm uses either the NPV or the APV method to evaluate foreign investment projects, a problem is typically encountered as to the accuracy of the cash flows that are expected to materialise in the future as a result of operating the project. Risk means that cash flows generated by the project may fluctuate far away from the expected value that would normally be used to calculate the NPV and the APV. If it is felt that this is the case then some adjustment may be made to account for risk. There are three methods to deal with risk in situations like these.

The risk-adjusted discount rate
The greater the risk associated with future cash flows the greater should be the discount rate used to calculate the present value of the future cash flows. This is why the discount rate may differ from the cost of capital. This is also the reason why different discount rates are used in the APV formula (equation 12.9), as more risky cash flows are discounted at higher rates. For example, cash flows associated with tax saving from depreciation and interest payments to creditors are less risky than operating cash flows, and this is why the latter are discounted at a higher rate.

This approach to the adjustment for risk is easy to implement, but it is criticised as being somewhat arbitrary. Furthermore, it does not take into account changes in the riskiness of cash flows from one time period to another, since the discount rate is assumed to be constant across time for a class of cash flows with a particular degree of risk. An example of changes in the riskiness of cash flows over time is changes in blocked funds and tax laws in a country with a high degree of political risk. Despite these shortcomings, the method of adjusting the discount rate is used because of its simplicity. It is also arguable that this technique is more appropriate to deal with country risk than with market risk.

Risk-adjusted cash flows
Some economists argue that adjusting the cash flows is more appropriate than adjusting the discount rate, particularly if the project involves market risk. This approach, it is arguable, allows the parent firm to reflect more specifically the impact of the risk during the investment. Shapiro (1992) also argues that better information is available on the effect of risk on cash flows than on the discount rate.

This approach is normally known as the certainty equivalent approach, as it is based on a reduction of risky future cash flows to a lower level that is accepted by the market. This adjustment is made separately for each period of the project’s life. The adjusted risk-free cash flows are then discounted at the risk-free discount rate to estimate the NPV of the project. The difference between this method and the adjusted discount rate method is that this method considers time and risk separately, whereas the previous method treats them jointly. Although this method is theoretically more appealing, it is
not widely used because there are practical problems in identifying the equivalent risk-free cash flows.

In this case the NPV formula is modified to the following:

\[ NPV = -X_0 + \sum_{t=1}^{n} \frac{\lambda_t X_t}{(1+k)^t} + \frac{\phi V_n}{(1+k)^n} \]  

(12.10)

where \(0 \leq \lambda_t \leq 1\) is the certainty equivalent factor applicable to cash flow \(X_t\) and \(0 \leq \phi \leq 1\) is the certainty equivalent factor applicable to the salvage value, \(V_n\). Because it is invariably the case that risk rises the further into the future the cash flow is, it follows that \(\lambda_1 > \lambda_2 > \ldots > \lambda_n\).

Sometimes a problem arises as to accounting for risk by adjusting cash flows or adjusting the discount rate. The likelihood of a bad outcome should be allowed for in the calculation of the cash flows rather than by adjusting the discount rate. Suppose, for example, that there are two possible outcomes, good and bad. If the cash flows under the good and bad outcomes are \(X_{i,t}\) and \(X_{j,t}\) arising with probabilities \(p\) and \((1-p)\) respectively, then the cash flow \(pX_{i,t} + (1-p)X_{j,t}\) should be discounted at the unadjusted discount rate. It would be wrong in this case to discount \(X_{i,t}\) at a higher discount rate to reflect the possibility of a bad outcome.

Sensitivity analysis
Sensitivity analysis entails the use of “what if” scenarios, which are implemented by changing the input variables, including the exchange rate. If the NPV or APV remains positive for several scenarios, then the parent firm should become more comfortable with the project. Sensitivity analysis can be applied to the discount rate or rates as well.

Simulation
Simulation can be used to generate a probability distribution for the NPV or the APV based on various combinations of the values of input variables. Consider, for example, a situation in which the exchange rate is forecast to be within a certain range such that any value within this range can materialise with equal probability. A large number of iterations are performed: in each iteration the value assumed by the exchange rate is picked randomly. This value is then used to calculate the cash flows and subsequently the NPV or APV. Each iteration produces a value for the NPV or APV, and after a large number of iterations we end up with a probability distribution for the APV or the NPV. From the probability distribution it is possible to estimate the probability with which the NPV or APV will be positive.

Break-even analysis
In break-even analysis, the focus is placed on the point at which the NPV or the APV switches from positive to negative. The initial cash flow, \(X_0\), does not
depend on sales revenue, whereas the present value of subsequent cash flows is a positive function of sales revenue. The equality of the initial cash flow and the present value of subsequent cash flows defines the level of sales that generates a zero NPV, the break-even level of sales. Below this level, the NPV is negative, and above this level the NPV is positive.

**Incorporating country risk analysis in capital budgeting**

Country risk analysis can also be incorporated in capital budgeting analysis, as suggested by Robock (1971), Kobrin (1979), Sethi and Luther (1986) and Clark (1997). One way to do this is by adjusting the discount rate or the cash flows. The higher the country risk is, the higher the discount rate applied to the project’s cash flows. If, for example, blocked funds are anticipated then the discount rate may be raised from 10 to 13%. The problem with this procedure is that there is no precise formula for adjusting the discount rate for country risk, which makes adjustment rather arbitrary. The use of a shorter payback period may be resorted to for the same purpose. However, Haendel (1979) argues that neither of these two methods provides a detailed examination of the risk involved or a true reflection of the investor’s fear. This is why it may be preferable to incorporate country risk analysis by adjusting the cash flows as suggested by Shapiro (1992).

Suppose that a project is analysed under three scenarios derived from country risk analysis: (i) that nothing will happen, (ii) that the host country will block a certain percentage of the funds to be transferred to the parent firm, and (iii) that the project will be confiscated after few years. Suppose also that these scenarios produce three net present values ($NPV_1$, $NPV_2$ and $NPV_3$) with probabilities of $p_1$, $p_2$ and $p_3$ respectively. The NPV of the project in this case should be calculated as the expected NPV, which is the weighted average of $NPV_1$, $NPV_2$ and $NPV_3$, where the weights are the probabilities. Hence

$$NPV = \sum_{i=1}^{3} p_i NPV_i$$  \hspace{1cm} (12.11)

Levi (1990) has suggested the following formalisation of the process whereby country risk is allowed for in capital budgeting. Let $X_t$ be the cash flow expected to arise from a project in the absence of country risk. Assume that country risk is present such that the project would cease to exist (for example, because of a takeover by the host government) at year $t$ with a probability $p$. Hence the probability that a cash flow arises in each individual year is $1 - p$. This means that the probability that cash flows arise for $t$ years is $(1 - p)^t$. The expected value of the cash flow in year $t$ is $X_t(1 - p)^t$. If $X_t$ is constant such that $X_t = \bar{X}$, then the present value of the cash flow is given by

$$PV = \sum_{t=1}^{n} \frac{(1-p)^t X_t}{(1+k)^t} = \bar{X} \sum_{t=1}^{n} \frac{(1-p)^t}{(1+k)^t}$$  \hspace{1cm} (12.12)
As \( n \to \infty \), we obtain
\[
\sum_{t=1}^{n} \frac{(1-p)^t}{(1+k)^t} = \frac{1-p}{k+p}
\] (12.13)

Hence
\[
PV = \frac{X(1-k)}{k+p}
\] (12.14)

In the absence of country risk, we have
\[
PV = \frac{X}{k}
\] (12.15)

It is obvious that the present value of the cash flows in the absence of country risk is greater than what is obtained when country risk is present.

Agmon (1985) has suggested a workable and comprehensive way to integrate country risk into capital budgeting. Agmon’s method is based on the proposition that the potential dependence of a project on the external environment is divided into two components: vulnerability and cost. Vulnerability is expressed in terms of the probability that an event that is likely to affect the project (such as tax changes) will occur. Vulnerability is also defined in terms of a probability distribution, and for simplicity it is assumed that the distribution can be fully described by its first and second moments. Cost, on the other hand, is measured as the actual impact on the cash flows of the project if a given event occurs. The distinction between vulnerability and cost is crucial because firms do not need to be concerned with all possible events. Only the non-trivial effects on cash flows have to be weighted by the probabilities that they will take place.


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